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Study of Thermoelastic Problem of a Thick Annular Disc Using Finite Integral Transform Techniques

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ABSTRACT: This paper is concerned with the determination of temperature, displacement and thermal stresses of a thick annular disc occupying the space $D: a \le r \le b, -h \le z \le h$. . The governing heat conduction equation has been solved by using Marchi- Zgrablich transform and Marchi Fasulo transforms techniques. The main aim of this paper is to study the thermo elastic response of a thick annular disc in which sources are generated according to linear function of the temperature with stated boundary conditions of the third kind, radiation type. The material of the disc is isotropic, homogeneous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type, where the stresses are required to be determined. The results are obtained in series form in terms of Bessel's function. In this article, we analysed unsteady state thermo elastic problem of temperature, displacement and thermal stresses of a thick annular disc due to heat generation. The statement of the problem is related to annular disc occupying the space $D: a \le r \le b \quad -h \le z \le h$ and an attempt is made to study the unsteady state thermo elastic problem to determine the temperature, displacement and stress functions of the thick annular disc with stated boundary conditions using Marchi- Zgrablich transform and Marchi Fasulo transform to find the solution of the problem. Conditions stated in the problem is of third kind boundary conditions at $r = a, r = b, z = h \; \& \; z = -h$. We successes fully established and obtained the temperature distribution using Marchi- Zgrablich transform and Marchi Fasulo transform techniques by applying to governing heat conduction equation. The displacement function in cylindrical co-ordinate system which are represented by the Goodier thermo elastic function ϕ and Michell's function M Further, by using expression for temperature, we obtained the expression for thermo elastic displacement Ur, Uz and stress functions $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz}$.

I. INTRODUCTION

Nowacki[1] has determined steady state thermal stresses in a thick circular plate subjected to an axi symmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Wankhede[5] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there are not many investigations on transient state Roy Chaudhari[4] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In a recent work, some problems have been solved by Noda et.al.[6] and Deshmukh et.al.[9] In all a fore mentioned investigations an axi symmetrically heated plate has been considered. Recently, Nasser[7,8] proposed the concept of heat sources in generalized thermo elasticity and applied to a thick plate problem. They have not however considered any thermo elasticity problem with boundary condition of radiation type in which sources are generated according to the linear function of the temperatures, which satisfies the time dependent heat conduction equation.

This paper is concerned with the transient thermo elastic problem of the annular disc in which sources are generated according to the linear function of temperature occupying the space $D: a \le r \le b, -h \le z \le h$ with radiation type boundary conditions.

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II. RESULT REQUIRED

A. FINITE MARCHI ZGRABLICH INTEGRAL TRANSFORM-

The finite Marchi Zgrablich integral Transform of f(r) is defined as

$$\bar{f}_p(n) = \int_a^b r f(r) S_p(\alpha, \beta, \mu_n r) dr$$

Where α_1, α_2, β_1, and β_2 are the constants involved in the boundary conditions

$$\alpha_1 f(r) + \alpha_2 f'(r)\big|_{r=a} = 0 \text{ and}$$

$$\beta_1 f(r) + \beta_2 f'(r)\big|_{r=b} = 0$$

For the differential equation $f''(r) + \frac{1}{r} f'(r) - \left(P^2 / r^2\right) f(r) = 0$

$\bar{f}_p(n)$ is the transform of f(r) with respect to Kernel $S_p(\alpha, \beta, \mu_n r)$ and weight function r

The inversion of equation is given by

$$f(r) = \sum_{n=1}^{\infty} \frac{\bar{f}_p(n) S_p(\alpha, \beta, \mu_n r)}{C_n}$$

Where $C_n = \frac{b^2}{2}\left\{S_p^2(K_1, K_2, \mu_n b) - S_{p-1}(K_1, K_2, \mu_n b) S_{p+1}(K_1, K_2, \mu_n b)\right\}$

$$- \frac{a^2}{2}\left\{S_p^2(K_1, K_2, \mu_n a) - S_{p-1}(K_1, K_2, \mu_n a) S_{p+1}(K_1, K_2, \mu_n a)\right\}$$

Kernel function $S_p(\alpha, \beta, \mu_n r)$ can be defined as

$$S_p(\alpha, \beta, \mu_n r) = J_p(\mu_n r)\left[Y_p(\alpha, \mu_n a) + Y_p(\beta, \mu_n b)\right]$$

$$- Y_p(\mu_n r)\left[J_p(\alpha, \mu_n a) + J_p(\beta, \mu_n b)\right]$$

And $J_p(\mu r)$ and $Y_p(\mu r)$ are Bessel's function of first and second kind respectively.

B. OPERATIONAL PROPERTY

$$\int_a^b r^2 \left[f''(r) + \left(\frac{1}{r}\right) f'(r) - \left(P^2 / r^2\right) f(r) \right] S_p(\alpha, \beta, \mu_n r) dr$$

$$= \left(\frac{b}{\beta_2}\right) S_p(\alpha, \beta, \mu_n b) \left[\beta_1 f(r) + \beta_2 f'(r)\right]_{r=b}$$

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$$-\left(\frac{\alpha}{\alpha_2}\right)S_p(\alpha,\beta,\mu_n a)\left[\alpha_1 f(r)+\alpha_2 f'(r)\right]_{r=a}$$

$$-\mu_n{}^2 \bar{f}_p(n)$$

C. FINITE MARCHI FASULO INTEGRAL TRANSFORM-

The finite Marchi Fasulo i integral Transform of f(z), -h<z<h is defined to be

$$\bar{F}(m)=\int_{-h}^{h} f(z)P_m(z)dz$$

Then at each point of (-h, h) at which f(z) is continuous

$$f(z)=\sum_{m=1}^{\infty}\frac{\bar{F}(m)}{\lambda_m}P_m(z)$$

Where

$$P_m(z)=Q_m\cos(a_m z)-W_m\sin(a_m z)$$

$$Q_m=a_m(\alpha_1+\alpha_2)\cos(a_m h)+(\beta_1-\beta_2)\sin(a_m h)$$

$$W_m=(\beta_1+\beta_2)\cos(a_m h)+(\alpha_2-\alpha_1)a_m\sin(a_m h)$$

$$\lambda_m=\int_{-h}^{h}P_m^2(z)dz=h[Q_m^2+W_m^2]+\frac{\sin(2a_m h)}{2a_m}[Q_m^2-W_m^2]$$

The eigen values a_m are the solution of the equation

$$[\alpha_1 a\cos(ah)+\beta_1\sin(ah)]\times[\beta_2\cos(ah)+\alpha_2 a\sin(ah)]$$

$$=[\alpha_2 a\cos(ah)-\beta_2\sin(ah)]\times[\beta_1\cos(ah)-\alpha_1 a\sin(ah)]$$

$\alpha_1,\alpha_2,\beta_1 and \beta_2$ are constants.

The sum must be taken on n corresponding to the positive roots of the equation. Moreover the integral transform has the following property

$$\int_{-h}^{h}\frac{\partial^2 f(z)}{\partial z^2}P_m(z)dz$$

$$=\frac{P_m(h)}{\alpha_1}\left[\beta_1 f(z)+\alpha_1\frac{\partial f(z)}{\partial z}\right]_{z=h}-\frac{P_m(-h)}{\alpha_2}\left[\beta_2 f(z)+\alpha_2\frac{\partial f(z)}{\partial z}\right]_{z=-h}-a_m{}^2\bar{F}(m)$$ Sometimes an image

III .STATEMENT OF THE PROBLEM

Consider thick annular disc of thickness 2h occupying the space $D: a\le r\le b,\ -h\le z\le h$ The material is homogenous and isotropic. The differential equation governing the displacement potential function ϕ (r,z,t) as Nowacki [1] is

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$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t.\mathrm{T} \tag{1}$$

Where, υ and α_t are poisons ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as Noda [6] is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\chi(r,z,t)=\frac{1}{K}\frac{\partial T}{\partial t} \tag{2}$$

Subject to initial condition

$$\mathrm{T}(\mathrm{r},\mathrm{z},0)=T_0 \quad \text{For all } \mathrm{a}\le \mathrm{r}\le \mathrm{b}, -\mathrm{h}\le z\le h \tag{3}$$

The boundary conditions are

$$\left[\mathrm{T}+\mathrm{K}_1\frac{\partial T}{\partial r}\right]_{r=a}=g_1(z,t) \tag{4}$$

$$\left[\mathrm{T}+\mathrm{K}_2\frac{\partial T}{\partial r}\right]_{r=b}=g_2(z,t) \tag{5}$$

$$\left[\mathrm{T}+\mathrm{K}_3\frac{\partial T}{\partial z}\right]_{z=h}=f_1(r,t) \tag{6}$$

$$\left[\mathrm{T}+\mathrm{K}_4\frac{\partial T}{\partial z}\right]_{z=-h}=f_2(r,t) \tag{7}$$

Where, k is thermal diffusivity of material of the plate. $k=\frac{\lambda}{\rho C}$, λ being the thermal conductivity of the material, ρ is the density and C is the calorific capacity, assumed to be constant. The displacement function in the cylindrical co-ordinate system are represented by Michell's function as

$$U_r=\frac{\partial\phi}{\partial r}-\frac{\partial^2 M}{\partial r\partial z} \tag{8}$$

$$Uz=\frac{\partial\phi}{\partial z}+2(1-\upsilon)\nabla^2 M-\frac{\partial^2 M}{\partial z^2} \tag{9}$$

The Michell's function must satisfy $\nabla^2\nabla^2 M=0$ (10)

Where, $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$

The component of the stresses are represented by the thermo elastic displacement potential ϕ and Michell's function M as Noda [6] are

$$\sigma_{rr}=2G\left\{\left[\frac{\partial^2\phi}{\partial r^2}-\nabla^2\phi\right]+\frac{\partial}{\partial z}\left[\upsilon\nabla^2 M-\frac{\partial^2 M}{\partial r^2}\right]\right\} \tag{11}$$

$$\sigma_{\theta\theta}=2G\left\{\left[\frac{1}{r}\frac{\partial\phi}{\partial r}-\nabla^2\phi\right]+\frac{\partial}{\partial z}\left[\upsilon\nabla^2 M-\frac{1}{r}\frac{\partial^2 M}{\partial r^2}\right]\right\} \tag{12}$$

$$\sigma_{zz}=2G\left\{\left[\frac{\partial^2\phi}{\partial z^2}-\nabla^2\phi\right]+\frac{\partial}{\partial z}\left[(z-\upsilon)\nabla^2 M-\frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{13}$$

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$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left[(1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \quad (14)$$

For traction free surface, stress function

$\sigma_z = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick plate.

Equations (1) to (14) constitute the mathematical formulation of the problem under consideration.

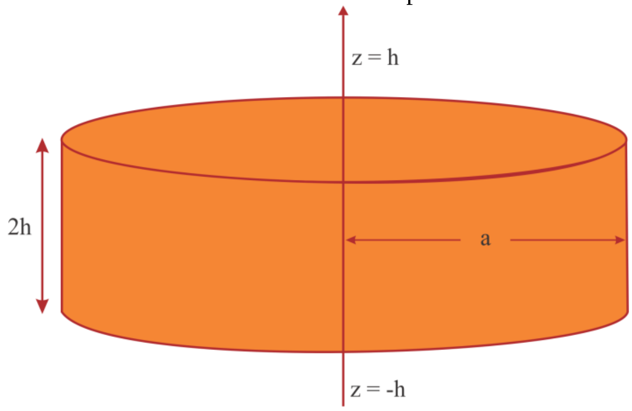

Fig. 1: Shows the geometry of the problem

IV. SOLUTION OF THE PROBLEM

In order to solve fundamental differential equation (2) under the boundary conditions (4) and (5). We first introduce the integral transform(2) of order n over the variable r. let n be the parameter of the transform then the integral transform and its inversion is giver by

$$\bar{f}(n) = \int_b^a rf(r)S_p(K_1, K_2, \mu_n r)dr \quad (15)$$

$$f(r) = \sum_{n=1}^{\infty}\left[\bar{f}(n)\Big/C_n\right]S_p(K_1, K_2, \mu_n r) \quad (16)$$

$$C_n = \frac{b^2}{2}\left\{S_p{}^2(K_1, K_2, \mu_n b) - S_{p-1}(K_1, K_2, \mu_n b)S_{p+1}(K_1, K_2, \mu_n b)\right\}$$
$$-\frac{a^2}{2}\left\{S_p{}^2(K_1, K_2, \mu_n a) - S_{p-1}(K_1, K_2, \mu_n a)S_{p+1}(K_1, K_2, \mu_n a)\right\} \quad (17)$$

Operational Property:

$$\int_b^a x\left\{\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} - \frac{p^2}{x^2}f\right\}S_p(K_1, K_2, \mu_n x)dx = \frac{b}{k_2}S_p(K_1, K_2, \mu_n b)\left\{f + k_2\frac{\partial f}{\partial x}\right\}_{x=b}$$
$$-\frac{a}{K_1}S_p(K_1, K_2, \mu_n a)\left\{f + k_1\frac{\partial f}{\partial x}\right\}_{x=a}$$
$$-\mu_n{}^2\bar{f}(n) \quad (18)$$

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Applying the transform defined in equation (2) to the equations (2), (3), (6) and (7) and taking into account equations (4) and (5) one obtains

$$\frac{d\overline{T}}{dz^2}(n,z,t) - \mu n^2\overline{T}(n,z,t) + \overline{\chi}(n,z,t) = \frac{1}{K}\frac{d\overline{T}(n,z,t)}{dt} \tag{19}$$

$$\overline{T}(n,z,0) = \overline{T}_0 \tag{20}$$

$$\left[\overline{T} + K_3\frac{d\overline{T}}{dz}\right]_{z=h} = \bar{f}_1(n,t) \tag{21}$$

$$\left[\overline{T} + K_4\frac{d\overline{T}}{dz}\right]_{z=-h} = \bar{f}_2(n,t) \tag{22}$$

Where $\overline{T}$ the transformed function of T and n is the transform parameter and μ_n are the positive roots of the characteristics equation.

$$J_0(K_1,\mu_n a)Y_0(K_2,\mu_n b) - J_0(K_2,\mu_n b)Y_0(K_1,\mu_n a) = 0$$

And g_1, g_2 are assumed to be zero

The Kernel function $S_0(K_1,K_2,\mu_n r)$ can be defined as

$$S_0(K_1,K_2,\mu_n r) = J_0(\mu_n,r)[Y_0(K_1,\mu_n a) + Y_0(K_2,\mu_n b)] - Y_0(\mu_n,r)[J_0(K_1,\mu_n a) + J_0(K_2,\mu_n b)]$$

With

$$\left.\begin{aligned} J_0(K_1\mu r) &= J_0(\mu r) + K_1\mu J_0^1(\mu r) \\ Y_0(K_1\mu r) &= Y_0(\mu r) + K_1\mu Y_0^1(\mu r) \end{aligned}\right\} \quad \text{for i=1,2}$$

In which $J_0(\mu r)$ and $Y_0(\mu r)$ are Bessel functions of first and second kind of order P=0 respectively

We introduce another integral transform [3] that responds to the boundary conditions of type (6) and (7)

$$\overline{g}(m,t) = \int_{-h}^{h} g(z,t)P_m(z)dz \tag{23}$$

$$g(z,t) = \sum_{m=1}^{\infty}\frac{\overline{g}(m,t)}{\lambda_m}P_m(z) \tag{24}$$

Where, $P_m(z) = Q_m\cos(a_m z) - W_m\sin(a_m z)$

In which

$$Q_m = a_m(K_3 + K_4)\cos(a_m h)$$

$$\lambda_m = \int_{-h}^{h} P_m^{\,2}(z)dz = h\left[Q_m^{\,2} + W_m^{\,2}\right] + \frac{\sin(2a_m h)}{2a_m}\left[Q_m^{\,2} - W_m^{\,2}\right]$$

The Eigen values a_m are the positive roots of the characteristics equation

$$[K_3 a\cos(ah) + \sin(ah)].[\cos(ah) + K_4 a\sin(ah)]$$
$$= [K_4 a\cos(ah) - \sin(ah)].[\cos(ah) - K_3 a\sin(ah)]$$

Further, Applying the transform defined in equation (23) to the equations (19), (20) and using equations (21), (22) one obtains

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$$\frac{d\overline{T}^*(n,m,t)}{dt} + KP^2\overline{T}^*(n,m,t) = \phi^* + \overline{\chi}^*$$

Where $P^2 = {\mu_n}^2 + {a_m}^2$

Where $\overline{T}^*$ denotes Marchi Fasulo integral transform of $\overline{T}$ and m is the transform parameter. Equation (25) is a linear equation whose solution is given by

$$\overline{T}^*(n,m,t) = e^{-kp^2t}\int_0^t \left(\phi^* + \overline{\chi}^*\right)e^{kp^2t'}dt' + Ce^{-kp^2t'} \qquad (25)$$

Using (3), we get $C = T_0^{\ *}(m,n)$

Applying inversion of Marchi Fasulo transform and Marchi Z. grablich transform to the equation (25) we get

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{e^{-kp^2t}\left[\int_0^t\left(\phi^* + \overline{\chi}^*\right)e^{kp^2t'} + T_0^{\ *}(m,n)\right]\right\}\frac{P_m(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r) \qquad (26)$$

Define, $M(r,z) = \sum_{n=1}^{\infty}\frac{1}{C_n}\psi\frac{P_m(z)}{\lambda_m}S_0\left(K_1,K_2,\mu_n r\right)$ (27)

Where, $\psi = e^{-kp^2t}\left[\int_0^t\left(\phi^* + \overline{\chi}^*\right)e^{kp^2t'} + T_0^{\ *}(m,n)\right]$

Also, $\phi = A\sum_{n=1}^{\infty}\frac{1}{C_n}\frac{P_m(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r)[\psi + B(t)]$ (28)

Where $A = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t$

And $B(t) = \int \psi dt$

V .DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (27) and (28) in equation (8), (9) we get,

$$U_r = A\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\frac{P_m(z)}{\lambda_m}S_0'\left(K_1,K_2,\mu_n r\right)[\psi + B(t)] - \sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\frac{P_m'(z)}{\lambda_m}S_0'\left(K_1,K_2,\mu_n r\right)\psi \qquad (29)$$

$$U_z = A\sum_{n=1}^{\infty}\frac{1}{C_n}\frac{P_m'(z)}{\lambda_m}S_0\left(K_1,K_2,\mu_n r\right)[\psi + B(t)] + 2(1-\upsilon)\sum_{n=1}^{\infty}\frac{{\mu_n}^2}{C_n}\psi\frac{P_m(z)}{\lambda_m}S_0''\left(K_1,K_2,\mu_n r\right)$$

$$+\frac{2(1-\upsilon)}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi\frac{P_m(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r)$$

$$+(1-2\upsilon)\sum_{n=1}^{\infty}\frac{1}{C_n}\psi\frac{P_{m''}''(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r) \tag{30}$$

VI. DETERMINATION OF THERMAL STRESSES

Substituting equation (27) and (28) in equation (11) to (14),we obtain

$$\sigma_{rr}=-\left\{\frac{2AG}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\frac{P_m(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r)[\psi+B(t)]\right\}$$

$$+\left\{2AG\sum_{n=1}^{\infty}\frac{1}{C_n}\frac{P_m''(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r)[\psi+B(t)]\right\}$$

$$+(\upsilon-1)\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}^{2}\psi\frac{P_m'(z)}{\lambda_m}S_0''(K_1,K_2,\mu_n r)$$

$$+\frac{\upsilon}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi\frac{P_{m'}'(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r)$$

$$+\upsilon\sum_{n=1}^{\infty}\frac{1}{C_n}\psi\frac{P_m'''(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r) \tag{31}$$

$$\sigma_{\theta\theta}=-\left\{2AG\sum_{n=1}^{\infty}\frac{\mu_n^2}{C_n}\frac{P_m(z)}{\lambda_m}S_0''(K_1,K_2,\mu_n r)[\psi+B(t)]\right\}$$

$$+\left\{2AG\sum_{n=1}^{\infty}\frac{1}{C_n}\frac{P_m''(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r)[\psi+B(t)]\right\}$$

$$+\left(\upsilon-\frac{1}{r}\right)\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}^{2}\psi\frac{P_m'(z)}{\lambda_m}S_0''(K_1,K_2,\mu_n r)$$

$$+\frac{\upsilon}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi\frac{P_m'(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r)$$

$$+\upsilon\sum_{n=1}^{\infty}\frac{1}{C_n}\psi\frac{P_m'''(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r) \tag{32}$$

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$$\sigma_{zz} = -\left\{2AG\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\frac{P_m(z)}{\lambda_m}S_0''(K_1,K_2,\mu_n r)[\psi + B(t)]\right\}$$
$$+\left\{\frac{2AG}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\frac{P_m(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r)[\psi + B(t)]\right\}$$
$$+(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi\frac{P_m'(z)}{\lambda_m}S_0''(K_1,K_2,\mu_n r)$$
$$+\frac{(1-\upsilon)}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi\frac{P_{m'}'(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r)$$
$$-\upsilon\sum_{n=1}^{\infty}\frac{1}{C_n}\psi\frac{P_m'''(z)}{\lambda_m}S_0(K_1,K_2,\mu_n r) \qquad (33)$$

$$\sigma_{rz} = 2AG\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\frac{P_m'(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r)[\psi + B(t)]$$
$$+(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n^3}{C_n}\psi\frac{P_m(z)}{\lambda_m}S_0'''(K_1,K_2,\mu_n r)$$
$$-\frac{(1-\upsilon)}{r^2}\sum_{n=1}^{\infty}\frac{\mu_n^2}{C_n}\psi\frac{P_m(z)}{\lambda_m}S_0''(K_1,K_2,\mu_n r)$$
$$-\upsilon\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi\frac{P_m''(z)}{\lambda_m}S_0'(K_1,K_2,\mu_n r) \qquad (34)$$

VII. NUMERICAL RESULTS AND DISCUSSION

To interpret the numerical computations, we consider material properties of Aluminum metal, which can be commonly used in both, wrought and craft farms, the low density of aluminum results in its extensive use in the aerospace industry and in other transportation fields. Its resistance to corrosion lead to its use in food and chemical handling (cookware, presser vessels etc.) and to architectural uses.

Table 1: Material properties and parameters used in this study. Property values are nominal

The foregoing analyses are performed by setting the radiation coefficients constants. Ki=0.86(i=1,3) and Ki=1(i=2,4) so, to obtain considerable mathematical simplicities. We can derived numerical results at t=0.25, 0.5, 0.75, 1.

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Modulus of Elasticity, E(dynes/cm^2)	6.9×10^{11}
Shear modulus, G(dynes/cm^2)	2.7×10^{11}
Poisson ratio, υ	0.281
Thermal expansion coefficient, $\alpha t(cm/cm-^0 C)$	25.5×10^{-6}
Thermal diffusivity, K(cm^2/sec)	0.86
Thermal conductivity , $\lambda[cal-cm/^0 C/\sec/cm^2]$	0.48
Inner radius, a(cm)	2
Outer radius, b(cm)	10
Thickness, h(cm)	3

VIII. CONCLUSION

In this article, the temperature distribution, displacement and thermal stresses of a thick annular disc are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series.

Any particular case of special interest can be assigned to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

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