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Magneto-Elastic Free Torsional Waves in a Non-homogeneous Orthotropic-Elastic Slab Having a Cylindrical Hole

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ABSTRACT: The aim of the present paper is to investigate Magneto-elastic free torsional waves in a non-homogeneous orthotropic-elastic slab having a cylindrical hole and obtain frequency equation. The elastic constants c_{ij} and density ρ of the slab are assumed to vary as $c_{ij} = A_{ij} + A'_{ij}\frac{\partial}{\partial t} + A''_{ij}\frac{\partial^2}{\partial t^2} + \dots\dots\dots\ \dots$ $+ A'^{n}_{ij}\frac{\partial^n}{\partial t^n}$ (i, j = 1,2,3.......n) and $\rho = \rho_0\left(\frac{r}{a}\right)^n$ respectively, where A'^{s}_{ij} and ρ_0 are constants, n any integer, a is the radius of the cylindrical hole and r is the radius vector. The frequency equation have been derived, numerical calculation have been made and graphs have been plotted to see the effects of variation of elastic constants and density on the produced torsional waves.

KEYWORDS: Torsional waves, Orthotropic-elastic material, Magnetic field, Frequency equation.

I. INTRODUCTION

During the last few years a great deal of activity has emerged in the study of interaction of elastic and magnetic fields. Chakravorti [4] discussed the propagation of torsional waves in a perfectly conducting elastic cylinder and a tube under the influence of uniform axial magnetic field. Narain [5] discussed torsional vibration of a non-homogeneous composite cylindrical shell subjected to a magnetic field. Narain [6] investigated torsional waves in a visco-elastic initially stressed cylinder placed in a magnetic field. Chandrashekharaiah [7] discussed propagation of torsional waves in magneto-visco-elastic solids. Narain and Kumar [8] discussed torsional deformation of a non-homogeneous magnetostrictve aeolotropic cylinder. Narain and Srivastava [9] discussed magneto-elastic torsional vibration of a non-homogeneous aeolotropic cylindrical shell of visco-elastic solid. Narain and Kaur [10] investigated magneto-elastic torsional vibration of a visco-elastic circular cylinder including strain and stress rate. Sequal to these the present paper is an attempt to investigate Magneto-elastic free torsional waves in a non-homogeneous orthotropic-elastic slab having a cylindrical hole. We assume that the non-homogeneity of the slab is due to the variable elastic constants c_{ij} and density ρ. The frequency equation in various cases have been derived and numerical calculation have been made. The graphs have been plotted to see the effects of variation of elastic constants and density on the produced torsional waves.

II. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

Consider a perfectly conducting non-homogeneous orthotropic-elastic slab with a cylindrical hole placed in a magnetic field and surrounded by vacuum. We suppose that the axis of z-coincides with the axis of cylindrical hole. Referred to the cylindrical co-ordinates (r, θ, z), the faces of the slab are $z = \pm h$ and cylindrical hole surface is given by $r = a$

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It is assumed that the density ρ of the material of the slab vary as $\rho=\rho_0\left(\frac{r}{a}\right)^n$, where n is an integer and r is the radius vector.

The stress-strain relations for an orthotropic material in cylindrical co-ordinates as given in Love [1] are

$$\begin{aligned}
\sigma_{rr} &= c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}\\
\sigma_{\theta\theta} &= c_{12}e_{rr}+c_{22}e_{\theta\theta}+c_{23}e_{zz}\\
\sigma_{zz} &= c_{13}e_{rr}+c_{23}e_{\theta\theta}+c_{33}e_{zz}\\
\sigma_{rz} &= c_{44}e_{rz}, \qquad \sigma_{\theta z}=c_{55}e_{\theta z}, \qquad \sigma_{r\theta}=c_{66}e_{r\theta}
\end{aligned} \tag{2.1}$$

The strain displacement relation is given by

$$2e_{ij}=u_{i,j}+u_{j,i} \tag{2.2}$$

where σ_{rr}, $\sigma_{\theta\theta}$ .... etc. are components of stress, e_{rr}, $e_{\theta\theta}$.........etc. and c_{11}, c_{12}.......etc. are components of strain and elastic constants respectively. The strain components are given by the relations,

$$\begin{aligned}
e_{rr}&=\frac{\partial u_r}{\partial r}, \qquad e_{\theta\theta}=\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}, \qquad e_{zz}=\frac{\partial u_z}{\partial z}\\
e_{r\theta}&=\frac{1}{2}\left(\frac{\partial u_\theta}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r}\right), \qquad e_{\theta z}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_z}{\partial\theta}+\frac{\partial u_\theta}{\partial z}\right)\\
e_{rz}&=\frac{1}{2}\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right)
\end{aligned} \tag{2.3}$$

and, the elastic constants are supposed to vary as

$$c_{ij}=A_{ij}+A'_{ij}\frac{\partial}{\partial t}+A''_{ij}\frac{\partial^2}{\partial t^2}+\dots\dots\dots+A'^{n}_{ij}\frac{\partial^n}{\partial t^n} \tag{2.4}$$

The stress equations of the motion as given in Kaliski are

$$\begin{aligned}
&\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}+\frac{H^2}{4\pi}\left(\frac{\partial^2u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}\right)=\rho\frac{\partial^2u_r}{\partial t^2}\\
&\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\partial\sigma_{\theta z}}{\partial z}+\frac{2}{r}\sigma_{r\theta}+\frac{H^2}{4\pi}\frac{1}{r}.\frac{\partial u_\theta}{\partial r}=\rho\frac{\partial^2u_\theta}{\partial t^2}\\
&\frac{\partial\sigma_{rz}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}}{\partial\theta}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{1}{r}\sigma_{rz}=\rho\frac{\partial^2u_z}{\partial t^2}
\end{aligned} \tag{2.5}$$

Maxwell's equations governing the electromagnetic field are,

$$\mathrm{curl}\vec{H} = 4\pi\vec{J}, \qquad \mathrm{curl}\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$
$$\mathrm{div}\,\vec{B} = 0, \qquad \vec{B} = \mu_e\vec{H} \tag{2.6}$$

where, the displacement current is neglected and Gaussian units have been used. Also by Ohm's law

$$\vec{J} = \sigma\left[\vec{E} + \frac{1}{c}\frac{\partial\vec{u}}{\partial t}\times\vec{B}\right] \tag{2.7}$$

In equations (2.6) and (2.7) $\vec{H}$, $\vec{B}$, $\vec{J}$, $\vec{E}$ respectively denote the magnetic intensity, magnetic induction , current density and electric intensity vector, μ_e and σ respectively denote magnetic permeability and electrical conductivity of the slab, $\vec{u}$ represents displacement vector in strained state and c is the speed of light.

The electromagnetic field equations in vacuum are,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}^* = 0, \qquad \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{h}^* = 0$$
$$\mathrm{curl}\,\vec{E}^* = -\frac{1}{c}\frac{\partial\vec{h}^*}{\partial t}, \qquad \mathrm{curl}\,\vec{h}^* = -\frac{1}{c}\frac{\partial\vec{E}^*}{\partial t} \tag{2.8}$$

where, $\vec{h}^*$ is the perturbation of the magnetic field and $\vec{E}^*$ the electric filed in vacuum.

Since we are considering torsional vibration, the displacement vector $\vec{u}[u_r, u_\theta, u_z]$ has only v as its non-vanishing component which is independent of θ in cylindrical co-ordinates i.e.

$$u_r = u_z = 0,$$
$$u_\theta = v = f(r,z)e^{i\omega t} \tag{2.9}$$

and, the magnetic intensity $\vec{H}$ has the components

$$H_r = H_\theta = 0,$$
$$H_z = H \text{ (constant).} \tag{2.10}$$

If the body is a perfect conductor of electricity, $\sigma \to \infty$ and the equation (2.7) gives,

$$\vec{E} = -\frac{1}{c}\frac{\partial\vec{u}}{\partial t}\times\vec{B}$$
$$\vec{E} = \left[-\frac{\mu_{eH}}{c}\frac{\partial v}{\partial t}, 0, 0\right] \tag{2.11}$$

Eliminating $\vec{E}$ from equations (2.6) and (2.11) and using equation (2.6) (iv) we get

$$\vec{h} = \left[0, H\frac{\partial v}{\partial z}, 0\right] \tag{2.12}$$

The equation (2.6) and (2.7) together with (2.12) gives

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$$\vec{J}\times\vec{B}=\left[0,-\frac{H^2}{4\pi}\frac{\partial^2 v}{\partial z^2},0\right] \tag{2.13}$$

The boundary condition for the slab with cylindrical hole is given by

$$\sigma_{r\theta}-T_{r\theta}-T^*_{r\theta}=0 \quad \text{on} \quad r=a \tag{2.14}$$

where $T_{r\theta}$, $T^*_{r\theta}$ are Maxwell's stress tensors in the body and in vacuum.

We can easily verify that

$$T_{r\theta}=T^*{}_{r\theta}=0 \tag{2.15}$$

and hence the boundary condition (2.14) reduces to

$$\sigma_{r\theta}=0 \quad \text{on } r=a \tag{2.16}$$

With the help of equations (2.3) and (2.4) the equations (2.1) can be written as

$$\sigma_{rr}=\sigma_{\theta\theta}=\sigma_{zz}=\sigma_{rz}=0$$

$$\sigma_{\theta z}=\frac{1}{2}\left(A_{55}+i\omega A'_{55}-\omega^2 A''_{55}+\dots\right)\frac{\partial f}{\partial z}e^{i\omega t} \tag{2. 17}$$

$$\sigma_{r\theta}=\frac{1}{2}\left(A_{66}+i\omega A'_{66}-\omega^2 A''_{66}+\dots\right)\left(\frac{\partial f}{\partial r}-\frac{f}{r}\right)e^{i\omega t}$$

With the help of the equation the equation (2.17) the equation (2.5) (ii) can be written as

$$\frac{\partial^2 f}{\partial r^2}+\frac{1-2\alpha}{r}\frac{\partial f}{\partial r}+\left(r^n\mu_1^2-\frac{3}{r^2}\right)f+k^2\frac{\partial^2 f}{\partial z^2}=0 \tag{2.18}$$

where,

$$\alpha=-\left[1+\frac{H^2}{4\pi\left(A_{66}+i\omega A'_{66}-\omega^2 A''_{66}+\dots\right)}\right]$$

$$\mu_1^2=\frac{2\rho_0\omega^2}{a^n\left(A_{66}+i\omega A'_{66}-\omega^2 A''_{66}+\dots\right)}$$

$$k^2=\frac{A_{55}+i\omega A'_{55}-\omega^2 A''_{55}+\dots}{A_{66}+i\omega A'_{66}-\omega^2 A''_{66}+\dots} \tag{2.19}$$

Assume that

$$f(r,z)=(A\cos pz+B\sin pz)F(r) \tag{2.20}$$

be the solution of the equation (2.18). Then using equations (2.18) and (2.20) we have,

$$\frac{\partial^2 F}{\partial r^2}+\frac{1-2\alpha}{r}\frac{\partial F}{\partial r}+\left(r^n\mu_1^2-\frac{3}{r^2}\right)F(r)-k^2p^2F(r)=0 \tag{2.21}$$

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III. METHODS OF SOLUTION

Special Cases: The equation (2.21) is very cumbersome to solve for general value of n and so we solve it for some particular value of n say $n = 0, -2$

Case-A For $n = 0$, the equation (2.21) becomes

$$\frac{\partial^2 F}{\partial r^2} + \frac{1-2\alpha}{r}\frac{\partial F}{\partial r} + \left(M^2 - \frac{3}{r^2}\right)F(r) = 0 \qquad (3.1)$$

where

$$M^2 = \mu_1^2 - k^2 p^2 \qquad (3.2)$$

Therefore, the solution of the equation (3.1) is

$$F(r) = r^{\alpha}\left[CJ_{\sqrt{3}}(Mr) + DY_{\sqrt{3}}(Mr)\right] \qquad (3.3)$$

where $J_{\sqrt{3}}$ and $Y_{\sqrt{3}}$ are Bessel Functions of first and second kind respectively of order $\sqrt{3}$.

From equations (3.3) and (2.20) we have

$$f(r,z) = r^{\alpha}(A\cos pz + B\sin pz)\left\{CJ_{\sqrt{3}}(Mr) + DY_{\sqrt{3}}(Mr)\right\} \qquad (3.4)$$

Hence, from the equations (3.4) and (2.9), we have

$$v = r^{\alpha}(A\cos pz + B\sin pz)\left\{CJ_{\sqrt{3}}(Mr) + DY_{\sqrt{3}}(Mr)\right\}e^{i\omega t}\ . \qquad (3.5)$$

But the surfaces $z = \pm h$ are free from stresses, hence

$$(\sigma_{\theta z})_{z=\pm h} = 0 \qquad (3.6)$$

Therefore, from the equations (3.6) and (2.17)(ii), we have

$$\left(\frac{\partial f}{\partial z}\right)_{z=\pm h} = 0 \qquad (3.7)$$

Hence, the equations (3.7) and (3.4) give

either, $B = 0$ and $p = \frac{m\pi}{h}$, m being an integer (3.8)

which is symmetric mode of vibration

or, $A = 0$ and $p = \frac{(2m+1)\pi}{2h}$ (3.9)

which is antisymmetric mode of vibration

Thus, from (3.5) the solution is of either of the following form

$$v = Ar^{\alpha}\cos pz\left\{CJ_{\sqrt{3}}(Mr) + DY_{\sqrt{3}}(Mr)\right\}e^{i\omega t} \text{ with } p = \frac{m\pi}{h} \qquad (3.10)$$

and

$$v = Br^{\alpha} \sin pz \{ C J_{\sqrt{3}}(Mr) + D Y_{\sqrt{3}}(Mr) \} e^{i\omega t} \quad \text{with} \quad p = \frac{(2m+1)\pi}{2h}. \tag{3.11}$$

For finite cylinder $Y_{\sqrt{3}}(Mr) = 0$

Therefore, equation (3.10) and (3.11) reduces to

$$v = AC\, r^{\alpha} \cos pz\, J_{\sqrt{3}}(Mr) e^{i\omega t} \tag{3.12}$$

and

$$v = BC\, r^{\alpha} \sin pz\, J_{\sqrt{3}}(Mr) e^{i\omega t} \tag{3.13}$$

Now, In view of equations (2.11) and (2.12), we take

$$\vec{E}^* = [E^*, 0, 0]$$

$$\vec{h}^* = [0, h^*, 0]$$

Hence, the equation (2.8) takes the following form

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) E^* = 0 \tag{3.14}$$

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial}{\partial t^2} \right) h^* = 0 \tag{3.15}$$

and

$$\frac{\partial h^*}{\partial t} = -\frac{c}{r} \frac{\partial E^*}{\partial Z} \tag{3.16}$$

For free torsional vibration we write the solution of (3.16) in the following forms

$$h^* = (A \cos pz + B \sin pz) h_0^*(r) e^{i\omega t} \tag{3.17}$$

and

$$E^* = (A \cos pz + B \sin pz) E_0^*(r) e^{i\omega t}.$$

Thus, the equation (3.16) together with (3.17) gives

$$\frac{d^2 h_0^*}{dr^2} + \frac{1}{r} \frac{dh_0^*}{dr} + \frac{\omega^2}{c^2} h_0^* = 0 \tag{3.18}$$

and

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$$\frac{d^2E_0^*}{dr^2}+\frac{1}{r}\frac{dE_0^*}{dr}+\frac{\omega^2}{c^2}E_0^*=0 \quad . \tag{3.19}$$

With the help of equations (2.1), (2.3), (2.16) we have

$$\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)_{r=a}=0\,. \tag{3.20}$$

From equations (3.20) and (3.12) we have

$$(Mr)J'_{\sqrt{3}}\ (Ma)+(\alpha-1)J_{\sqrt{3}}\ (Ma)=0. \tag{3.21}$$

Using recurrence formula

$$xJ'_n(x)=nJ_n(x)-xJ_{n+1}(x)$$

in equation (3.1) we have

$$\frac{J_{\sqrt{3}}\ (Ma)}{J_{\sqrt{3}+1}\ (Ma)}=\frac{Ma}{\alpha+\sqrt{3}-1}$$

or,

$$\tan\theta=\frac{Ma}{\alpha+\sqrt{3}-1} \tag{3.22}$$

where

$$\theta=Ma-\frac{\pi}{4}-\frac{\sqrt{3}}{2}\pi \tag{3.23}$$

If θ is small, $\tan\theta\approx\theta$ and then from equations (3.22) and (3.23) we have,

$$Ma\left(\alpha+\sqrt{3}-2\right)-\left(\frac{\pi}{4}+\frac{\sqrt{3}}{2}\pi\right)\left(\alpha+\sqrt{3}-1\right)=0 \tag{3.24}$$

From equations (2.19), (3.2) and (3.24), we have,

$$a\left(\alpha+\sqrt{3}-2\right)\left[\frac{2\rho_0\omega^2}{A_{66}+i\omega A'_{66}-\omega^2A''_{66}+\ldots.}-\frac{(A_{55}+i\omega A'_{55}-\omega^2A''_{55}+\ldots.)p^2}{A_{66}+i\omega A'_{66}-\omega^2A''_{66}+\ldots.}\right]^{1/2}$$

$$=\frac{\pi}{4}\left(1+2\sqrt{3}\right)\left(\alpha+\sqrt{3}-1\right) \tag{3.25}$$

If $A'_{55}, A''_{55}\ldots\ldots.\text{and}\, A'_{66}, A''_{66}\ldots\ldots.\text{etc.}$ are zero then the equation (3.25) reduces to

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$$\omega^2 = \frac{c_{66}}{2\rho_0}\left[p^2 + \frac{\pi^2\left(1+2\sqrt{3}\right)^2\left\{H^2+4\pi c_{66}\left(2-\sqrt{3}\right)^2\right\}^2}{16a^2\left\{H^2+4\pi c_{66}\left(3-\sqrt{3}\right)\right\}^2}\right] \tag{3.26}$$

which is the required frequency equation

where,

$p = \frac{m\pi}{h}$ i.e. in symmetric mode of vibration and hence the frequency equation (3.26) becomes

$$\omega^2 = \frac{c_{66}\pi^2}{2\rho_0}\left[\frac{m^2}{h^2} + \frac{\left(1+2\sqrt{3}\right)^2\left\{H^2+4\pi c_{66}\left(2-\sqrt{3}\right)\right\}^2}{16a^2\left\{H^2+4\pi c_{66}\left(3-\sqrt{3}\right)\right\}^2}\right] \tag{3.27}$$

Again when, $p = \frac{(2m+1)\pi}{2h}$, i.e. in antisymmetric mode of vibration, the frequency equation (3.26) becomes

$$\omega^2 = \frac{c_{66}\pi^2}{2\rho_0}\left[\frac{(2m+1)^2}{4h^2} + \frac{\left(1+2\sqrt{3}\right)^2\left\{H^2+4\pi c_{66}\left(2-\sqrt{3}\right)\right\}^2}{16a^2\left\{H^2+4\pi c_{66}\left(3-\sqrt{3}\right)\right\}^2}\right] \tag{3.28}$$

If there were no magnetic field, H=0, the equations (3.27) and (3.28) reduces to

$$\omega^2 = \frac{c_{66}\pi^2}{2\rho_0}\left[\frac{m^2}{h^2} + \frac{\left(1+2\sqrt{3}\right)^2\left(2-\sqrt{3}\right)^2}{16a^2\left(3-\sqrt{3}\right)^2}\right] \tag{3.29}$$

and,

$$\omega^2 = \frac{c_{66}\pi^2}{2\rho_0}\left[\frac{(2m+1)^2}{4h^2} + \frac{\left(1+2\sqrt{3}\right)^2\left(2-\sqrt{3}\right)^2}{16a^2\left(3-\sqrt{3}\right)^2}\right] \tag{3.30}$$

Case-B For $n = -2$, the equation (2.39) gives,

$$\frac{\partial^2 F}{\partial r^2} + \frac{1-2\alpha}{r}\frac{\partial F}{\partial r} - \left(k^2p^2 + \frac{\ell^2}{r^2}\right)F(r) = 0 \tag{3.31}$$

where,

$$\ell^2 = 3-\mu_1^2 \tag{3.31*}$$

The solution of the equation (3.31) is given by

$$F(r) = r^{\alpha}\left[P\,I_\ell(kpr) + Q\,K_\ell(kpr)\right] \tag{3.32}$$

where P and Q are constants, I_ℓ and K_ℓ are modified Bessel Functions of first and second kind respectively.

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From the equations (2.9), (2.20) and (3.32) we have,

$$v = r^{\alpha}(A\cos pz + B\sin pz)\ [P I_{\ell}(kpr) + Q K_{\ell}(kpr)]$$

For finite cylinder $K_{\ell} = 0$, then

$$v = P r^{\alpha}(A\cos pz + B\sin pz)\ I_{\ell}(kpr). \tag{3.33}$$

Using the conditions (3.6), (3.7), (3.8) and (3.9) in equation (3.33) we have,

either,

$$v = A P r^{\alpha} \cos pz\, I_{\ell}(kpr) e^{i\omega t} \tag{3.34}$$

or,

$$v = B P r^{\alpha} \sin pz\, I_{\ell}(kpr) e^{i\omega t} \tag{3.35}$$

From equations (3.20) and (3.34) we have,

$$2\ell^2 + 2\ell(\alpha - 1) = k^2 p^2 r^2 \tag{3.36}$$

From equations (2.34), (3.31)* and (3.36) we have,

$$\left[3 - \frac{2\rho_0\omega^2 a^2}{A_{66} + i\omega A'_{66} - \omega^2 A''_{66}}\right] - \left[3 - \frac{2\rho_0\omega^2 a^2}{A_{66} + i\omega A'_{66} - \omega^2 A''_{66}}\right]^{\frac{1}{2}}$$

$$\left[2 + \frac{H^2}{4\pi\left(A_{66} + i\omega A'_{66} - \omega^2 A''_{66} + ...\right)}\right] = \frac{p^2 r^2}{2}\left[\frac{A_{55} + i\omega A'_{55} - \omega^2 A''_{55} + ...}{A_{66} + i\omega A'_{66} - \omega^2 A''_{66} + ...}\right] \tag{3.37}$$

If $A'_{55}, A''_{55}, ...$ and $A'_{66}, A''_{66}, ...$ etc. are zero, $c_{55} = c_{66}$

then the equation (3.37) reduces to

$$\omega^2 = \frac{c_{66}}{\rho_0 a^2}\left[3 + \frac{2\pi c_{66}\left(p^2 r^2 + 6\right)}{H^2}\right]$$

On the surface of the cylinder, $r = a$ we have,

$$\omega^2 = \frac{c_{66}}{\rho_0 a^2}\left[3 + \frac{2\pi c_{66}\left(p^2 a^2 + 6\right)}{H^2}\right] \tag{3.38}$$

which is the required frequency equation

Let $p = \frac{m\pi}{h}$ i.e. in symmetric mode of vibration, the equation (3.38) becomes

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$$\omega^2 = \frac{c_{66}}{\rho_0 a^2}\left[3 + \frac{2\pi c_{66}}{H^2}\left(\frac{m^2\pi^2 a^2}{h^2} + 6\right)\right] \tag{3.39}$$

and for, $p = \frac{(2m+1)\pi}{2h}$ i.e. in antisymmetric mode of vibration, the equation (3.38) becomes

$$\omega^2 = \frac{c_{66}}{a^2\rho_0}\left[3 + \frac{2\pi c_{66}}{H^2}\left\{\frac{(2m+1)^2\pi^2 a^2}{4h^2} + 6\right\}\right] \tag{3.40}$$

which is the required frequency equation of the wave so generated.

IV. NUMERICAL CALCULATIONS

We calculate numerical value of the frequency of torsional wave for different materials in various particular cases. Case-A when n = 0, the frequency equation (3.27), (3.28), (3.29) and (3.30) are considered. In case-B when n = -2, the frequency equations (3.39) and (3.40) are considered. In both the cases the graphs between r and ω have been plotted for Beryllium, Rock-crystal and Zinc.

Table-1

Case-A when $n = 0,\ m = 0,\ H = 0.50$

(a) In the presence of magnetic field and case of symmetric mode of vibration.

S.No.	Material	ρ_0	c_{66}	a	ω
1.	Beryllium	2.70	26.94	-4	-0.41
				-3	-0.55
				-2	-0.83
				-1	-1.66
				0	∞
				1	1.66
				2	0.83
				3	0.55
				4	0.41
2.	Rock-crystal	2.70	12.73	-4	-0.28
				-3	-0.38
				-2	-0.57
				-1	-1.14
				0	∞
				1	1.14
				2	0.57
				3	0.38
				4	0.28
3.	Zinc	7.14	14.30	-4	-0.01
				-3	-0.06
				-2	-0.09
				-1	-0.19
				0	∞
				1	0.19
				2	0.09
				3	0.06
				4	0.01

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Table-2

when $n = 0,\ m = 0,\ h = 10,\ H = 0.50$

(b) In the presence of magnetic field and case of antisymmetric mode of vibration.

S.No.	Material	ρ_0	c_{66}	a	ω
1.	Beryllium	2.70	26.94	-4 -3 -2 -1 0 1 2 3 4	7.20 11.50 17.20 34.35 ∞ 34.35 17.20 11.50 7.20
2.	Rock-crystal	2.70	12.73	-4 -3 -2 -1 0 1 2 3 4	0.40 0.61 0.75 1.24 ∞ 1.24 0.75 0.61 0.40
3.	Zinc	7.14	14.30	-4 -3 -2 -1 0 1 2 3 4	0.32 0.40 0.48 0.81 ∞ 0.81 0.48 0.40 0.32

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Table-3

(c) If there is no magnetic field and case of symmetric mode of vibration.

S.No.	Material	ρ_0	c_{66}	m	a	ω
1.	Beryllium	2.70	26.94	0	-4 -3 -2 -1 0 1 2 3 4	-0.41 -0.55 -0.83 -1.65 ∞ 1.65 0.83 0.55 0.41
2.	Rock-crystal	2.70	12.73	0	-4 -3 -2 -1 0 1 2 3 4	-0.28 -0.38 -0.57 -1.14 ∞ 1.14 0.57 0.38 0.28
3.	Zinc	7.14	14.30	0	-4 -3 -2 -1 0 1 2 3 4	-0.18 -0.25 -0.37 -0.74 ∞ 0.74 0.37 0.25 0.18

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Table-4

when $n = 0,\ m = 0,\ h = 10,$

(d) In there is no magnetic field and case of antisymmetric mode of vibration.

S.No.	Material	ρ_0	c_{66}	a	ω
1.	Beryllium	2.70	26.94	-4 -3 -2 -1 0 1 2 3 4	1.35 1.78 2.64 5.24 ∞ 5.24 2.64 1.78 1.35
2.	Rock-crystal	2.70	12.73	-4 -3 -2 -1 0 1 2 3 4	0.93 1.22 1.80 3.6 ∞ 3.6 1.80 1.22 0.93
3.	Zinc	7.14	14.30	-4 -3 -2 -1 0 1 2 3 4	0.60 0.80 1.18 2.35 ∞ 2.35 1.18 0.80 0.60

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Table-5

Case-B when $n = -2,\ m = 0,\ H = 0.50$

(a) In the presence of magnetic field and case of symmetric mode of vibration.

S.No.	**Material**	ρ_0	c_{66}	**a**	ω
1.	Beryllium	2.70	26.94	-4 -3 -2 -1 0 1 2 3 4	-50.30 -67.12 -100.68 -201.35 ∞ 201.35 100.68 67.12 50.30
2.	Rock-crystal	2.70	12.73	-4 -3 -2 -1 0 1 2 3 4	-23.80 -31.70 -47.60 -95.20 ∞ 95.20 47.60 31.70 23.80
3.	Zinc	7.14	14.30	-4 -3 -2 -1 0 1 2 3 4	-16.44 -21.91 -32.80 -65.74 ∞ 65.74 32.80 21.91 16.44

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Table-6

when $n = -2,\ m = 0,\ h = 10,\ H = 0.50$

(b) In the presence of magnetic field and case of antisymmetric mode of vibration.

S.No.	Material	ρ_0	c_{66}	a	ω
1.	Beryllium	2.70	26.94	-4	51.96
				-3	68.34
				-2	108.60
				-1	201.77
				0	∞
				1	201.77
				2	108.60
				3	68.34
				4	51.96
2.	Rock-crystal	2.70	12.73	-4	24.56
				-3	32.30
				-2	49.98
				-1	95.38
				0	∞
				1	95.38
				2	49.98
				3	32.30
				4	24.56
3.	Zinc	7.14	14.30	-4	16.96
				-3	22.32
				-2	33.14
				-1	62.11
				0	∞
				1	62.11
				2	33.14
				3	22.32
				4	16.96

The graphs corresponding to the table (1), (2), (3), (4), (5) and (6) are drawn in the form of the graph (1), (2), (3), (4), (5) and (6) as under

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Table-6

when $n = -2,\ m = 0,\ h = 10,\ H = 0.50$

(b) In the presence of magnetic field and case of antisymmetric mode of vibration.

S.No.	Material	ρ_0	c_{66}	a	ω
1.	Beryllium	2.70	26.94	-4	51.96
				-3	68.34
				-2	108.60
				-1	201.77
				0	∞
				1	201.77
				2	108.60
				3	68.34
				4	51.96
2.	Rock-crystal	2.70	12.73	-4	24.56
				-3	32.30
				-2	49.98
				-1	95.38
				0	∞
				1	95.38
				2	49.98
				3	32.30
				4	24.56
3.	Zinc	7.14	14.30	-4	16.96
				-3	22.32
				-2	33.14
				-1	62.11
				0	∞
				1	62.11
				2	33.14
				3	22.32
				4	16.96

The graphs corresponding to the table (1), (2), (3), (4), (5) and (6) are drawn in the form of the graph (1), (2), (3), (4), (5) and (6) as under

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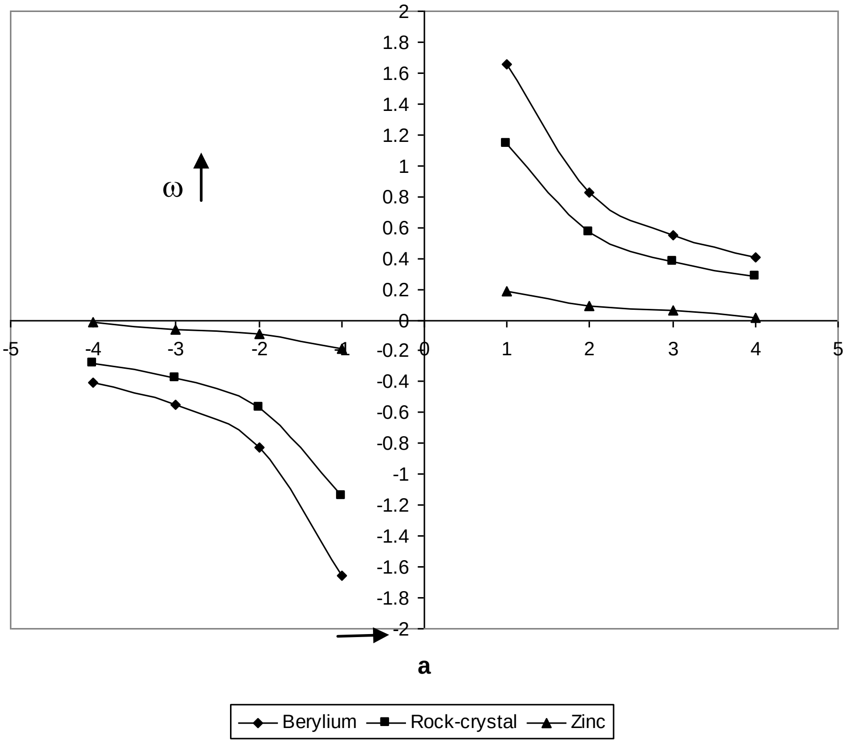
Graph-1



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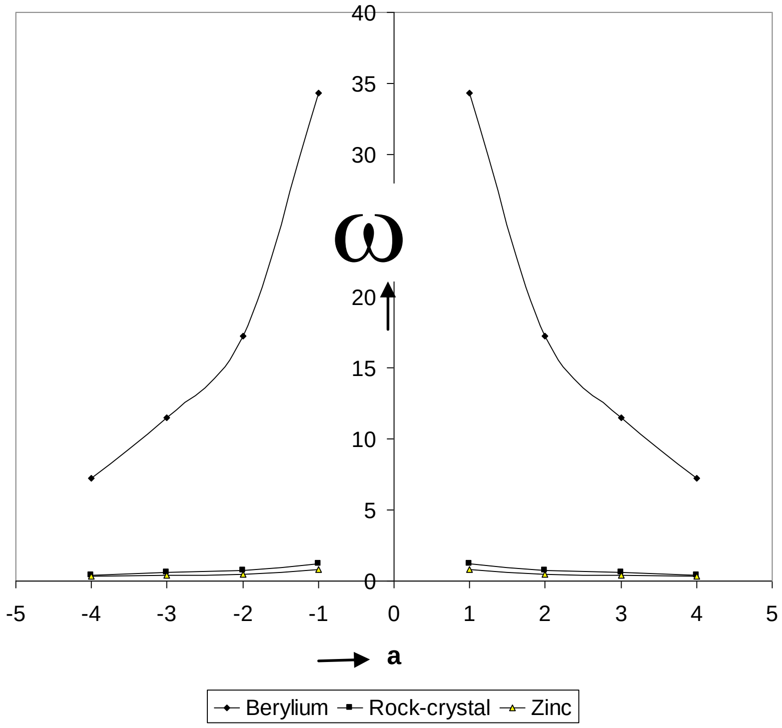
Graph-2



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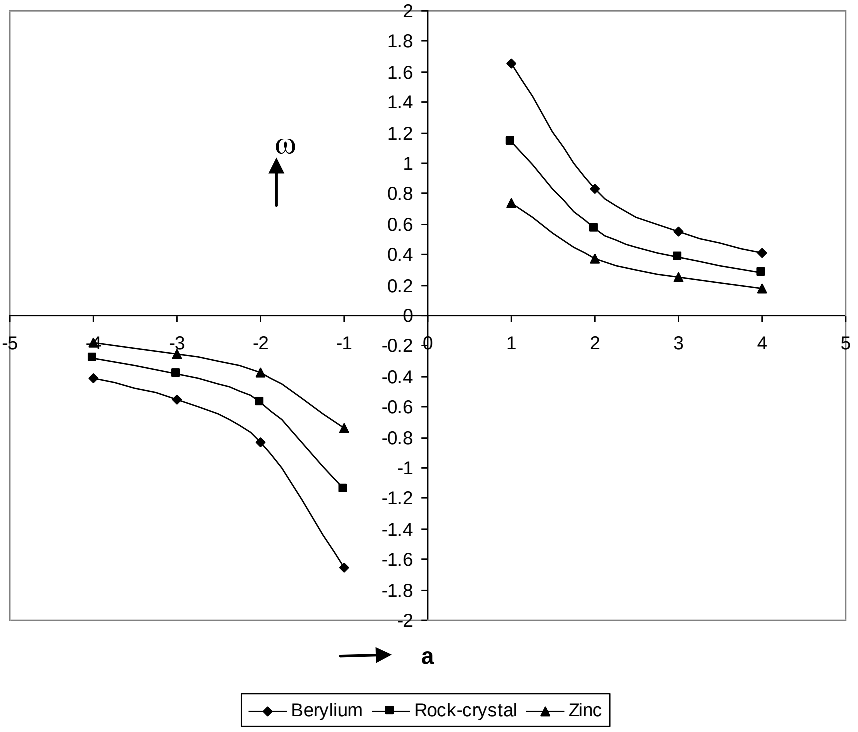
Graph-3



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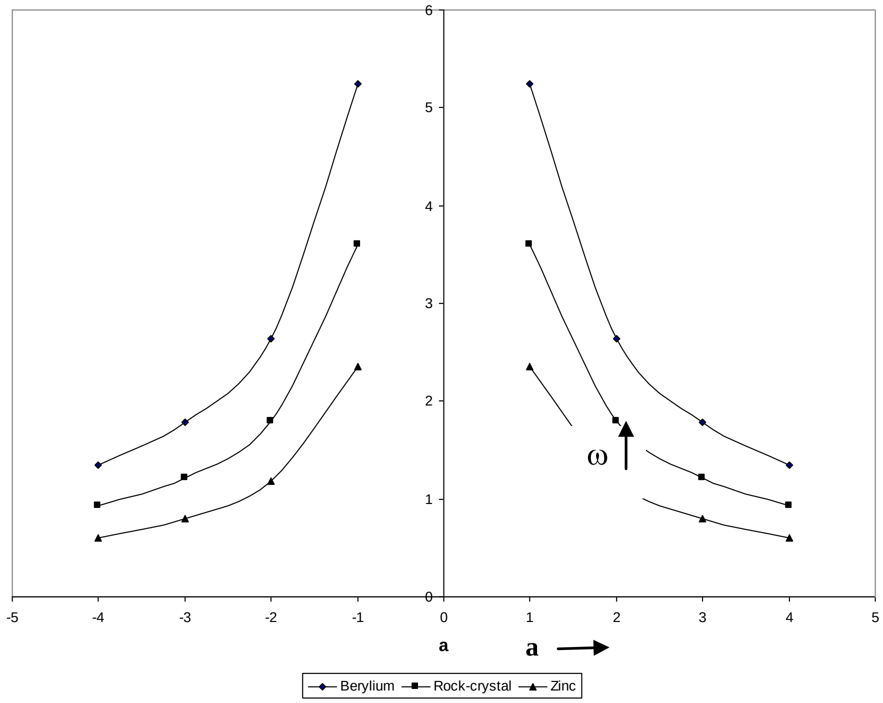
Graph-4



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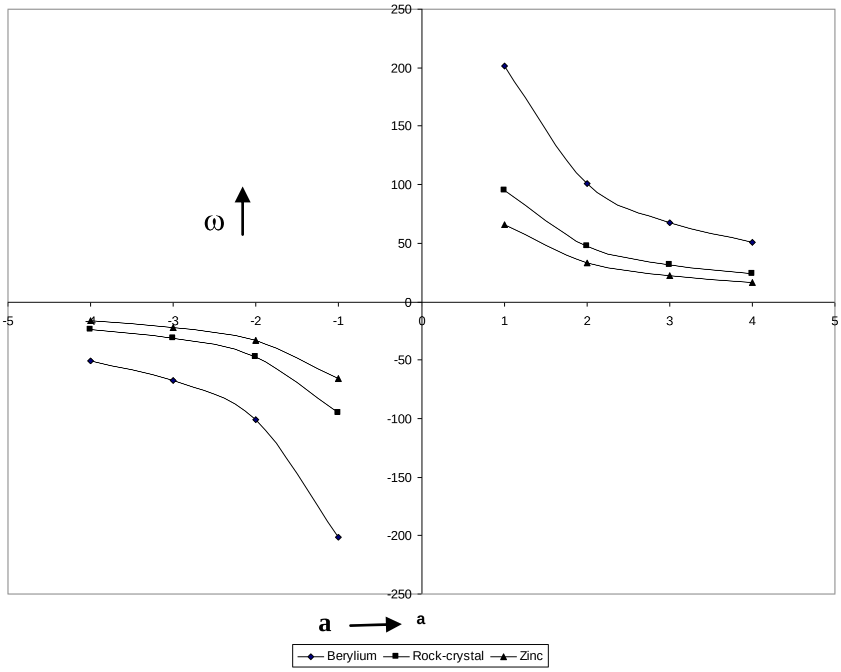
Graph-5



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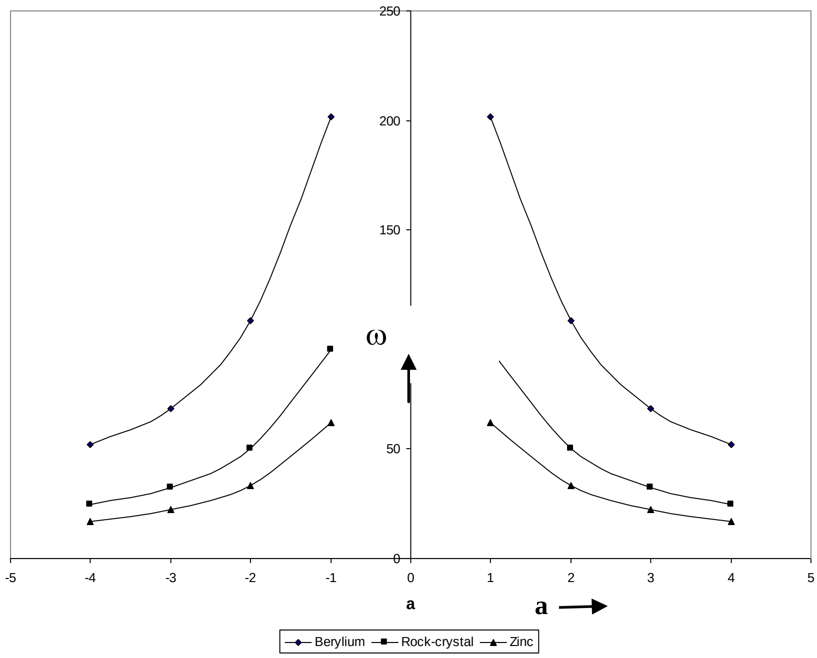
Graph-6



VI. CONCLUSION

Graphs drawn between the frequency ω and radius of the cylindrical hole a show that the frequency of all the three materials namely Beryllium, Rock-crystal and zinc increases when radius of the cylindrical hole decreases. Again, the frequency of these materials decreases when radius of the cylindrical hole increases. From the graphs (1), (3) and (5) we observed that the curves are mirror image about the line $y = -x$ whereas the graphs (2), (4) and (6) indicates it is mirror image about the line $x = 0$.

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