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# **Modeling and Robust $H_{\infty}$ Control of a Synchronous Machine with a Salient Rotor**

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**ABSTRACT:** In this paper, we introduce the modeling of a synchronous permanent magnet machine with a salient rotor and the development methodology of centralized and decentralized robust  $H_{\infty}$  control applied to this machine. The proposed system model is nonlinear, we linearize around a point of application. The resulting model will then be used to reproduce the dynamic behaviour of the machine as well as to the synthesis of robust control law. The order is based on the standard  $H_{\infty}$  to increase performance and reduce measurement noise. To illustrate the results, we made a comparison between a standard state feedback control and  $H_{\infty}$  infinite robust control. The simulation results show, firstly, that the system in case of technical placements poles loses classic performance measuring, the presence of noise and other conditions remain satisfactory in the case of a controller robust.

**KEYWORDS:** Robust control,  $H_{\infty}$  infinite control, Equation of Riccati, linear system, state feedback control.

## **I. INTRODUCTION**

In the presence of disturbances, the technique of classical state feedback control of linear systems appears insufficient and gives mediocre performance. This research proposes adaptive algorithms [4], other compensation for interference with additive terms in the control algorithm [5] or robust controllers.

The main advantage of Robust Control is to generate control laws which allow firstly shaping the response of the servo system to give it the desired behaviour and secondly maintain this behaviour to the vagaries and fluctuations that affect. In this paper we investigate a control strategy which is to develop centralized and decentralized controllers based on the  $H_{\infty}$  synthesis and each having the structure of a state observer estimates increased disturbances and commands generated by other controllers so to ensure a stable transfer between magnitudes and bounded external disturbances regulated and the acting on the whole system. The method is therefore primarily to solve a Riccati equation, and an equation of pseudo Riccati. We conclude by validating the results using a simulation that compares the state feedback controller and the classical robust control.

## **II. THE $H_{\infty}$ CONTROL [1]**

### **A. THE DECENTRALIZED $H_{\infty}$ -CONTROL**

It is to develop decentralized monovariable  $N$  controllers based on  $H_{\infty}$  synthesis to ensure a stable transfer and bounded between the quantities regulated and external disturbances acting on the system overall.

The system is represented by the following model:

$$\dot{x} = Ax + \sum_{i=1}^q B_i u_i + Gw_0 \quad (1)$$

$$y_i = C_i x + w_i \quad i \in (1, 2, \dots, q) \quad (2)$$

$$Z = \begin{bmatrix} Hx \\ u_1 \\ \vdots \\ u_q \end{bmatrix} \quad (3)$$

$u_i$  is the  $i$ th local control and  $y_i$  is the  $i$ th measured local output.  $Z$  represents the output controlled. Here the vector of differences  $Z$  is increased by the control  $u$  as limiting the biasing of the actuators.

Assume that the pair  $(A, H)$  is detectable and we adopt the following notations:

$$\sum_{i=1}^q B_i u_i = [B_1 \ B_2 \ \dots \ B_q] u = Bu \quad (4)$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_3 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_3 \end{bmatrix} x + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_3 \end{bmatrix} + Cx + w \quad (5)$$

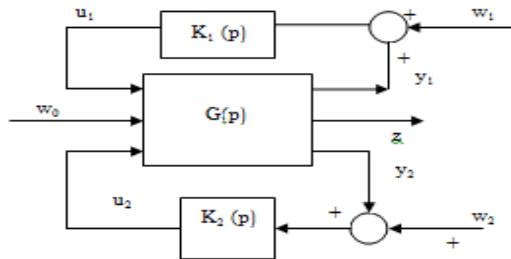


Fig.1 Base case decentralized

The problem is to develop a  $q$  decentralized controllers using the local output signals  $y_i$  and generating local controls  $u_i$  of the form:  $u_i = k_i x$   $i \in (1, 2, \dots, q)$

We define  $q$  monovaryable controllers each having  $n$ -dimensional structure of a state observer full-order:

$u_i = k_i \hat{x}_i$  With

$$\dot{\hat{x}}_i(t) = Ax_i + B_i u_i + \sum_{j=1, j \neq i}^q B_j \hat{u}_j^i + G \hat{w}_0^i + L_i (y_i - C_i \hat{x}_i) \quad (6)$$

Where

$\hat{u}_j^i = K_j \hat{x}_i$  is the estimate of the command generated by the  $j$ th controller.

$\hat{w}_0^i = K_d \hat{x}_i$  is the estimate of the disturbance  $w_0$ ,  $K_d$ 's gain.

$L_i$  represents the gain observer status.

The equation becomes:

$$\dot{\hat{x}}_i = (A + BK + GK_d - L_i C_i) \hat{x}_i + L_i y_i \quad (7)$$

$$u_i = K_i x_i \text{ with } i \in (1, 2, \dots, q)$$

$K_i, K_d, \text{ et } L_i$  constitute the parameters of synthesis of decentralized control law.

$$\text{Let } A_a = A + BK + GK_d$$

$$y_i = C_i x + w_i$$

Then:

$$\dot{\xi}_i = (A_a - L_i C_i) \xi_i + L_i C_i x + L_i w_i$$

$$u_i = K_i \xi_i \text{ With } i \in (1, 2, \dots, q)$$

We define the state vector closed-loop:

$$x_e = (x^T \ \xi^T)^T, \xi = (\xi_1^T \ \xi_2^T \ \dots \ \xi_q^T)^T \quad (8)$$



The purpose of control is to determine the feedback gain  $K_1, L_i$  observer gain and the estimated gain of the disturbance  $K_d$  as the closed loop system satisfies:

$$\|T\|_\infty \leq \alpha, T(p) = H_e(pI - F_e)^{-1}G_e \quad (9)$$

Where  $\alpha$  is a specified number.

The following lemma and theorem plays an important role in the theory of synthesis  $\|T\|_\infty$  and forms the basis of the approach we will develop:

Lemma 1: proof see [1]

Let  $T(p) = H(pI - F)^{-1}G$  transfer function of a triple  $(F, G, H)$ ,  $(F, H)$  is detectable pair; the two following properties are equivalent:

$F$  is Hurwitz and  $\|H\|_\infty \leq \varepsilon, \exists X \geq 0$  and a scalar  $\alpha \geq 0$  such as:

$$F^T X + X F + \frac{1}{\alpha^2} X G G^T X + H^T H \leq 0 \quad (10)$$

Theorem 1: Consider the system:

$$\begin{aligned} \dot{x} &= Ax + Bu + w_0 \\ y &= Cx + w \\ z &= \begin{bmatrix} Hx \\ u \end{bmatrix} \end{aligned}$$

$(A, H)$  being a pair detectable. On defines the Riccati equation:

$$A^T X + X A + \frac{1}{\alpha^2} X G G^T X - X B B^T X + H^T H = 0 \quad (11)$$

Where  $\alpha$  is a strictly positive real.

Assume that there exists a positive symmetric solution of  $X$  and take:

$$K = -B^T X \quad K_d = \frac{1}{\alpha^2} G^T X \quad (12)$$

With  $A_a = A + BK + GK_d$  is Hurwitz.

$A + GK_d$  has no eigenvalues on the imaginary axis.

We define the pseudo-riccati equation:

$$W A_c^T + A_c + W + \frac{1}{\alpha^2} W K_c^T K_c W - W C_c^T C_c W + G_c G_c^T + (W - W_D) C_c^T C_c (W - W_D) = 0 \quad (13)$$

Where  $W$  is a positive definite matrix:

$$W = \begin{bmatrix} W_{11} & \dots & W_{1q} \\ W_{q1} & \ddots & W_{qq} \end{bmatrix} \quad (14)$$

$$W_D = \text{diag}(W_{11}, W_{22}, \dots, W_{qq})$$

The observer gain is:

$$L_i = W_{ii} C_i^T i \in (1, 2, \dots, q) \quad (15)$$

Decentralized control law:  $u_i = k_i \xi_i$

$$\dot{\xi}_i = (A + BK + GK_d - L_i C_i) \xi_i + L_i C_i x + L_i w_i \quad (16)$$

With  $i \in (1, 2, \dots, q)$

Stabilizes the system and satisfies:

$$\|T\|_\infty \leq \alpha \quad T(p) = \frac{w_e(p)}{z(p)} = H_e(pI - F_e)^{-1}G_e \quad (17)$$

So, the solution of the problem requires solving two algebraic equations one of which is Riccati type and the other is called pseudo-Riccati equation to obtain compensators such as the transfer function of the closed loop system satisfies to:  $\|T\|_\infty \leq \alpha$

Monovariable compensators are defined as state observer plus disturbance estimation and commands generated by other controllers.

## B. THE CENTRALIZED $H_\infty$ -CONTROL

The centralized case [7], [8], [9], [10] can be viewed as a special case of decentralized design.

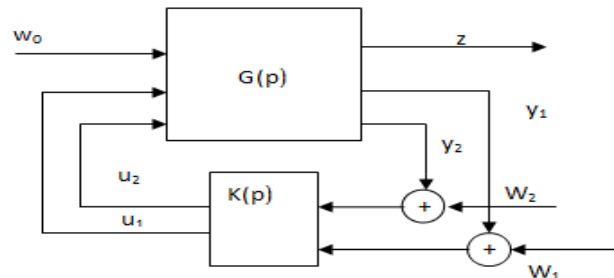


Fig.2 Base case centralized

(A, H) is a detectable pair. We define the Riccati equation:

$$A^T X + XA + \frac{1}{\alpha^2} XGG^T X - XBB^T X + H^T H = 0 \quad (18)$$

Where  $\alpha$  is a strictly positive real.

Assume that there exists a positive symmetric solution of X and take:

With  $A_a = A + BK + GK_d$  is Hurwitz; suppose also

$$L = (I - \alpha^{-2} YX)^{-1} YC^T \quad (19)$$

Where  $Y > 0$  satisfies the observer design ARE

$$AY + YA^T + \frac{1}{\alpha^2} YH^T H Y + YC^T + GG^T = 0 \quad (20)$$

So, the solution of the problem requires solving two algebraic equations Riccati type to obtain a compensator such that the transfer function of the closed loop system satisfies:

$$\|T\|_{\infty} \leq \alpha$$

This compensator is multivariable as defined state observer plus disturbance estimation.

After applying all these 4 steps, we get a filtered image that contains only text regions.

### III. MODELING OF THE MACHINE

With simplifying assumptions concerning the PMSM, the motor model expressed in the benchmark Park, in the form of state can be written [2] [3]:

$$\begin{cases} \dot{X} = A(x) + Bu \\ Y = C(x) \end{cases}$$

$$\begin{aligned} \text{With: } \dot{x}_1 &= \frac{d}{dt} i_d = -\frac{R}{L_d} i_d + \frac{L_q}{L_d} P \Omega i_q + \frac{u_d}{L_d} \\ \dot{x}_2 &= \frac{d}{dt} i_q = -\frac{R}{L_q} i_q - \frac{L_d}{L_q} P \Omega i_d - \frac{\phi f}{L_q} P \Omega + \frac{u_q}{L_q} \\ \dot{x}_3 &= \frac{d}{dt} \Omega = \frac{P}{J} [\phi f i_q + (L_d - L_q) i_d i_q] - \frac{f}{J} \Omega \end{aligned}$$

The system is rewritten as suggested linearization:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \\ \Omega \end{bmatrix} ; u = \begin{bmatrix} u_d \\ u_q \end{bmatrix} F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} a_1 x_1 + a_2 x_2 x_3 \\ b_1 x_2 + b_2 x_1 x_3 + b_3 x_3 \\ c_1 x_3 + c_2 x_1 x_2 + c_3 x_2 \end{bmatrix}$$

With:

$$a_1 = -\frac{R}{L_d} ; a_2 = \frac{L_q}{L_d} P ; b_1 = -\frac{R}{L_q} ; b_2 = -\frac{L_d}{L_q} P ; b_3 = -\frac{\phi f}{L_q} P ; c_1 = -\frac{f}{J} ; c_2 = \frac{P}{J} (L_d - L_q) ; c_3 = \frac{P}{J} \phi f$$

The following parameters of the PMSM are [6]:

$$L_d = 4\text{mH}; L_q = 2.8\text{mH}; \phi f = 0.12\text{wb}; J = 1.1 \times 10^{-3} \text{Kg.m}^2; F = 1.4 \times 10^{-3} \text{N.m.s/rad}; R = 0.6 \Omega; P = 4$$

This model is linearized about an operating point:

$$x_0 = \begin{bmatrix} i_{d0} \\ i_{q0} \\ \Omega_0 \end{bmatrix}; u_0 = \begin{bmatrix} u_{d0} \\ u_{q0} \end{bmatrix}$$

We can calculate the matrix of this system:

$$\begin{aligned} \dot{x} &= Ax + Bu + Gw_0 \\ y &= Cx + w \\ z &= \begin{bmatrix} Hx \\ u \end{bmatrix} \end{aligned}$$

Which gives us:

$$A = \begin{bmatrix} -\frac{R}{L_d} & \frac{L_q}{L_d} P \Omega_0 & \frac{L_q}{L_d} P i_{q0} \\ -\frac{L_d}{L_q} P \Omega_0 & -R/L_q & -\frac{L_d}{L_q} P i_d - \frac{\phi f P}{L_q} \\ \frac{P}{J} ((L_d - L_q) i_q & \frac{P}{J} (\phi f + (L_d - L_q) i_d) & -\frac{f}{J} \end{bmatrix}$$

$$B = \begin{bmatrix} 1/L_d & 0 \\ 0 & 1/L_q \\ 0 & 0 \end{bmatrix}; C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}; H = [0 \ 0 \ 1] G = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J} \end{bmatrix} C_r$$

Finally, the operating point and the vector control selected are:

$$\begin{bmatrix} i_{d0} \\ i_{q0} \\ \Omega_0 \end{bmatrix} = \begin{bmatrix} 0.16 \\ 0.16 \\ 55.75 \end{bmatrix}; \begin{bmatrix} u_{d0} \\ u_{q0} \end{bmatrix} = \begin{bmatrix} 0 \\ 27 \end{bmatrix}$$

1. Application of controls:

Application of state feedback control:

The choice of the gain matrix K based on the location of the poles that wants to impose the system, we take the following clusters: [-10,-0.200,-5275]

$$\text{What gives us: } K = \begin{bmatrix} -0.5985 & 0.7417 & 0.0095 \\ -0.8097 & 14.1901 & 0.4623 \end{bmatrix}$$

Application of H $\infty$  algorithm:

By applying the algorithm previously mentioned we come to find the values of X which we calculate the value of gains K, K<sub>d</sub>, and L<sub>i</sub> which are the synthesis parameters of the law of decentralized control.

$$\text{We have: } K = -B^T X \quad ; K_d = \frac{1}{\alpha^2} G^T X$$

In the decentralized case, L<sub>i</sub> obtained by an algorithmic aspect which solves the pseudo-Riccati equation.

$$\text{Decentralized case: } L_1 = \begin{bmatrix} 0.0010 \\ 0.0010 \\ -0.0027 \end{bmatrix}; L_2 = \begin{bmatrix} -0.0027 \\ -0.0036 \\ 0.0106 \end{bmatrix}$$

$$\text{In centralized case: } L = (I - \alpha^{-2} YX)^{-1} Y C^T$$

$$L = \begin{bmatrix} -0.0027 & 0.0010 \\ -0.0036 & 0.0010 \\ 0.0106 & -0.0027 \end{bmatrix}$$

## IV. SIMULATION

## A. SIMULATION WITH STATE FEEDBACK:

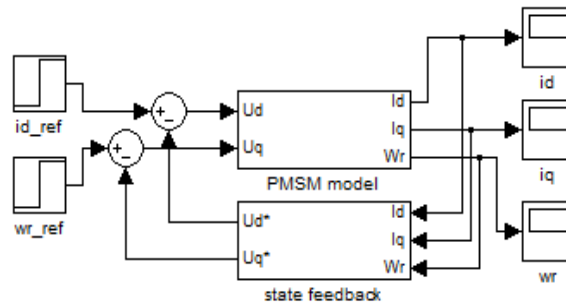
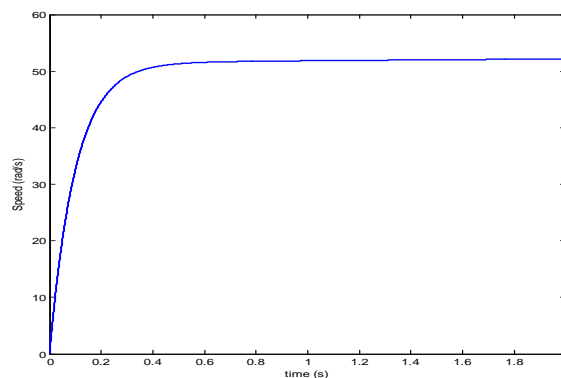


Fig.3:PMSMwith state feedback

Fig.4: speed  $\Omega$  without noise

The Figure 4 shows that the speed  $\Omega$  in the noiseless case tends to the reference value "55.75rad / s" but does not reach. Existence of a static error "3.75 rad /s." if you want to remedy this problem, you can use a pre-compensator gain.

With noise:

Applying a white noise on the measurement of the speed of the machine, it is clear that in the conventional regulator Fig.IV.3 pole placement loses its performance and the system becomes unstable.

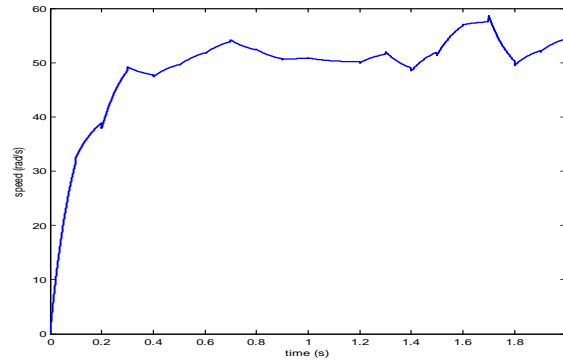


Fig.5: speed  $\Omega$  noisy

In the presence of disturbance classical state feedback control loses these performances.

**B. SIMULATION WITH CONTROLLER  $H_{\infty}$ :**

With noise:

We will present the simulation made of robust (decentralized/centralized) control with added noise and see its influence on the regulated output, which is the speed of the PMSM.

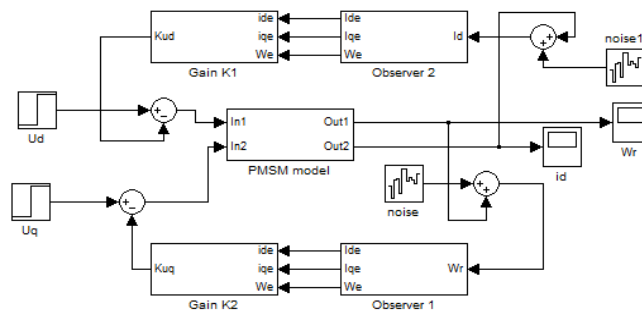


Fig.6: Representation of Model PMSM + Observer + Gain (decentralized case)

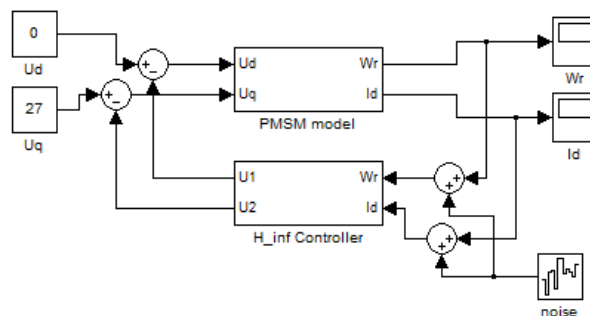
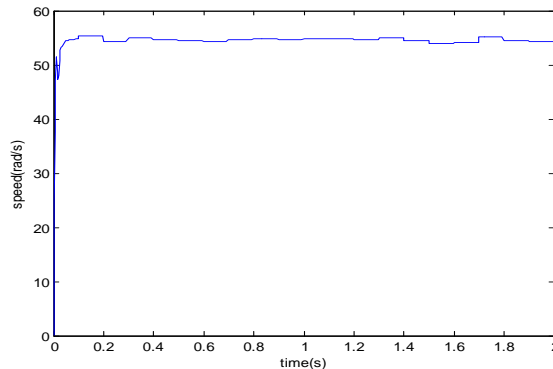


Fig.7: Representation of Model PMSM +  $H_{\infty}$  controller (centralized case)

Fig.8: speed  $\Omega$  noisy

The robust controller (decentralized/centralized) lessens the noise, and keeps the same performance which gives conclusive results

The differences between the decentralized case and centralized case can't be viewed, because of the lower order of the PMSM.

#### C. COMPARISON OF THE TWO COMMANDS:

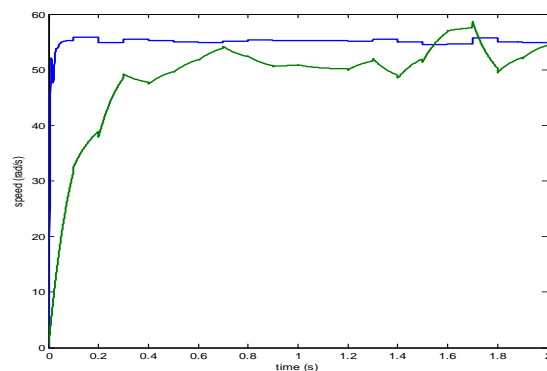


Fig.9: speed with noise power=0.01

In green the speed  $\Omega$  with the state feedback controller, in blue the speed  $\Omega$  with the robust  $H^\infty$  controller.

#### V.CONCLUSION

We presented in this article the application of the robust control of a permanent magnet synchronous machine with a salient rotor.

We compare the classical control state feedback with the  $H^\infty$  control, and notice their effects on the speed.

The robust  $H^\infty$  control clearly shows that it presents superior performances to the classical control state feedback that it is in performances or in the noises rejections.

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