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Series Solution of Multifluid Miscible Fluid Flow through Homogeneous Porous Media

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ABSTRACT: In this paper, the phenomenon of longitudinal dispersion in the flow of two miscible fluids through porous media is discussed. The phenomenon of longitudinal dispersion is the process by which miscible fluids in the laminar flow mix in the direction of the flow has been discussed. A well-ordered pattern of flow in which fluid motion occurs in layer. i.e. one layer sliding smoothly over the adjacent layer , without mixing with each other is called laminar flow. Any tendency towards instability is damped out by viscous forces that also resist relative motion of adjacent layers. Laminar flow is governed by Newton's law of viscosity. The hydrodynamic dispersion is the macroscopic outcome of the actual movements of individual tracer particles throughout the pores and various physical and chemical phenomenon that take place within the pores. This phenomenon simultaneously occurs due to molecular diffusion and convection.

KEYWORDS: Multifluid; Miscible; Porous Media.

I. INTRODUCTION

This phenomenon plays an important role in the sea water intrusion into reservoir at river mouths and in the underground recharge of waste water. Immiscible flooding, that is, the oil is displaced by one of the LPG (liquid petroleum gas) products, Ethane, Propane or Butane. If the reservoir conditions are such that the LPG is in the liquid phase then it is miscible with the oil and theoretically all residual oil can be recovered.
Multiphase flow is further classified as miscible or immiscible according as the flowing phases are completely mixing into each other without an identifiable interface or the phases are non-mixing with an identifiable interface.
In the miscible fluids flow the interfacial tension between two fluids is zero and there is no identifiable interface between them whereas in immiscible fluid flow, interfacial tension between the fluids is non-zero and there is a distinct (identifiable) interface between them.
e.g. Water and ink is miscible while oil and water is immiscible flow.
This problem has been discussed by several authors from different viewpoints, namely Scheidegger [1], Greenkorn [2], Schwartz [3] etc.
The governing equation of this case is partial differential equation. It is converted in Ordinary differential equation using similarity transformation and then solution is obtained by Power series method [4].

II. STATEMENT OF THE PROBLEM

Many Important problems in water resources engineering involve the mass-transport of a miscible fluid in a flow. A fluid is considered to be a continuous material and hence in addition to the velocity of a fluid element, the molecules in this element have random motion. As a result of the random motion, molecules of a certain material in high concentration at one point will spread with time. So the velocity considered here is time dependent. The net molecular motion from a point of higher concentration to one of lower concentration is called molecular diffusion.
Fluid flows in nature are usually turbulent, but we have considered the porous medium through which the fluid flows, to be homogeneous and for this reason, in the direction of flow, we assume laminar flow in which miscible fluids mix.
In moving through the random passages of the medium, two fluid elements adjacent to each other at one time will separate, as they may take different routes. The geometrical dispersion is coupled with molecular diffusion and dispersion due to no uniformity of the velocity across the cross-section of the passages. By considering the passage as randomly connected tubes, the dispersion in an isotropic medium can be described with a coefficient D for longitudinal Dispersion in the direction of seepage velocity.

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III.MATHEMATICAL FORMULATION OF THE PROBLEM

The equation of continuity for the mixture is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad (1)$$

Where ρ is the density for mixture and $\vec{v}$ is the pore (seepage) velocity (vector).
The equation of diffusion for a fluid flow through homogeneous porous medium with no addition or subtraction of the dispersing material, is given by

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\vec{v}) = \nabla \cdot \left[\rho \bar{D} \nabla \left(\frac{c}{\rho}\right)\right] \qquad (2)$$

Where c is the concentration of the fluid A in the other host fluid B (i.e. c is the mass of A per unit volume of the mixture), $\bar{D}$ is the tensor co-efficient of dispersion with nine components D_{ij} .
In a laminar flow through homogeneous porous medium at constant temperature, ρ may be considered to be constant. The equation (3.9.1) gives

$$\nabla \cdot \vec{v} = 0$$

And equation (2) becomes

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\vec{v}) = \nabla \cdot [\bar{D}\, \nabla c] \qquad (3)$$

When the seepage velocity $\vec{v}$ is along the X-axis, the non-zero components are $D_{11} = D_L$ and $D_{22} = D_T$ (coefficient of transverse dispersion), and other D_{ij} are zero. Thus equation (3), in this case, becomes,

$$\frac{\partial c}{\partial t} + u\,\frac{\partial c}{\partial x} = D_L\,\frac{\partial^2 c}{\partial x^2} \qquad (4)$$

Where u is the component of velocity along X-axis which is time dependent and D_L is the longitudinal dispersion co-efficient.

An appropriate initial and boundary conditions are :

$$c(x,0) = \gamma_0\,, 0 \le x \le L\;,\; c(0,t) = \gamma_1 \text{ and } c(L,t) = \gamma_2\; t > 0 \qquad (5)$$

Where γ_0 is the initial concentration of the tracer
γ_1 is the concentration at $x = 0$
γ_2 is the concentration at $x = \text{L}$

Since u is the cross-sectional time dependent flow velocity through porous medium, it is regarded as $-\frac{1}{\sqrt{t}}$ (where negative sign shows the decreasing of velocity in the direction of flow) for definiteness. Again by setting dimensionless variables $\text{X} = \frac{x}{L}$, the equation (4) together with (5) becomes

$$\frac{\partial c}{\partial t} - \frac{1}{L\sqrt{t}}\,\frac{\partial c}{\partial X} = \frac{D_L}{L^2}\,\frac{\partial^2 c}{\partial X^2} \qquad (6)$$

And $c(X,0) = \gamma_0\,, 0 \le X \le, c(0,t) = \gamma_1$ and $c(1,t) = \gamma_2, t\; > 0$ **(7)**

Now we use α similarity (Boltzmann) transformation [8] to convert equation (6) into an ordinary different equation. For that we consider

$$\eta = X^{\propto} t^{\beta} \qquad \textbf{(8)}$$

Where $\propto$ and β are to be determined so as the resulting equation in η will be free from X and t. Substituting (8) in (6) , we get

$$\beta X^2\;\frac{\eta}{t} c' - \frac{\propto}{L\sqrt{t}} \eta X c' = \frac{D_L}{L^2}[\propto(\propto -1)\eta c' + \propto^2 \eta^2 c''] \qquad (9)$$

Now the right hand side of equation (9) is free of X and t, thus for the full equation to be a function only of η. That is $\frac{X^2}{t} = f(\eta)$ which determines the values of $\propto$ and β.

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Here, we choose $f(\eta) = \eta^2$; so that $\propto = 1$ and $\beta = -\frac{1}{2}$. The transformation (8) is called the Boltzmann transformation, and under it equation (6) becomes

$$D_L \frac{d^2c}{d\eta^2} + \left[\frac{L^2}{2}\eta + L\right]\frac{dc}{d\eta} = 0, \quad \text{Where } D_L \neq 0 \tag{10}$$

Under the transformation (8), the first boundary condition becomes (since $t > 0$)

$$c(0) = \gamma_1 \tag{11}$$

And the initial and second auxiliary conditions are consolidate only if $\gamma_0 = \gamma_2$ and hence we have

$$c(1) = \gamma_2 \text{ , } c(L) = \gamma_2 \tag{12}$$

IV. MATHEMATICAL SOLUTION OF PROBLEM

Using series solution for (10)

$$\text{Let } c = \sum_0^\infty a_k \eta^k \text{ , } a_0 \neq 0 \tag{13}$$

Substituting (13) in (10) , we get

$$\sum_2^\infty D_L\, k(k-1)a_k\eta^{k-2} + \sum_1^\infty \left(\frac{L^2\eta}{2} + L\right) k a_k \eta^{k-1} = 0$$

$$\sum_2^\infty D_L\, k(k-1)a_k\eta^{k-2} + \sum_1^\infty \frac{L^2}{2} k a_k \eta^{k} + \sum_1^\infty L\, k a_k \eta^{k-1} = 0$$

$$\sum_2^\infty D_L\, k(k-1)a_k\eta^{k-2} + \sum_3^\infty \frac{L^2}{2}(k-2)\, a_{k-2}\eta^{k-2} + \sum_2^\infty L\,(k-1)a_{k-1}\eta^{k-2} = 0$$

$$2a_2 D_L + La_1 + \sum_3^\infty \begin{bmatrix} D_L\, k(k-1)a_k + \frac{L^2}{2}(k-2)a_{k-2} \\ + L(k-1)a_{k-1} \end{bmatrix} \eta^{k-2} = 0$$

Comparing co-efficients of both sides , we get ,

$$2a_2 D_L + La_1 = 0 \Rightarrow a_2 = -\frac{La_1}{2D_L} \tag{14}$$

$$a_k = -\frac{L}{D_L k} a_{k-1} - \frac{L^2}{2D_L k(k-1)} a_{k-2} \text{ , } k \geq 3 \tag{15}$$

$$k = 3, a_3 = \frac{L^2}{6D_L}\left[\frac{1}{D_L} - \frac{1}{2}\right] a_1 \tag{16}$$

$$k = 4, a_4 = -\frac{L^3}{24D_L}\left[\frac{1}{D_L} - \frac{3}{2}\right] a_1 \tag{17}$$

Substituting (14), (16) and (17) in (13) we get

$$c = a_0 + a_1\left[\eta - \frac{L}{2D_L}\eta^2 + \frac{L^2}{6D_L}\left(\frac{1}{D_L} - \frac{1}{2}\right)\eta^3 - \frac{L^3}{24D_L}\left(\frac{1}{D_L} - \frac{3}{2}\right)\eta^4 + \dots\dots\right] \tag{18}$$

$$c(0) = \gamma_1 \Rightarrow a_0 = \gamma_1,$$

$$c(1) = \gamma_2 \Rightarrow a_1 = \frac{\gamma_2 - \gamma_1}{y}$$

$$\text{Where } y = \left[1 - \frac{L}{2D_L} + \frac{L^2}{6D_L}\left(\frac{1}{D_L} - \frac{1}{2}\right) - \frac{L^3}{24D_L}\left(\frac{1}{D_L} - \frac{3}{2}\right) + \dots\dots\dots.\right] \neq 0$$

From (18), we get

$$c = \gamma_1 + \frac{\gamma_2 - \gamma_1}{y}\left[\frac{x}{L\sqrt{t}} - \frac{L}{2D_L}\frac{x^2}{L^2 t} + \frac{L^2}{6D_L}\left(\frac{1}{D_L} - \frac{1}{2}\right)\frac{x^3}{L^3 t^{3/2}} - \frac{L^3}{24D_L}\left(\frac{1}{D_L} - \frac{3}{2}\right)\frac{x^4}{L^4 t^2} \dots\dots\dots.\right] \quad \left[\because \eta = \frac{x}{L\sqrt{t}}\right]$$

Therefore, we have,

$$c = \gamma_1 + \frac{\gamma_2 - \gamma_1}{yL}\left[\frac{x}{\sqrt{t}} - \frac{x^2}{2D_L t} + \left(\frac{1}{D_L} - \frac{1}{2}\right)\frac{x^3}{6D_L t^{3/2}} - \left(\frac{1}{D_L} - \frac{3}{2}\right)\frac{x^4}{24D_L t^2} + \dots\dots\dots.\right] \tag{19}$$

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V. CONCLUSION

Equation (19) is a series solution of the problem (4) together with the conditions (5) .

(a) As time (t) tends to ∞ , concentration (c) tends to γ_1

i.e. after long time concentration reach to concentration at the boundary $x = 0$.

(b) As x tends to 0 , c tends to γ_1

(c) As x tends to L, c tends to $\gamma_2 (= \gamma_0)$. (choose t=1)

From (a),(b), (c) we can conclude that the obtained solution satisfies all given conditions. So we can conclude that the solution obtained here is suitable to our problem.

VI. SCOPE OF THE PROBLEM

When sewage is discharged into a body of water, it is important to certain how the sewage is dispersed in the receiving water. To determine the rate of evaporation from the surface of a reservoir, it is necessary to know the rate at which water vapor near this surface is carried into the air above.

This type of problem also has an importance in spreading of contaminant in a canal, evaporation of water vapour from water surface, contaminant of an aquifer.

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