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Numerical Study of Steady MHD Plane Poiseuille Flow and Heat Transfer in an Inclined Channel

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ABSTRACT: In this paper, a numerical study of the Poiseuille flow of an electrically conducting fluid between two inclined parallel plates is presented. The plates are non-conducting as well as isothermal. The viscous and Ohmic dissipation terms are taken into account in the energy equation. The coupled nonlinear equations of momentum and energy are solved numerically using finite difference scheme. The effects of various parameters such as Hartmann number, Grashof number, Prandtl number, Eckert number and angle of inclination on the velocity and temperature distributions are discussed numerically and presented through graphs.

KEYWORDS: Poiseuille flow, viscous dissipation, Ohmic dissipation, finite difference technique, inclined channel.

I. INTRODUCTION

The study of magnetohydrodynamics (MHD) flow and heat transfer in channel has been a topic of great interest for many researchers due to its important applications in many engineering devices concerning plasma physics, geophysics, petroleum industries, MHD pump, MHD flowmeter, cooling of nuclear reactors and nuclear fusion technology. In 1937, Hartmann [1] carried out the pioneering work in the study of steady MHD channel flow of an electrically conducting fluid under a uniform magnetic field tranverse to insulated channel wall. Alireza and Shai [2] investigated the effect of temperature-dependent transport properties on the developing MHD flow and heat transfer in parallel channel whose walls are held at constant and equal temperature. Siegel [3], Perlmutter and Siegel [4], Alpher [5] and Osterle and Young [6] studied heat transfer analysis in Hartmann problem for free and forced convective flow to an electrically conducting fluid in a channel. Chutia and Deka [7] presented a numerical study on steady laminar MHD flow and heat transfer of an electrically conducting fluid in a rectangular duct in the presence of oblique transverse magnetic field.
Hartmann flow in a horizontal channel cited above cannot be used to understand the flow characters if the horizontal pipe or channel is tilted because in this case due to earth's gravitation, there will be an external body force on the flow. Basically, inclined geometry has enormous applications in heat transfer technology like solar collector [8]. Taking into account these facts, Malashetty and Umavathi [9] studied two phase fluid flow and heat transfer in an inclined channel. Later on, Umavathi et al. [10] investigated the Poiseuille-Couette flow of two immiscible fluids between inclined parallel plates, where one of the fluids was assumed to be electrically conducting while the other fluid and channel walls were assumed to be electrically insulating. The viscous and Ohmic dissipation terms were taken into account in the energy equation. Sharma and Singh [11] investigated numerically transient free convective flow of a viscous incompressible electrically conducting fluid along an inclined isothermal non-conducting plate in the presence of transverse magnetic field with viscous and Ohmic dissipations. Goyal and Kumari [12] studied the free convection heat and mass transfer MHD oscillatory flow of visco-elastic fluid between two inclined porous plates in the presence of radiation absorption, chemical reaction and thermal radiation. Daniel and Daniel [13] studied convective flow of two immiscible fluids and heat transfer with porous along an inclined channel with pressure gradient. Singh and Singh [14] studied MHD flow heat transfer of a dusty visco-elastic liquid down an inclined channel in porous medium. Badari Narayana et al. [15] studied numerically steady flow of a Jeffrey fluid in an inclined two-dimensional channel by finite difference analysis. Oulaid et al. [16] presented a numerical study of simultaneous heat and mass transfer with phase change in an inclined channel formed by two parallel plates. Mekheimer [17] investigated the nonlinear

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peristaltic transport of a magnetic field in an inclined planar channel. Krishna Kumari et al. [18] studied the peristaltic pumping of a Casson fluid in an inclined channel under the effect of a magnetic field.
Aim of the present paper is to investigate the Poiseuille flow of an electrically conducting fluid between two isothermal non-conducting inclined parallel plates in the presence of transverse magnetic field with viscous dissipation and Ohmic dissipation. The governing equations of momentum and energy are solved using finite difference scheme. The velocity and temperature distributions are discussed numerically and presented in terms graphics for various values of physical parameters.

II. MATHEMATICAL MODEL

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid between two infinite inclined parallel plates extending in the z and x-directions, making an angle ϕ with the horizontal direction. The two plates are non-conducting and maintained at different constant temperatures T_0 and T_1. The x-axis is taken parallel along the axis of the channel and y-axis is normal to the plates. A magnetic field of uniform intensity B_0 is applied in y-direction which is normal to the plates. The flow is assumed to be unidirectional, steady, laminar and fully developed. It is assumed that the magnetic Reynolds number is sufficiently small so that the induced magnetic field can be neglected, and the induced electric field is assumed to be negligible. It is also assumed that the electrical field due to polarization of charges and Hall Effect are neglected. The Oberbeck-Boussinesq approximation is employed for density variation and the flow is assumed to be driven by the constant pressure gradient $(-\partial p/\partial x)$. Under these assumptions, the governing equations of momentum and energy are given below (Umavathi et al. [10]) :

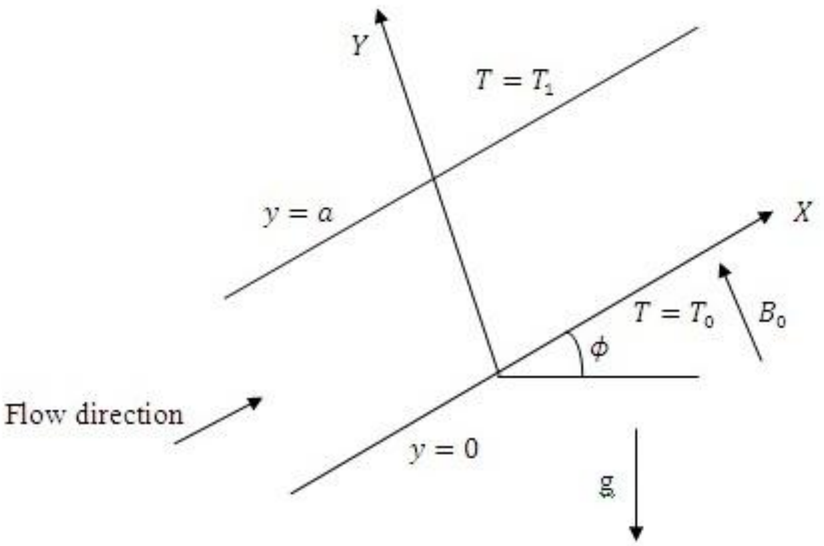

Fig. 1: The physical configuration of the problem

$$\mu \frac{d^2u}{dy^2} + \rho g \beta (T - T_0) Sin\phi - \sigma B_0^2 u = \frac{dp}{dx} \quad (1)$$

$$k \frac{d^2T}{dy^2} + \mu \left(\frac{du}{dx}\right)^2 + \sigma B_0^2 u^2 = 0 \quad (2)$$

where u is the x-component of velocity, T is the fluid temperature, μ is the coefficient of viscosity, g is the acceleration due to gravity, β is the thermal expansion coefficient, σ is the electrical conductivity and k is the thermal conductivity.

The boundary conditions on velocity and temperature are

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$$\left.\begin{array}{l} u = 0, T = T_0 \text{ at } y = 0 \\ u = 0, T = T_1 \text{ at } y = a \end{array}\right\} \quad (3)$$

Eqs. (1) and (2) along with boundary conditions (3) are made dimensionless by using the following transformations:

$$u^* = \frac{u}{u_0},\ y^* = \frac{y}{a},\ T^* = \frac{T - T_0}{T_1 - T_0} \quad (4)$$

Where $u_0 = -\frac{a^2}{\mu}\frac{dp}{dx}$

Using Eq. (4) into Eqs. (1) and (2), and removing asterisks, the non-dimensional governing equations become

$$\frac{d^2u}{dy^2} + \frac{Gr}{Re}(Sin\phi)T - M^2u + 1 = 0 \quad (5)$$

$$\frac{d^2T}{dy^2} + EcPr\left(\frac{du}{dy}\right)^2 + EcPrM^2u^2 = 0 \quad (6)$$

Where

$Gr = \frac{g\beta a^3(T_1 - T_0)}{\upsilon^2}$, $Re = \frac{u_0 a}{\upsilon}$, $M = B_0 a\left(\frac{\sigma}{\mu}\right)^{1/2}$, $Ec = \frac{u_0^2}{C_p(T_1 - T_0)}$ and $Pr = \frac{\mu C_p}{k}$ are the Grashof number, the Reynolds number, the Hartmann number, the Eckert number and the Prandtl number, respectively.

The corresponding boundary conditions (3) on velocity and temperature become

$$\left.\begin{array}{l} u = 0, T = 0 \text{ at } y = 0 \\ u = 0, T = 1 \text{ at } y = 1 \end{array}\right\} \quad (7)$$

III. NUMERICAL SOLUTIONS

The governing differential Eqs. (5) and (6) are to be solved subject to the boundary conditions (7) for the velocity and temperature distributions. These equations are coupled and nonlinear because of the inclusion of the viscous and Ohmic dissipation terms in the energy equation. Hence the closed form solutions are difficult to obtain. Here, an attempt has been made to solve these equations numerically employing finite difference technique. Replacing the derivatives with corresponding central difference approximations of second order accuracy, we obtain finite difference equations of Eqs. (5) and (6) as following :

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{2h} + \frac{Gr}{Re}(Sin\phi)T_i - M^2u_i + 1 = 0 \quad (8)$$

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{2h} + EcPr\left(\frac{u_{i+1} - u_{i-1}}{2h}\right)^2 + EcPrM^2u_i^2 = 0 \quad (9)$$

Simplifying Eqs. (8) and (9) for velocity and temperature, we obtain

$$u_i = C_5(u_{i+1} + u_{i-1}) + C_6T_i + C_7 \quad (10)$$

$$T_i = 0.5(T_{i+1} + T_{i-1}) + C_8(u_{i+1} + u_{i-1}) + C_9u_i^2 \quad (11)$$

Where

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$C_1 = \frac{Gr}{Re} Sin\phi$, $C_2 = M^2$, $C_3 = EcPr$, $C_4 = C_2C_3$, $C_5 = \frac{1}{2+h^2C_2}$, $C_6 = \frac{h^2C_1}{2+h^2C_2}$, $C_7 = \frac{h^2}{2+h^2C_2}$, $C_8 = \frac{C_4}{4}$ and $C_9 = \frac{h^2C_5}{8}$ are constants.

The discretized boundary conditions on velocity and temperature are

$$\left.\begin{array}{l} u_i = 0, \quad T_i = 0 \quad \text{at } i = 1 \\ u_i = 0, T_i = 1 \text{ at } i = m+1 \end{array}\right\} \tag{12}$$

where index i refers to y and m denotes the number of grids inside the computational domain in the direction of Y.

IV. RESULTS AND DISCUSSION

The Poiseuille flow of an electrically conducting fluid between two inclined parallel plates in the presence of transverse magnetic field is investigated numerically by developing finite difference codes in Matlab programming. Computational domain is divided into 100 uniform grids. The computed values of u_i and T_i appearing in the discretized Eqs. (10) and (11) subject to the boundary conditions (12) with selecting non-dimensional parameters M, Gr, Re, Ec, ϕ and Pr have been iterated to a suitable number so that the convergent solutions of u_i and T_i are considered to be achieved when the maximum differences between two successive iterations are less than a tolerance, 10^{-7}. Numerical results for velocity and temperature distributions are presented graphically by using the software package "Matlab R2008b" in Figs. 2 to 9.

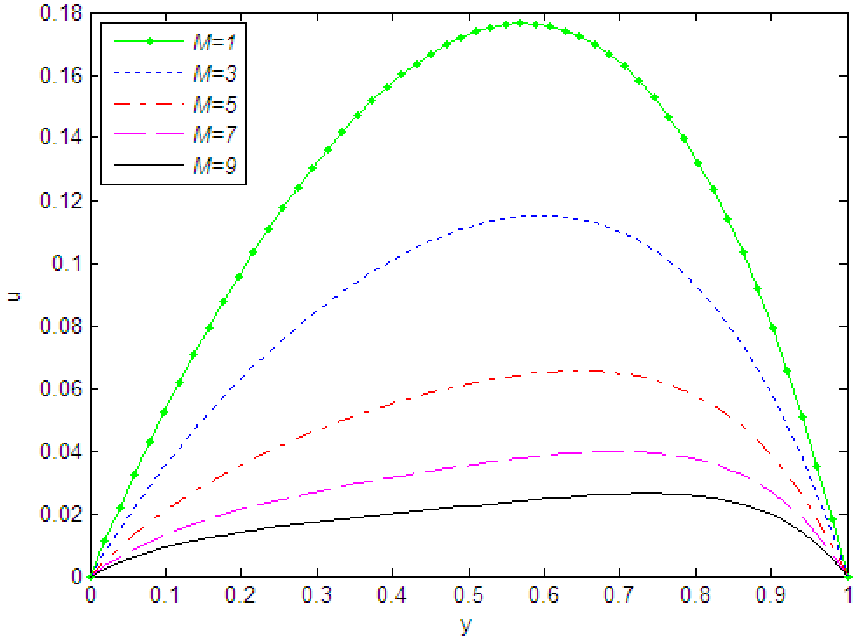

Fig. 2: Velocity distributions at various Hartmann number M for fixed Re = 1, Gr = 3, ϕ = 45^0, Ec = 0.1 and Pr = 0.71

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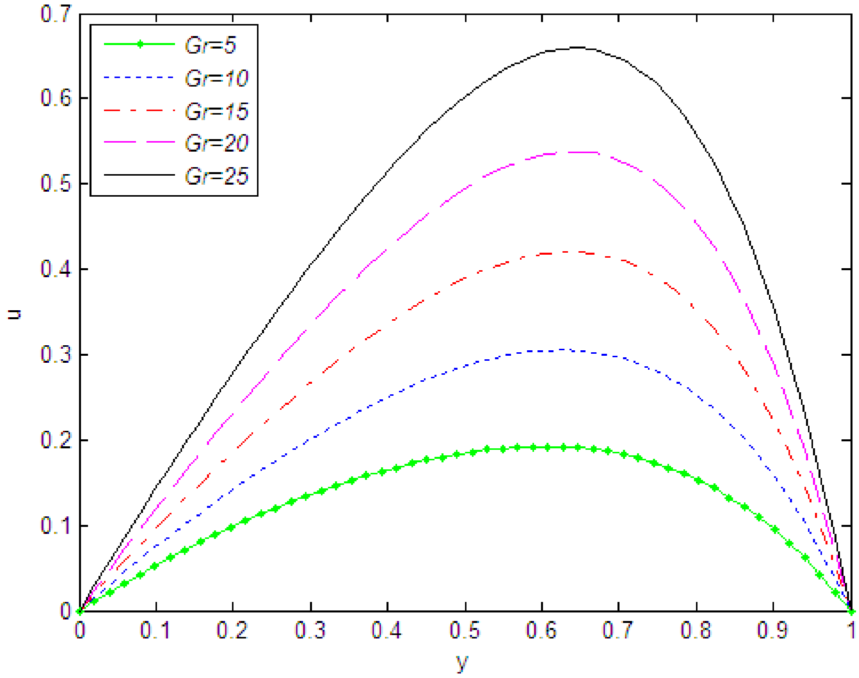

Fig. 3: Velocity distributions at various Grashof number Gr for fixed Re = 1, M = 2, $\phi = 45^0$, Ec = 0.1 and Pr = 0.71

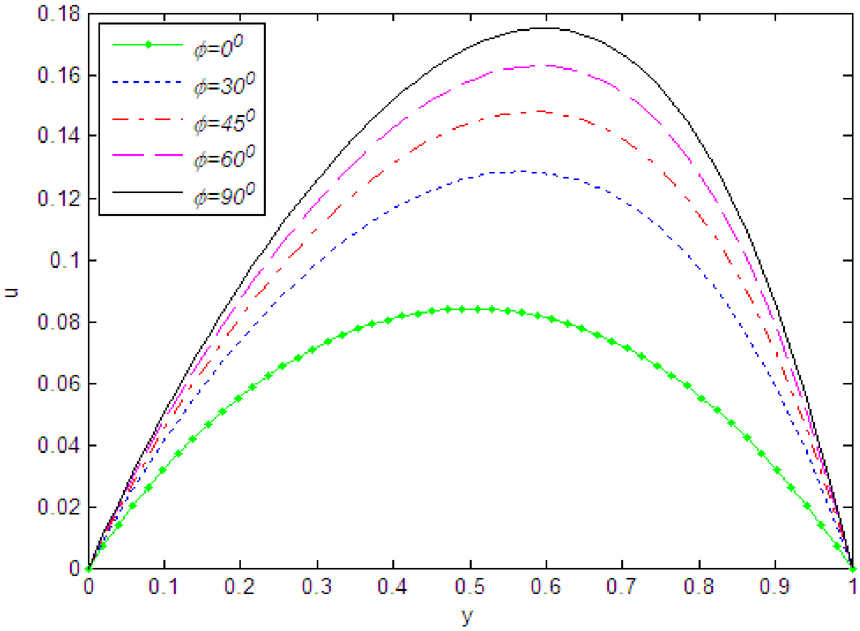

Fig. 4: Velocity distributions at various inclination angles ϕ for fixed Re = 1, Gr = 3, M = 2, Ec = 0.1 and Pr = 0.71

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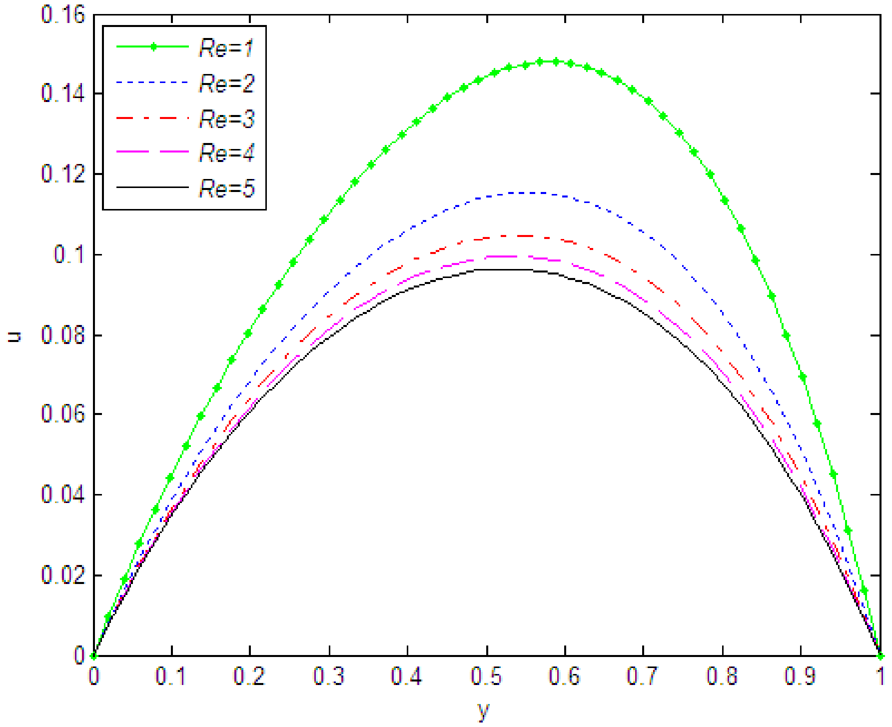

Fig. 5: Velocity distributions at various Reynolds number Re for fixed Gr = 3, M = 2, $\phi = 45^0$, Ec = 0.1 and Pr = 0.71

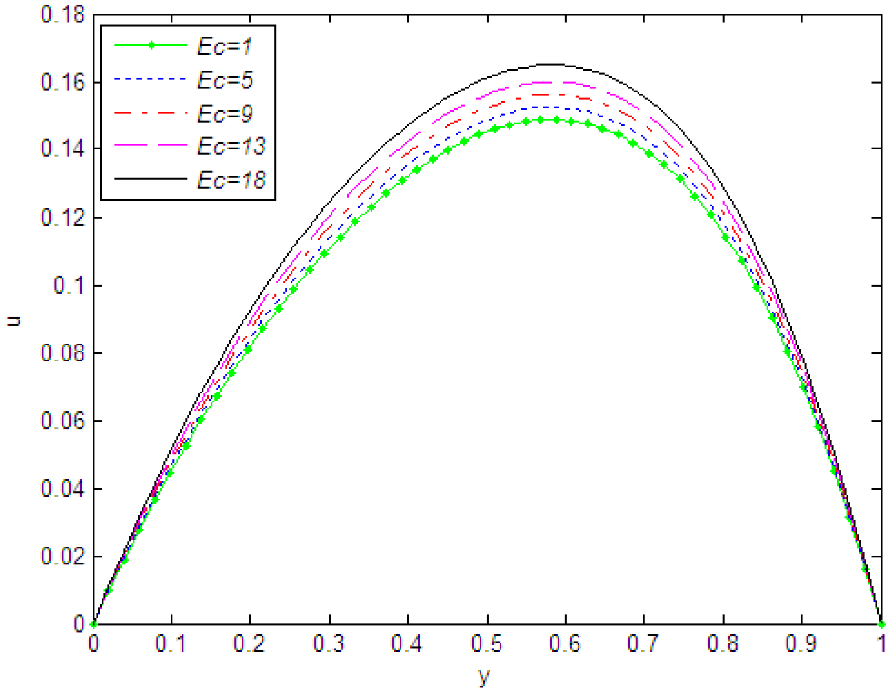

Fig. 6: Velocity distributions at various Eckert number Ec for fixed Re = 1, Gr = 3, M = 2, $\phi = 45^0$ and Pr = 0.71

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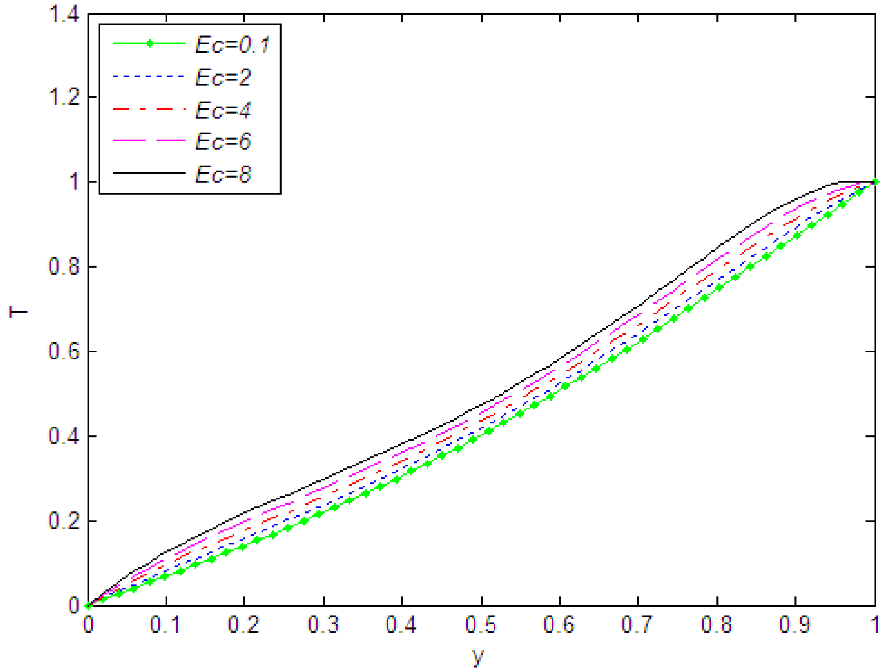

Fig. 7: Temperature distributions at various Eckert number Ec for fixed, Re = 1, Gr = 3, M = 2, $\phi = 45^0$ and Pr = 0.71

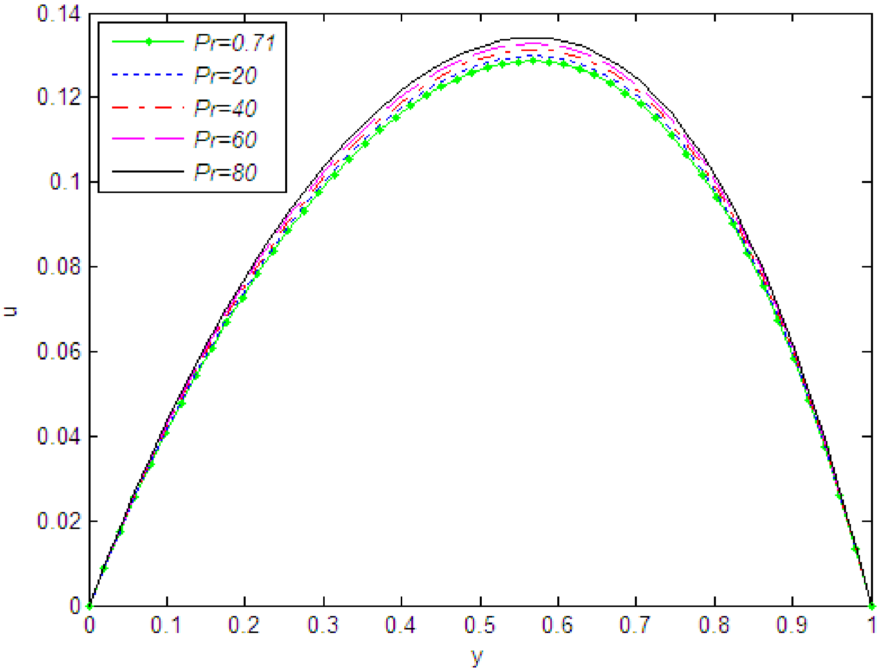

Fig. 8: Velocity distributions at various Prandtl number Pr for fixed, Re = 1, Gr = 3, M = 2, $\phi = 45^0$ and Ec = 0.1

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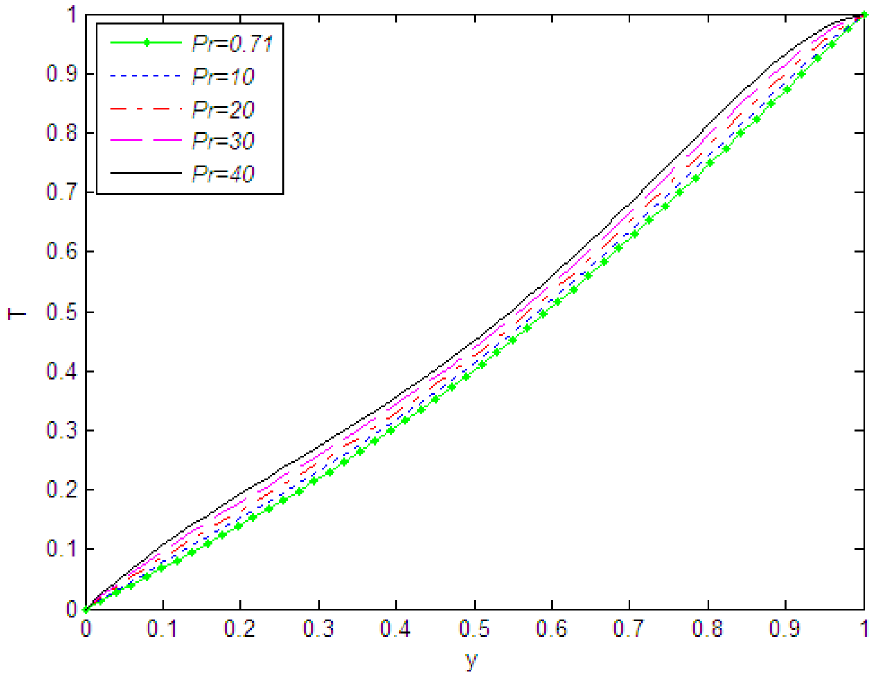

Fig. 9: Temperature distributions at various Prandtl number Pr for fixed $\mathrm{Re} = 1$, $\mathrm{Gr} = 3$, $\mathrm{M} = 2$, $\phi = 45^0$ and $\mathrm{Ec} = 0.1$

In Fig. 2, the effect of Hartmann number on velocity distribution is shown. It is evident from this figure that velocity distribution decreases for increasing values of Hartmann number M. Increasing value of the Hartmann number has a tendency to retard the fluid moving forward, this is due to fact that the Hartmann number represents the ratio of the Lorentz force to the viscous force, implying that the larger the Hartmann number, the stronger the retarding effect on fluid velocity. It reduces the volumetric flow rate in the channel and the wall friction.
The effect of Grashof number on the velocity distribution is shown in Fig. 3. It is noticed that velocity distribution increases for increasing values of Grashof number Gr. Physically, an increase in the value of Grashof number indicates an increase of buoyancy forces which support the flow.
The effect of inclination angle ϕ on velocity is depicted in Fig. 4; it is observed that the increase in the value of ϕ increases the fluid velocity. This is due to the fact that magnitude of the driving force increases with the increase in inclination angle.
Fig. 5 shows the effect of Reynolds number Re on velocity. Velocity is found to be decreases gradually with the increasing value of Re.
We infer from this study that the maximum velocity does not occur in the middle of the channel but moves towards the upper wall as the value of Gr and ϕ increase.
In Figs. 6 and 7 the effects of Eckert number on velocity and temperature are presented respectively. Both the velocity and temperature increases with an increase in Ec. The increase of temperature resulting from the increasing dissipation effect due to large Ec, and as a consequence the velocity increases for the increasing buoyancy force in the momentum equation.
The effects of Prandtl number on velocity and temperature are depicted in Figs. 8 and 9 respectively, both velocity and temperature increases as Prandtl number Pr increases.

V. CONCLUSION

A numerical investigation has been performed for steady Poiseuille flow of an electrically conducting fluid between two isothermal non-conducting inclined parallel plates in the presence of transverse magnetic field with viscous dissipation and Ohmic dissipation. The governing equations of momentum and energy are solved using finite difference scheme. We conclude from this investigation that fluid velocity decreases for increasing values of Hartmann M and

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Reynolds number Re; and increases for increasing values of Grashof number Gr, Eckert number Ec, Prandtl number Pr and angle of inclination ϕ. It is also observed that velocity profile is not parabolic and moves towards the upper heated wall as the values of Gr, Ec, Pr and ϕ increases. Whereas temperature field increases from lower plate to upper plate as Ec and Pr increases.

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