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Mathematical Modeling of EPS System

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ABSTRACT:The purpose of this research is to drive the mathematical equation for electric power steering (EPS) control logic which integrates base-assist, damping, return, and inertia control logic. In this we have studied the electric power steering system for a front wheel- steered, rear-wheel driven four wheels vehicles. Forms of the electric power steering system according to the location of assist motor are discussed, one of this forms is used to build mathematical model. We have derived the mathematical model of column assisted front wheel steering on the basis of Newton's laws of motion. We designed a controller for our model with the help of assist characteristics curves and PID action. We have carried out simulation for different types of vehicle path which include circular and moose test path. We choose two different cases of steering and for each case we have carried out simulation by considering the two paths states.

KEYWORDS: Electric power steering, Characteristics curve and PID action, simulation technique, circular and moose test path

I. INTRODUCTION

A steering system is a significant subsystem for a vehicle operation. Stiffness of steering shaft is very important parameter in designing an electric power steering system as it affects returnability considerable. Returnability means ability of the steering wheel to come its center position after completing a turn. When the steering wheel is turned and then released during cornering, it returns to the center position by the torque exerted on the tires by road, which is called as self-aligning torque. This self-aligning torque helps in returnability. The electric power steering system according to the location of assist motor are discussed, one of this forms is used to build mathematical model. Finally operating model of the electric power steering is explained.Building a mathematical model is very necessary to study the effect of stiffness damping and moment of inertia of steering column and assist motor on driver's torque.

II. MATHEMATICAL EQUASION FOR EPS

A. MODEL ASSUMPTION

If all the masses and moment of inertia of various spring and damping element that appear in the system are considered in relation to their interaction with then system will become complex. Some assumptions are made to simplify the system [1].

1. All mechanical connections are rigid
2. All elements behave like a spring mass damper system
3. The EPS system is composed of three basic elements: steering column, steering rack, and assist motor
4. Neglect the masses of tie-rods and tire

B. MATHEMATICAL EQUASION

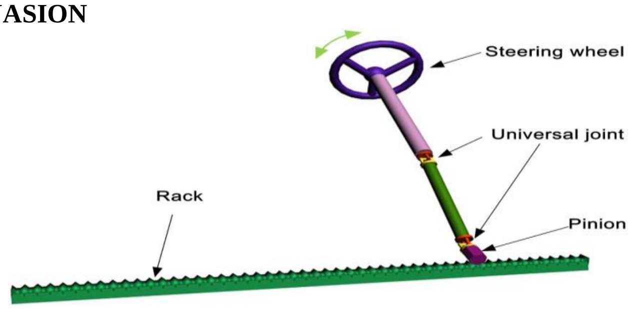

Fig.1 Manual steering configuration[2]

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Vol. 3, Issue 9 , September 2016

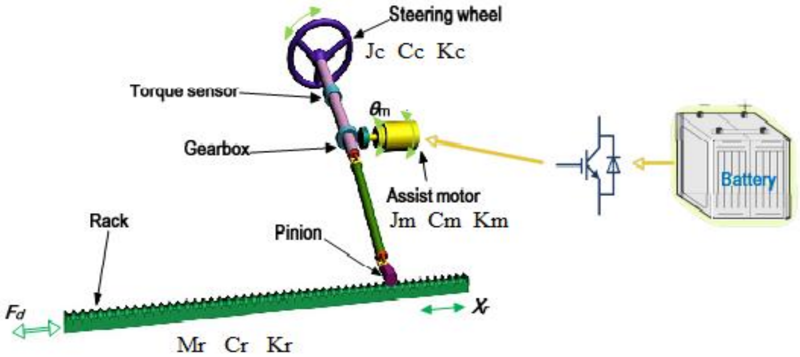

Fig .2 EPS steering model[2]

Using newton's laws of motion and with the help of assumption made the equation of EPS can be derived. Consideration the moment of the steering wheel and the viscous damping of input axle, we get,

$$J_c\ddot{\theta}_c + C_c\,\dot{\theta}_c = T_d + T_{sen} \qquad 1$$

Where,T_d is the driver torque or steering torque working on input wheel, T_{sen} is the anti-torque working on the torsion bar.

$$T_{sen} = K_C(\theta_C - \frac{X_r}{R_p}) \qquad 2$$

The motor for is a permanent magnetic field DC motor. The relationship among voltage V across it, inductance L, armature resistance R, back electromotive force K_b , motor speed $\frac{d\theta_m}{dt}$,current I and time t is,

$$V_m = \text{LI} + \text{RI} + K_b\,\dot{\theta}_m \qquad 3$$

Torque generated by the motor

$$T_m = K_a I \qquad 4$$

Where, K_a is torque co-efficient of motor,

To analyze the force working on the mechanism for the motor, an equation can be obtain as follows,

$$J_m\ddot{\theta}_m + C_m\,\dot{\theta}_m = T_m - T_a \qquad 5$$

Where T_a is assist torque and it is given by,

$$T_a = K_m(\theta_m - G\frac{X_r}{R_p}) \qquad 6$$

Where, G is motor gear ratio. When a vehicle steers, steering resistance is primarily influenced by the friction created by the tire and the surface of ground and various and other friction, it is given by,

$$F_{TR} = K_r X_r + F_d \qquad 7$$

Where F_d is the random signal from the road and it is proportional to rack displacement.

ISSN: 2350-0328
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Combining equation (1) to (7) we get

$$J_c\ddot{\theta}_c + C_c\dot{\theta}_c + K_c\theta_c = T_d + K_c\frac{X_r}{R_p} \qquad 8$$

$$J_m\ddot{\theta}_m + C_m\dot{\theta}_m + K_m\theta_m = T_m + K_m(G\frac{X_r}{R_p}) \qquad 9$$

$$M_r\ddot{X}_r + C_r\dot{X}_r + K_rX_r = \frac{K_r}{R_p}\left(\theta_c - \frac{X_r}{R_p}\right) + G\frac{K_m}{R_p}(\theta_m - G\frac{X_r}{R_p}) + F_d \qquad 10$$

This is a simple mathematical model of electric power steering. In (8), (9) and (10),J_c, C_c, K_c,T_d are steering column and steering wheel rotational moment of inertia, viscous damping coefficient of steering column, torsion bar stiffness, driver torque respectively, of steering column,J_m, C_m, K_m, θ_m,T_m, are motor rotational moment of inertia, viscous damping coefficient of motor shaft, motor shaft stiffness, motor rotation angle, motor torque respectively ,of motor is gear ratio, T_d is driver torque, T_a is assist torque, and L is Motor induction, I is motor current, V_m is motor terminal voltage, R is Armature resistance, V is Armature resistance.

III. ASSIST CHARACTERISTIC CURVES

The major object of EPS system is to provide a proper amount of assist torque by the electric motor to reduce the effort of the driver when the vehicle is cornering. For generating a suitable torque output from the motor to assist the driver, assist characteristic curves are needed. According to assist curves, the core controller of the EPS system receives signals form the torque sensor and vehicle speed sensor and calculates a suitable amount of assist gain to output to motor controller. Therefore, the motor can provide a proper torque to assist the driver to turn the steering wheel [4].

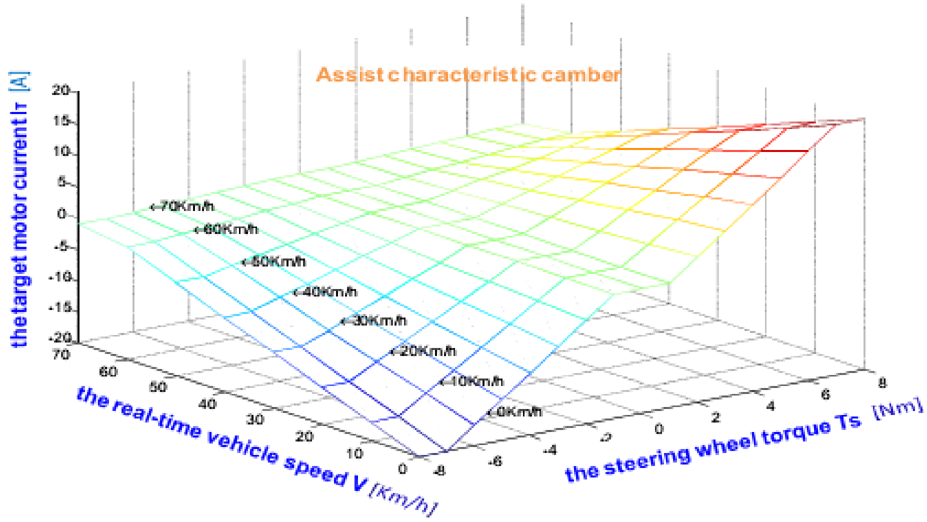

Fig. 3Assist characteristic curves of EPS[4]

The power-assisted characteristic of EPS has three typical kinds of characteristic curves; the straight line type, the broken line type, and the curve type. Each characteristic curves can be divided into three areas; the no power assisted area, the changing area of power-assisted and the fixed area. In comparison to the three kinds of power-assisted characteristic curves, the straight-line characteristic curve is the simplest, which is convenient to design and adjust the control system easily. The curvilinear power-assisted characteristic curve is so complicated.

ISSN: 2350-0328
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Vol. 3, Issue 9 , September 2016

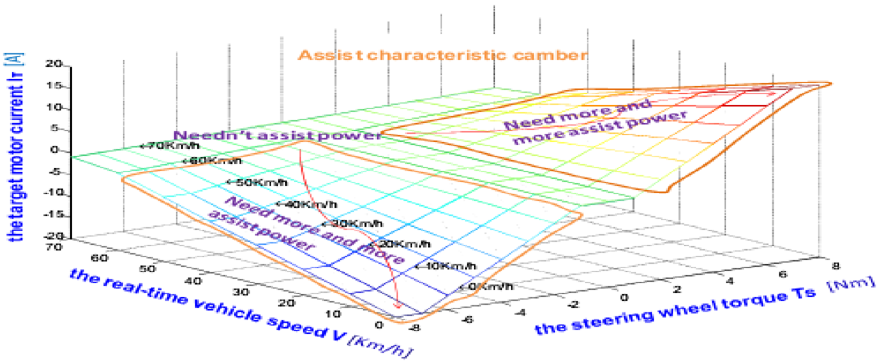

Fig .4Explanation of Assist characteristic curve[5]

The target motor current which is determined by the assist characteristic based on the input of torque sensor and vehicle speed sensor. The relationship of the real-time vehicle speed is derived from vehicle speed sensor and the steering wheel torque is expressed by the following function$\boldsymbol{I_T} = \boldsymbol{f}(\boldsymbol{V}, \boldsymbol{T_S})$. By using the Lookup Table (2-D) block in MATLAB/Simulink, the assist characteristic can be realized as shown in Fig 4 [5]

Fig.5shows the torque flow of the steering system. The driver's torque (T_d) actuates the motor which sends the assist torque to displace the rack. Output y is the actual torque acting on the rack to displace it which should be identical to driver torque.

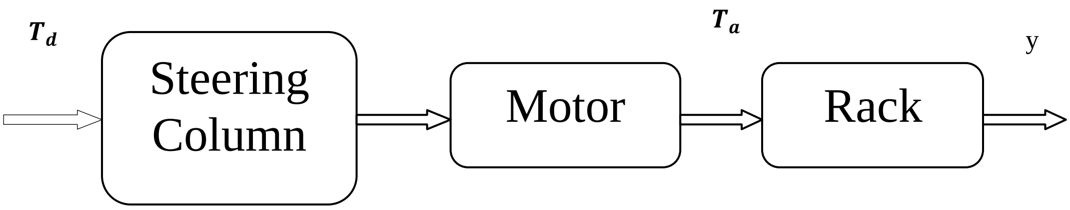

Fig .5 Torque flows of EPAS [3]

IV. SIMULATION RESULTS

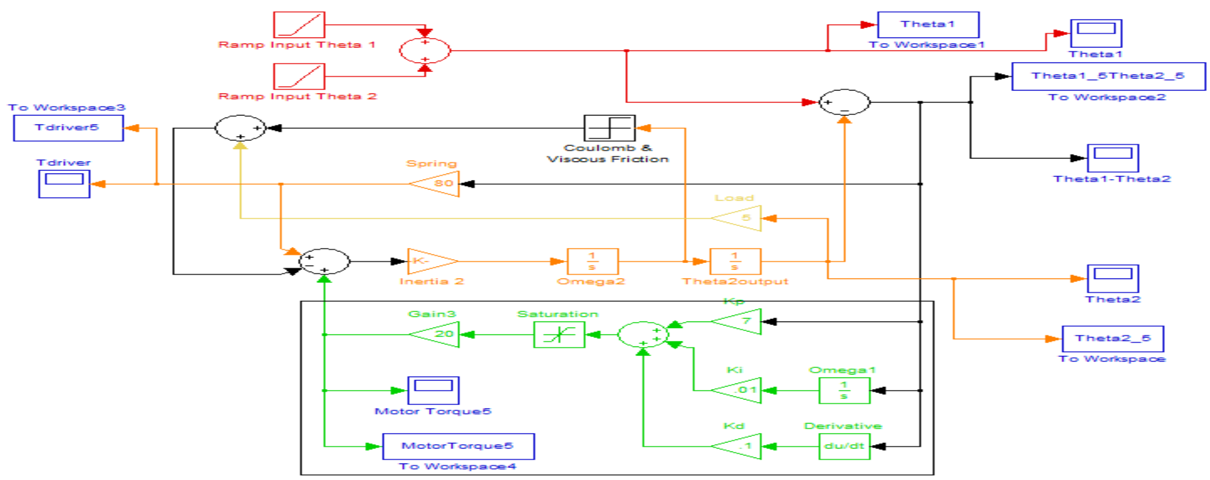

Fig .6 Simulink control block for EPS

Simulations were performed in Matlab /Simulink in order to evaluate assist characteristic of EPS and torque sensor responses. The simulations were carried out for the vehicle following a particular path. Two type of path were

ISSN: 2350-0328
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considered i.e. (1) circular path and (2) moose test path. Also simulations were carried to observe, how assist is vary with load of the vehicle. PID controller is used with different values of K_p, K_i, K_d. The controller takes signals from torque and speed sensor. According to the signal input, controller decides how much torque is required, according to that it sends the current signal to the assist motor as shown in fig 6.

A. TYPE OF VEHICLE PATH CONSIDERED FOR SIMULATION

Let us consider two types of path followed by a vehicle for simulations, to verify the proposed assist logic for electrical power steering control.

B.CIRCULAR PATH

The desired vehicle path is circular as shown in fig 8. In this case vehicle enters a circle first and then follows a circular path with a constant steering wheel angle as shown in fig 9.

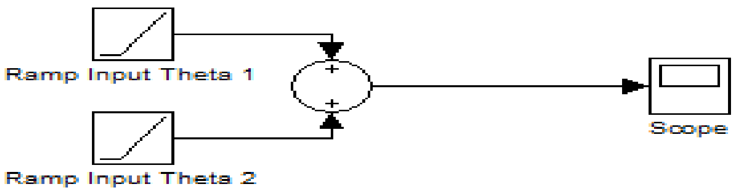
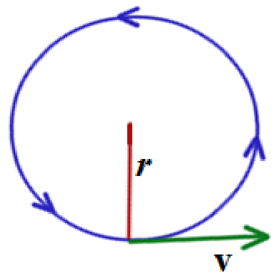


Fig 7.simulink block for circular path



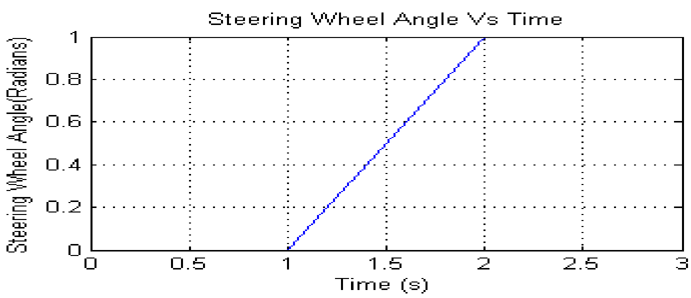


Fig .8Circular path

Fig .9 steering wheel angle input for circular path

C. MOOSE TEST PATH

Moose test is used to test how much a certain vehicle acts when avoiding a certain danger e.g. goat, dog and cow. It is also called as elk test .this test is performed on dry road surface. In this test traffic cones are set up in S-shape to simulated obstacle, the road and its edges, with presence of person at each available place in vehicle and weight in trunk to achieve maximum load. Path followed by the vehicle is as shown in fig. 11 and steering wheel angle is as shown in fig.12.

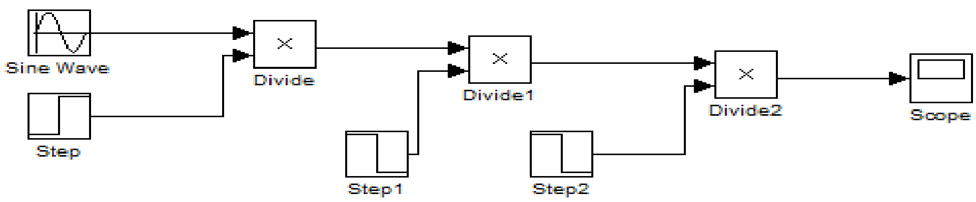


Fig .10 Simulink block for noose test path

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Vol. 3, Issue 9 , September 2016

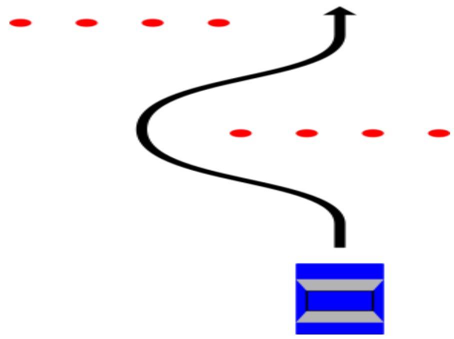

Fig. 11 Moose test path

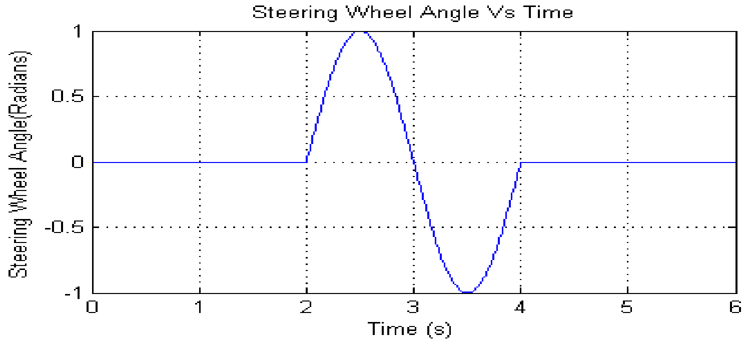

Fig.12 Steering wheel angle in a Moose test path

When the driver comes on the track, he/she quickly swerves in to the oncoming lane to avoid the obstacle and then immediately swerves back to avoid oncoming traffic.

D. CONSIDER TWO CASES OF STEERING FOR SIMULATIONS

1. With EPS and without EPS
2. Assist varying with load

Simulation was performed for each case with circular path and moose test path.

E. WITH EPS AND WITHOUT EPS FOR CIRCULAR PATH

Fig.13comparisons of driver torque with EPS and without EPS. In case of EPS driver torque is less than 5 N-m while in case of without EPS it is more than 15 N-m. This shows that driver needs to apply more torque in case of without EPS. EPS case we take gear ratio is G=20, $\boldsymbol{K_p = 8, K_i = .1, K_d = .1}$, spring constant $\boldsymbol{K_s = 80}$ and without EPS case we take gear ratio G= 0,$\boldsymbol{K_p = 0, K_i = 0, K_d = 0}$, spring constant $\boldsymbol{K_s = 1120}$ and spring damper is 10.

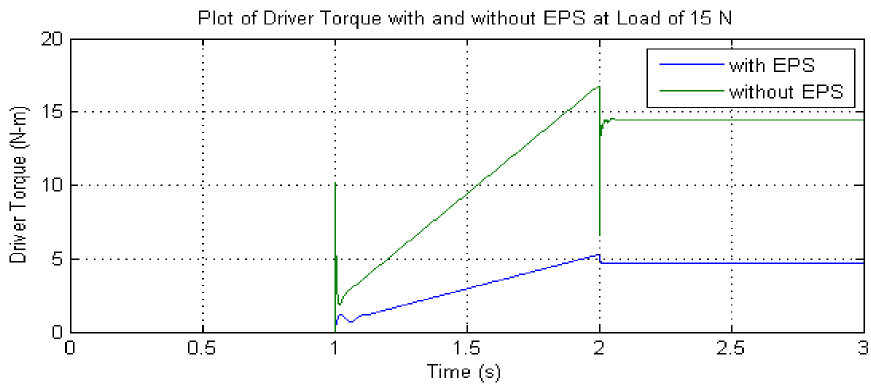

Fig.13: Driver torque with and without EPS for circular path at 10 N Load

F. WITH EPS AND WITHOUT EPS FOR MOOSE TEST PATH

Fig.14. comparisons of driver torque with EPS and without EPS. In case of EPS driver torque is less than 10 N-m while in case of without EPS it is more than 13 N-m. This shows that driver needs to apply more torque in case of without EPS. For this case we take gear ratio is 20, $\boldsymbol{K_p = 4, K_i = 1, K_d = 1}$ and spring constant , $\boldsymbol{K_s = 80}$ and without EPS case we take gear ratio G= 0,$\boldsymbol{K_p = 0, K_i = 0, K_d = 0}$, spring constant $\boldsymbol{K_s = 1120}$ and spring damper is 1.

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Vol. 3, Issue 9 , September 2016

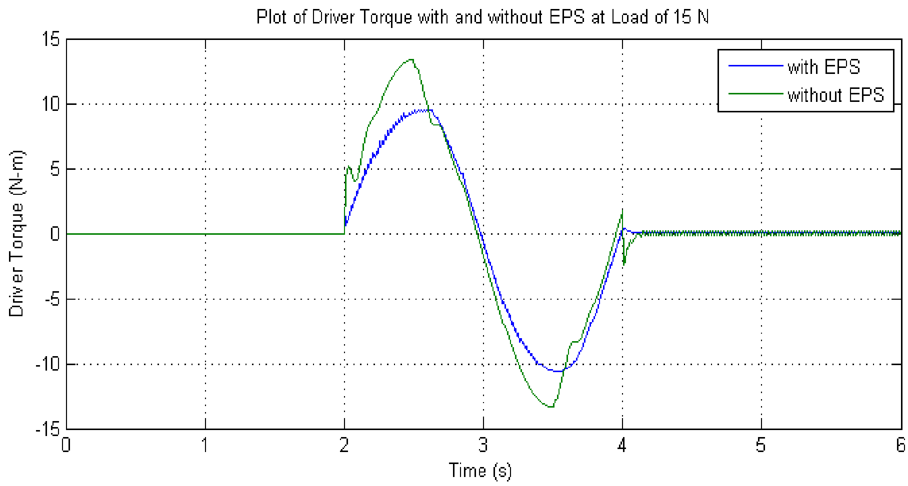

Fig.14. Driver torque with and without EPS for moose test path at 15 N Load

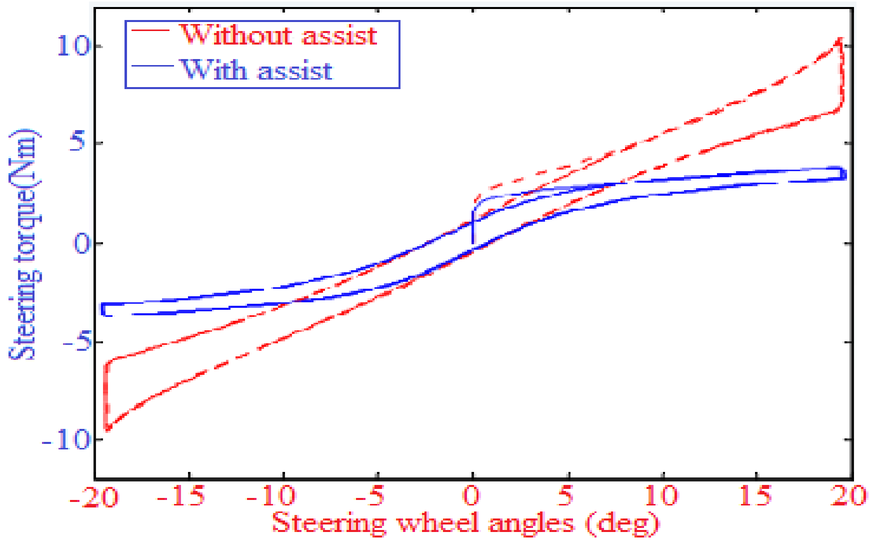

Fig.15 Torque-angle plots of on-center handling test

V. CONCLUSION AND FUTURE WORK

We have derived the mathematical model of column assisted front wheel steering on the basis of Newton's law of motion. We designed a controller for our model with the help of assist characteristics curves and PID controller. We have carried out simulation for different types of vehicle path which include circular and moose test path. We choose two different cases of steering and for each case we have carried out simulation by considering the two paths states above.

The simulation result shows that the introducing of a feedback to EPS is an effective way to improve the vehicle performance. EPS with torque control play a significant role in portability when the vehicle turning at low speed and cornering. We can say that our controller can provide good steering efforts.

Though the EPS is very useful and efficient, it needs complex algorithms to design, torque sensors used in EPS are very expensive, and this increases the cast of practical implementation. It is a challenge in front of designers to design a simple and low cost EPS for practical application. Future work may include the following,

1. Conducting the experiments on test bench and actual vehicle to find out controller performances.

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International Journal of Advanced Research in Science, Engineering and Technology

Vol. 3, Issue 9 , September 2016

2. This may include some modifications in controller depending on vehicle instrumentation and feedback available.
3. Improving the considered model by including the effort of lateral slip on steering rack.
4. In the future, hardware implementation of the PID controlled EPS is expected to be completed.
5. We also plan to test this control system by conducting real-world road test.

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