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Graphoidal Cover Independence Number of a Graph

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ABSTRACT: Let G be a graph. Let ψ be any acyclic graphoidal cover of G. The minimum cardinality of the maximum ψ -independence set is called graphoidal cover independence number of G and is denoted by $\beta_{o\psi}$.

In this paper, we find graphoidal cover independence number for some standard graphs and some interesting results.

I. INTRODUCTION

The concept of graphoidal cover and graphoidal covering number of a graph G was introduced by Acharya and Sampathkumar [2].

A graphoidal cover of a graph G is a collection of ψ of paths(not necessarily open) in G satisfying the following conditions.

(i) Each path in ψ has at least two vertices.

(ii)Every vertex of G is an internal vertex of at most one path in ψ .

(iii)Every edge of G is in exactly one path in ψ .

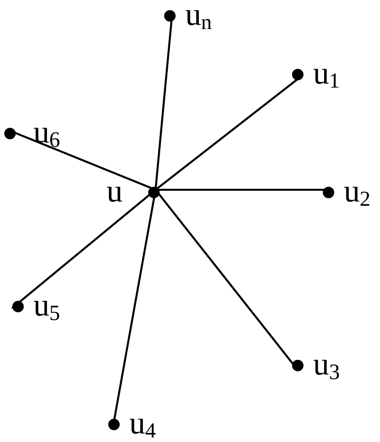
Arumugam and Suresh Suseela [4] introduced the concept of acyclic graphoidal cover and acyclic graphoidal covering number of a graph G.

A graphoidal cover ψ of a graph G is called an acyclic graphoidal cover if every member of ψ is a path. Let ψ be any acyclic graphoidal cover of G.The maximum cardinality of maximal ψ -independence set is called ψ -independence number of a graph G and is denoted by $\beta_{o\psi}$ (G).

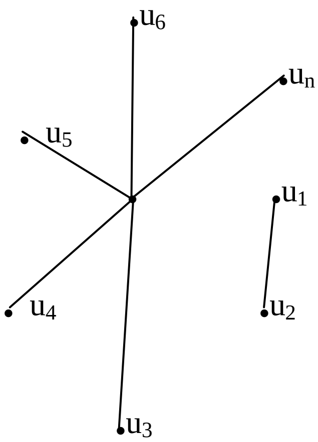
II.MAIN RESULTS

Example 1.1.

Consider the graph G and the corresponding ψ -graph . Let ψ = {(u_1uu_2)} ∪ E(G) . Then $\beta_{o\psi}$ (G) = n –1.



G



$G(\psi)$

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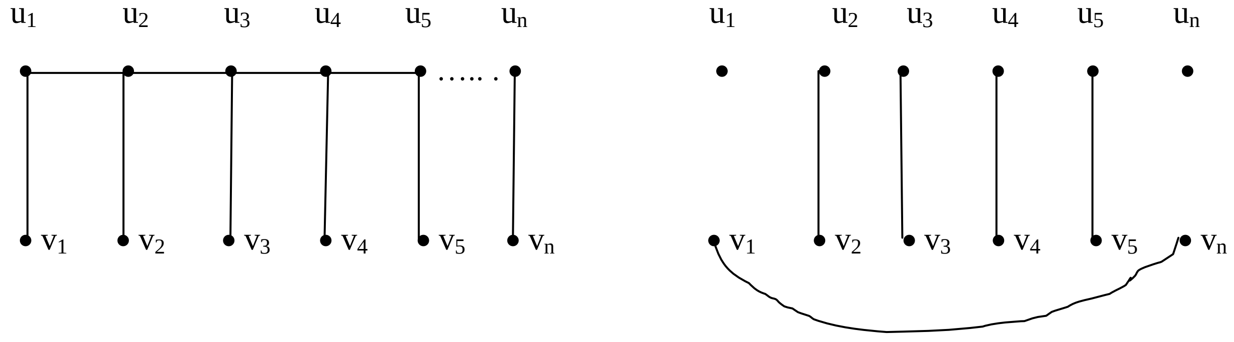
Example 1.2.

Consider the graph G and the corresponding ψ -graph Let ψ = {($v_1u_1u_2$ ……u_nv_n)} ∪ E(G) . Then $\beta_{o\psi}$ (G) = n +1.



G $G(\psi)$

Remark 1.3: From the above examples, we can conclude that for any acyclic graphoidal cover ψ , there is no relation between $\beta_0(G)$ and $\beta_{0\psi}(G)$.

In the following theorems, we prove that the differences $\beta_0 - \beta_{0\psi}$ and $\beta_{0\psi} - \beta_0$ can be made arbitrarily large.

Theorem 1.4. *Given any positive integer n, there exists a graph G and an acyclic graphoidal cover ψ of G such that* $\beta_0(G) - \beta_{0\psi}(G) = n$

Proof. We construct a graph G as follows. Let G be the graph obtained from the cycle C = ($v_1, v_2, \ldots, v_n, v_1$) by attaching three pendant edges to every vertex of C .Let x_i, y_i and z_i (1≤ i≤ n) be the pendant vertices which are adjacent to v_i..

Then ψ = $\{x_iv_iy_i : 1 \le i \le n\} \cup \{z_iv_i : 1 \le i \le n\} \cup E(G)$ is an acyclic graphoidal cover of G. Further G(ψ) is isomorphic to (C • K_1) ∪ n K_2 so that $\beta_{o\psi}$ (G) = 2n. Also $\beta_0(G)$ = 3n and we have $\beta_0(G) - \beta_{0\psi}(G)$ = n.

Theorem 1.5. *Given any positive integer n, there exists a graph G and an acyclic graphoidal cover ψ of G such that* $\beta_{0\psi}(G) - \beta_0(G) = n$

Proof. We construct a graph G as follows. Let H = $K_{1,n}$. Let V(H)=$V(H) = \{v, v_1, v_2 \ldots v_n\}$ be the vertex set of H and v be the centre vertex of H. Subdivide H by once and $u_1, u_2, u_3 \ldots u_n$ be the subdividing vertices and attach a pendant to the vertex v and call it as w and denote the graph as G. Now $|G| = 2n+2$ .Let ψ *be* any acyclic graphoidal cover of G.Let S be the set of vertices which are interior to ψ *.Let* $S = \{u_1, u_2, u_3 \ldots u_n\}$ *and* $\psi = \{vu_iv_i / 1 \le i \le n\} \cup \{vw\}$. *Then* G($\psi$) is isomorphic to $K_{1,n+1}$ ∪ (nK_1) and $\beta_{o\psi}$ (G) = n +1 + n = 2n +1 and $\beta_0(G)$ = n + 1,so that $\beta_{o\psi}$ (G) $- \beta_0(G)$ = 2n + 1 − (n + 1) = n

Problem 1.6. Does there exist a graph G such that

(i) $\beta_{o\psi}$ (G) $- \beta_0(G)$ > n

(ii) $\beta_0(G) - \beta_{o\psi}$ (G) < n

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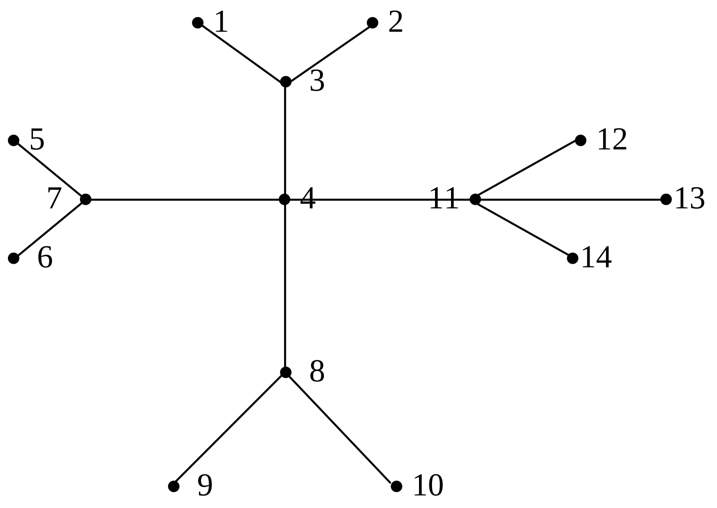
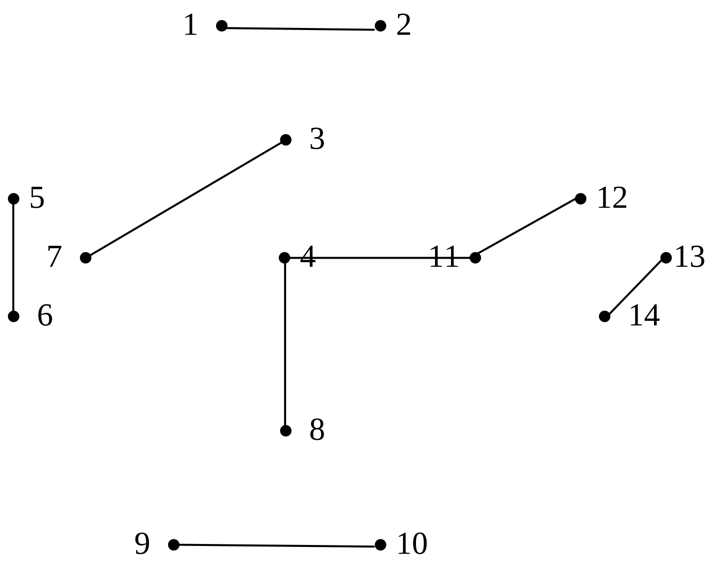
Remark 1.7.*For any two minimum graphoidal covers* ψ_1 *and* ψ_2, $\beta_{0\psi_1} \neq \beta_{0\psi_2}$ *and not isomorphic also. For example, consider the graph G,*



G

Let $\psi_1 = \{(1\ 3\ 2)(5\ 7\ 6)(9\ 8\ 10)(13\ 11\ 14)(7\ 4\ 3)(8\ 4)(4\ 11)(11\ \ 12)\}$,then G(ψ_1) is given below.



G(ψ_1) $\beta_0(\psi_1)$ = 7

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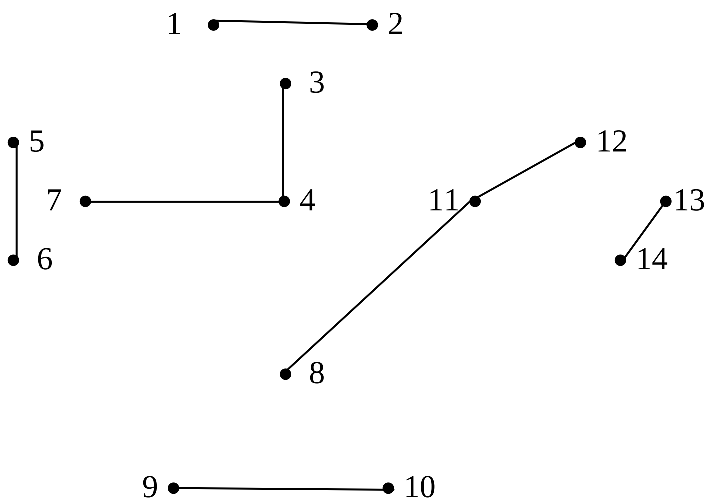
Let $\psi_2 = \{(1\ 3\ 2)(5\ 7\ 6)(9\ 8\ 10)(13\ 11\ 14)(8\ 4\ 11)(11\ 12)(7\ \ 4)(4\ \ 3)\}$,then G(ψ_2) is given below.



G(ψ_2) $\beta_0(\psi_2)$ = 8

Theorem 1.8. *Let T be a tree. Then for any minimum acyclic graphoidal cover ψ of T, $\beta_{o\psi}$ (T) = n–1, if and only if T= P_n, where n is the number of vertices.*

Proof. Let T = P_n then obviously $\beta_{o\psi}$ *(T) = n —1.If* $\beta_{o\psi}$ *(T) = n —1,then we have to prove that T =* P_n .Suppose T ≠ P_n .Then at least one vertex of degree ≥ 3.Then all vertices of degree ≥ 2 are interior to ψ , since ψ is a minimum graphoidal cover and G(ψ) is isomorphic to (n — 4) $K_1 \cup 2 K_2$ and $\beta_{o\psi}$ (T) = n — 2 which is a contradiction. This completes the proof.

Problem 1.9. *Characterise the graphs G for which* $\beta_{o\psi}$ *(G) = n —1.*

Theorem 1.10. *For any acyclic graphoidal cover ψ of the* path *G= P_n on n vertices,*

$$\beta_{0\psi}(G) = n-(k+1)+\left\lceil \frac{k+1}{2} \right\rceil \text{ where } k = |\psi|.$$

Proof. Let G = P_n be a path on n vertices. It is clear that for any graphoidal cover ψ of G,G(ψ) is isomorphic to

$P_{k+1} \cup (n-(k+1))K_1$ where k = $|\psi|$,so that $\beta_{0\psi} = n-(k+1)+\left\lceil \frac{k+1}{2} \right\rceil$.

Theorem 1.11. *For any acyclic graphoidal cover ψ of the cycle* G = C_n *on n vertices,* $\beta_{0\psi}(G) = n-k+\left\lceil \frac{k}{2} \right\rceil$ *where* k = $|\psi|$.

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Proof. Let G = C_n be a cycle on n vertices. It is clear that for any graphoidal cover ψ of G,G(ψ) is isomorphic to $C_k \cup (n-k)K_1$ where $k = |\psi|$,so that $\beta_{0\psi}(G) = n-k+\left\lceil \frac{k}{2} \right\rceil$

Theorem 1.12. *Let T be any tree with n pendant vertices and let 'r' denote the number of vertices of degree two. Then for any minimum acyclic graphoidal cover ψ , $\beta_{o\psi}$ (T) $\leq n + r - 1$.*

Proof. Let $\psi = \{P_1, P_2, P_3 ... P_{n-1}\}$ be *a minimum acyclic graphoidal cover of T. Let* $v_i, 1 \leq i \leq n-1$ *be an end vertex of* P_i. *Then* $D = \{v_1, v_2, v_3 ... v_{n-1}\} \cup \{u_1, u_2, u_3 ... u_r\}$ *where* $u_1, u_2 ...... u_r$ *are the* vertices *of degree two is an independence of T(ψ).* Hence $\beta_{o\psi}$ *(T)* $\leq n - 1 + r = n + r - 1$

Problem1.13. C*haracterize the graphs G for which* $\beta_{o\psi}$ *(G) = n+ r −1*

III.CONCLUSION

We find graphoidal cover independence number for some standard graphs and some interesting results and also we can find graphoidal cover independence number for any graph.

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