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# Algorithms of Adaptive Parametric Identification of Nonlinear Objects of Control

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**ABSTRACT:** The problems of formation and construction of algorithms for adaptive parametric identification of nonlinear objects of control that guarantee a given time of parametric estimation are considered. When constructing identification algorithms to ensure the stabilization of the solution, the concepts of decomposition of symmetric matrices are used based on the use of their factorized forms and singular decomposition. This approach allows to provide numerical stability of the procedure for inversion of matrices and, thus, to increase the accuracy and reliability of the results obtained due to a certain complication of the algorithms for processing initial data.

**KEYWORDS:** Nonlinear object of control, parametric identification, adaptive model, decomposition of symmetric matrices, singular decomposition.

## I. INTRODUCTION

In control systems with configurable models, the model is a dynamic link that has a structure similar to the control object, but with the coefficients available to the change. The customization algorithm should approximate the behavior of the model to the behavior of the object based on the current information about the input and output signals of the object. The resulting coefficients of the model can be considered as estimates of the coefficients of the object. To construct the regulator, methods of the classical control theory are also used, assuming an exact knowledge of the parameters of the object. In this case, instead of unknown parameters, their estimates obtained using the model are substituted into the law of regulation. Like the reference model, the customizable model can be explicitly implemented in the system or be present implicitly in it, as a set of coefficients of the custom equation.

When constructing adaptive systems with customizable models for which identification is part of the adaptation process, an important issue is the choice of the identification method. By this time, a large number of works have been published in which different approaches, methods, algorithms and computational schemes are used to identify objects [1-10]. To apply these methods, it is necessary to have as an a priori information the equation of the model of the object, in which only parameters can be unknown. Depending on the identification criterion or the applied computational algorithm, these methods received a different name. These are LSM, generalized LSM, maximum likelihood method (MLM), Bayesian estimates (BE), instrumental variable method (IVM), parameter estimation using the Kalman filter (KF), etc. [1,4,5,6].

The actual task is the development or modification of methods and algorithms for identifying the parameters of nonlinear systems on the basis of a customizable model that guarantees the specified identification time.

## II. FORMULATION OF THE PROBLEM

We shall consider a process described by a nonlinear differential equation of the form

$$\dot{x} = Cx + \psi(x, f, t) + A\varphi(x, t), \qquad x(0) = x_0,$$

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where $x$ –( $n\times1$)-is the extended state vector of the process; $f=f(t)$ – a vector of input effects of dimension ( $k\times1$), $k\le n$; $A$ is a ( $n\times p$ ) - matrix containing $m\le np$ unknown constant parameters, the remaining elements of it are zeros; $C$ is a given, constant, Hurwitz ( $n\times n$ )- matrix with real coefficients of the form $c_1,c_2,...,c_n$; $\psi(\cdot)$, $\varphi(\cdot)$ – are known vector-valued functions of size ( $n\times1$) and ( $p\times1$).

In addition to the state vector of the object, the state vector of the regulator is also included in the extended process state vector. Vectors $x$, $f$ are measured by sensors, and unknown elements of matrix $A$ can not be directly measured.

The adaptive model of the process is taken in the form:

$$\dot{y}=Cy+\psi(x,f,t)+B(t)\varphi(x,t),\ \ y(0)=y_0\,,$$

where $y-(n\times1)$ is the model state vector; $B(t)$ – ( $n\times p$ ) matrix of customizable parameters of the model. The number of them in $B(t)$ equal to $m$. They are located on the same name with unknown parameters of the matrix.

The generalized error vector $\varepsilon=x-y$ satisfies equation

$$\dot{\varepsilon}=C\varepsilon+\Phi(x,t)\alpha,\ \ \varepsilon(0)=\varepsilon_0\,, \tag{1}$$

where $\Phi(x,t)$ is the ( $n\times m$ ) - matrix; $\alpha=\alpha(t)$ – ( $m\times1$ ) is the vector of parametric mismatches formed from $m$ nonzero elements of the matrix $A-B(t)$ and it is arranged so as to satisfy the relation

$$[A-B(t)]\varphi(x,t)=\Phi(x,t)\alpha\,.$$

As an algorithm for adjusting the parameters of the adaptive model, we take

$$\dot{\alpha}=W(t)[\varepsilon-\exp(Ct)\varepsilon_0],\ \ \alpha(0)=\alpha_0, \tag{2}$$

where $\exp(Ct)$ – the fundamental matrix of equation

$$\dot{\varepsilon}=C\varepsilon\,.$$

This formulation of the problem and those close to it were considered in [4-6,9,11]. In [11] the problem of finding an unknown ( $m\times n$ ) - a matrix $W(t)$ in which the system (1), (2) the parametric mismatches α has transient

$$\alpha=\exp(-\Lambda t)\alpha_0\,, \tag{3}$$

where $\Lambda=diag(\lambda_1,...,\lambda_m)$ with given, real and constant coefficients $\lambda_i>0\ \ (i=1,...,m)$.

When $m\le n$, the solution of the problem is achieved by means of an algorithm

$$\dot{\alpha}=-\Lambda\left(R^TR\right)^{-1}R^T\varepsilon,\ \ \alpha(0)=\alpha_0,$$

where «$T$» is the symbol of transposition; $-\Lambda(R^TR)^{-1}R^T=W$; $R$ is a ( $n\times m$ ) - matrix satisfying condition

$$\dot{R}=CR+R\Lambda+\Phi(x,t),\quad R(0)=0\,. \tag{4}$$

Thus, the desired matrix $W(t)$ is determined by the ratio

$$W=-\Lambda(R^TR)^{-1}R^T\,,\ \text{if}\ m<n\,, \tag{5}$$

$$W=-\Lambda R^{-1}\,,\ \text{if}\ m=n\,.$$

When $np\ge m>n$ the algorithm of determination has the form

$$\dot{\alpha}=-\Lambda R^T\left(RR^T\right)^{-1}[\varepsilon-\exp(Ct)\varepsilon_0],\ \ \alpha(0)=\alpha_0.$$

In this case, the solution by the method of least squares is proposed to be sought in the form:

$$\alpha=R^T\left(RR^T\right)^{-1}[\varepsilon-\exp(Ct)\varepsilon_0]\,.$$

Thus, the required matrix $W(t)$ in the case $np\ge m>n$ has the form:

$$W=-\Lambda R^T(RR^T)^{-1}\,. \tag{6}$$

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Taking into account that $\Lambda = diag(\lambda_1,...,\lambda_m)$, equation (4) can be written in the form of the following system of $m$ equations [11]:

$$\dot{R}_i = (C + \lambda_i E)R_i + \Phi_i(x,t), \quad R_i(0) = 0, \quad (i = 1,...,m), \tag{7}$$

where $R_i = (r_{1i}, r_{2i},...,r_{ni})^T$ – $i$-th column of the matrix $R$; $E$ – identity matrix; $\Phi_i(x,t)$ - $i$-th column of the matrix $\Phi(x,t)$.

It is required that the real parts of the eigenvalues of the matrix $C + \lambda_j E$ be negative, i.e.

$$\operatorname{Re} v_{ji} < 0, \quad (j = 1,...,n), \tag{8}$$

where $v_{1i}, v_{2i},...,v_{ni}$ – $n$ eigenvalues of the matrix $C + \lambda_j E$.

The requirement (8) is completely natural, since it is a necessary and sufficient condition for the stability of systems (7) according to Hurwitz [12].

Conditions (8) can be written in the following form

$$\max_j \{\operatorname{Re} \mu_j\} + \max_i \{\lambda_i\} < 0, \tag{9}$$

where $\mu_j = \lambda_i - v_{ji}$.

Expression (9) establishes a connection between the real parts of the eigenvalues of the matrix $C$ and the eigenvalues of the matrix $\Lambda$ on which the transient process (3) depends. Thus, the choice of the coefficients $c_1,...,c_n$ of the matrix $C$ for given $\lambda_i$, as well as the choice $\lambda_i$ for fixed $c_i$, can not be arbitrary, but is subordinated to the condition (9).

Obtaining pre-selected characteristics of transient processes by custom parameters is designed to improve the quality of management processes in the management systems of technical and technological objects.

In the implementation of algorithms for calculating matrices $W$ on the basis of expressions (5) and (6), computational difficulties arise due to the possible poor conditionality of the matrix $R$. In this connection, it becomes necessary to stabilize the desired solution $W$ of the identification equation as an inverse problem of the dynamics of controllable systems [10, 13, 14]. One of the possible effective solutions to this problem is the use of algorithms for calculating the matrix $G = (R^T R)^{-1}$, using certain expansions of the matrix $R$ [12, 15-19]. The use of matrix decomposition methods makes it possible to increase the accuracy and reliability of the results obtained due to a certain complication of the algorithms for processing initial data.

## III. SOLUTION OF THE TASK

We will consider an expansion of the form

$$QR = \begin{bmatrix} F \\ 0 \end{bmatrix},$$

where the matrix $Q$ is an orthogonal matrix, $F$ is an upper triangular matrix of order $n \times n$.

Then the following equality holds

$$R^T R = F^T F,$$

and if $rankR = n$, then

$$G = (R^T R)^{-1} = F^{-1}(F^{-1})^T.$$

Then, from the identity $F^{-1}F = I$ and the fact that both the matrices $F$ and $F^{-1}$ the upper triangular matrices, we obtain the equations [17,18]

$$\sum_{l=i}^{j} t_{il} r_{lj} = \delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \quad 1 \le i \le n, \quad i \le j \le n,$$

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where $t_{ij}$ – are the elements of the matrix $F^{-1}$.

Solving relatively $t_{ij}$, we find

$$t_{ij} = \begin{cases} r_{jj}^{-1}, & j = i, \\ -r_{jj}^{-1}, \quad \sum_{l=i}^{j-1} t_{il} r_{lj}, & j > i. \end{cases}$$

These expressions should be used as follows:

$$t_{ii} = r_{ii}^{-1}, \quad i = 1, \ldots, n,$$

$$t_{ij} = -t_{jj} \sum_{l=i}^{j-1} t_{il} r_{lj}, \quad j = i+1, \ldots, n, \quad i = 1, \ldots, n-1.$$

When solving the problem under consideration, it is also expedient to use an expansion of the form

$$\tilde{Q} RP = \begin{bmatrix} \tilde{F} \\ 0 \end{bmatrix},$$

where $\tilde{Q}$ and $P$ – orthogonal matrices, at that $P$ – permutation matrix $\tilde{F}_{n\times n}$ – upper triangular matrix of order $n \times n$.

Then the following relations hold:

$$R^T R = P^T \tilde{F}^T \tilde{F} P,$$

$$G = (R^T R)^{-1} = P \tilde{F}^{-1} (\tilde{F}^{-1})^T P^T.$$

When solving problems of the method of least squares in order to estimate the number of conditionality of the matrix, it becomes necessary to calculate singular numbers. However, it is often desirable to obtain an estimate for the conditioning number without calculating the singular numbers. The algorithms considered in this paper generate, as an intermediate result, a non-degenerate triangular matrix $F$ having the same nonzero singular numbers as the original matrix $R$. In the general case, a non-degenerate triangular matrix is the best starting point in the evaluation of conditionality than the filled matrix [16-18].

Thus, for example, to calculate the lower bound of $\rho$ for the condition number $\kappa = s_1 / s_n$ of a non-degenerate triangular $n \times n$ matrix $F$, one can use an estimate of the form

$$\kappa \geq \rho \equiv \frac{\max_{i,j} |r_{ij}|}{\min_i |r_{ii}|},$$

where $s_1 \geq \max_{i,j} |r_{ij}|$ and $s_n \leq \min_i |r_{ii}|$ are estimates for the largest and smallest singular numbers of the matrix $F$.

This lower bound can not, in general, be regarded as a reliable estimate for $\kappa$. In fact, $\kappa$ can be much larger than $\rho$. It can be shown [17, 19, 20] that a more realistic estimate of the condition number is an estimate of the form

$$\kappa = s_1 / s_n \geq 2^{n-2}.$$

Since triangular matrices that arise in algorithms for solving linear systems are considered, the following relations [17,19], connecting the smallest singular number of the matrix $R$ and the last diagonal element $r_{nn}$ of the matrix $F$, and also the singular numbers of the matrix $R$ with the diagonal elements $r_{ii}$ of the matrix $F$, respectively:

$$s_n \leq |r_{nn}|, \quad s_n \geq 2^{1-n} |r_{nn}|,$$

$$2^{1-i} |r_{ii}| \leq s_i \leq (n-i+1)^{1/2} |r_{ii}|, \quad i = 1, \ldots, n.$$

In solving this problem, the concepts of singular expansion of matrices are very effective [17-20]. Of the numerous matrix expansions, the singular decomposition, which is the factorization of the matrix $R$ into a product $U \Sigma V^T$, where

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$U, V$ are unitary matrices, and a $\Sigma$ is a diagonal matrix, is given a special place. This is due to the fact that the singular expansion is stable, i.e. small perturbations of the initial matrix $R$ correspond to small perturbations of the diagonal matrix $\Sigma$ and vice versa [17-21].

If $rank(R_{m\times n}) = n$, then $L = R^T R$. It is symmetric and positive definite, hence has a complete set of real positive eigenvalues. Taking into account that $R = U\Sigma V^T$, we rewrite $L$ in the form

$$L = (U\Sigma V^T)^T U\Sigma V^T = V\Sigma^T \underbrace{U^T U}_{I} \Sigma V^T = V\Sigma^2 V^T .$$

$L = V\Sigma^2 V^T$ – the spectral decomposition of the matrix $L$, where $V$ – matrix consisting of columns of eigenvectors, and $\Sigma^2$ – diagonal matrix consisting of eigenvalues. The matrix $\Sigma = \sqrt{\Sigma^2}$ is a diagonal matrix with elements equal to the square root of the elements of the matrix $\Sigma^2$. The matrix $U$ can be found explicitly from

$$R = U\Sigma V^T , \qquad (10)$$

i.e.

$$U = RV\Sigma^{-1} ,$$

where $U, V$ are orthogonal matrices, and $\Sigma$ is a diagonal matrix, $\Sigma^{-1}$ is a diagonal matrix with elements equal to the reciprocal values of the elements of the matrix $\Sigma$.

If $rank(R_{m\times n}) = m$, then $L = RR^T$. Taking (10) into account, we find a representation for $L$:

$$L = U\Sigma V^T (U\Sigma V^T)^T = U\Sigma \underbrace{V V^T}_{I} \Sigma^T U^T = U\Sigma^2 U^T .$$

Then from (10) we can find $V$ by explicit substitution $V = (\Sigma^{-1} U^T R)^T$.

## IV. CONCLUSION

The above computational procedures make it possible to stabilize the solution of the problem of synthesis of algorithms for estimating the parameters of a nonlinear object in adaptive control systems with a tunable model and to raise the qualitative indicators of control processes of dynamic objects under conditions of parametric uncertainty.

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