ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 4, Issue 12 , December 2017

Estimation of Dynamic State Variables Using Laguerre Series Approximation

Aksoy S. , Bolat M.

Full Professor, Department of Electrical Electronic Engineering, Faculty Of Engineering and Architecture,Siirt, Turkey.

P.G. Student, Department of Electrical and Electronic, Faculty Of Engineering and Architecture,Siirt, Turkey

ABSTRACT: In this study, an estimation algorithm is proposed to estimate the state variables of linear time-invariant multi input-multi output systems using only input and output measurements. The proposed recursive algorithm is based on the Laguerre series approximation and uses some important properties of the Laguerre series. When the number of elements of series is sufficiently large, the Laguerre series approximation gives results very close to exact solution.

KEYWORDS: State estimation, laguerre series, state observer.

I. INTRODUCTION

State variables that determine a system's dynamics should be known for analysis and control of dynamical systems (Daughlas B. Miron, 1989;Chen Chi-Tsong, 1984; Brasch, F.M.,and Pearson, J.B. 1970; Phillips, Y.A., 1985). Specifically, dynamic feedback for pole placement is required. Furthermore, estimation of state variables in real time is a very important problem in adaptive control applications (Aström, K.J., and Wittermark, B., 1989).Unfortunately, all of the state variables cannot be measured in practice. As a result, use of a suitable state observer or estimator is unavoidable in order to obtain unmeasurable state variables. There exist variety of state observers in the literature (Kailath, T., 1983;Reilly, J.,1983). The gain matrix **G**, which is necessary for state observer, can be calculated by using one of the methods such as Ackerman, Bass Gura etc. (Reilly, J.,1983). Implementation of state observers that use only input and output measurements of the system is carried out via solution of the state and error integral equations pertinent to the observer. There are several numerical solution algorithms for solution of the state and error integral equations in the literature (Hildebrand,F.B.,1937; Roltson A., 1978; Sheid F. J., 1968).Even though the Runge-Kutta numerical integration algorithm is frequently used for this purpose, it has several drawbacks that depend on the step-size T. First, accuracy gets poorer as T increases. Second, computation time becomes an issue if T is too small. Third, round-off errors may become important for small values of T because the number of cycles required to cover the desired time interval [0,t] increases. In numerical integration, similar approaches such as the Picard iteration and the Parker-Sochacki methods are sometimes used for computation of the state and error integral equations of observers (Kendall E. Atkinson, 1978; Gwynne A. Evans, 1995).Note that equations are evaluated for each t in the interval [0,t] in all of the above mentioned algorithms.

In this study, a new general algorithm that uses only input and output measurements is proposed for state variables estimation of linear, time-invariant multi-input multi output systems. The proposed algorithm is based on the Laguerre series approximation and has an analog solution. Hence, it is not effected by rounding errors. As a consequence, accuracy is not a function of the step size. The solution that results from the proposed algorithm gets closer to the true solution when more and more terms are kept in the Laguerre series. Finally, the proposed method gives the approximate solution of the estimation vector $\hat{x}(t)$ as a function of time in the interval $t \in [0,\infty)$. Consequently, computation of the state and error integral equations for each t is eliminated.

The Laguerre series are defined on the interval $t \in [0,\infty)$ and have the orthogonality property like the Walsh, Chebyshev and Legendre series (Stavroulakis P., and Tzafestas,S., 1977; Cheng-Chilan Liu and Y. P. Shih, 1983; G. Sansone, 1991). The proposed algorithm uses some important properties such as the operational matrix of integration for Laguerre vector (Hwang, C., and Shih, Y.P., 1981; Chyi Hwang and Y. P. Shih, 1982; M. H. Perng , 1986).

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 12 , December 2017

The algorithm consists of three steps. In the first step, the feedback gain matrix $\mathbf{G}$, which will force the estimation error to go to zero in a short time, is determined by using a suitable method [6]. In the second step, the state and error equations are converted into integral equations by integrating the terms on either side of the equations. Then, unknown state and error vectors together with the Laguerre series approximation of known input vectors are substituted in the integral equations. After some algebraic manipulations, the time dependent terms on either side of the integral equations are removed. Hence, the problem is reduced to a set of nonlinear equations with constant coefficients. Finally, in the last step, nonlinear equations are converted into a recursive form whose solution can be obtained easily by a computer program. Unknown series expansion coefficients for estimation of state variables vector are easily calculated.

The proposed estimation algorithm was implemented in MATLAB™ and it was applied to different cases. Results obtained by the proposed algorithm are in harmony with the real results.

II. THE LAGUERRE SERIES

Let f(t) be a square integrable function in the interval $t \in [0,\infty)$. Then, the Laguerre series approximation of f(t) with r terms is given by

$$f(t) \cong \sum_{k=0}^{r-1} a_k \lambda_k(t) = a^T \lambda(t) \tag{1}$$

where a and λ are the Laguerre polynomial coefficient vector and the Laguerre polynomial vector, respectively defined as

$$\mathbf{a}^T = [a_0 \quad a_1 \quad .... \quad a_{r-1}] \tag{2}$$

$$\boldsymbol{\lambda}^T(t) = [\lambda_0(t) \quad \lambda_1(t) \quad .... \quad \lambda_{r-1}(t)] \tag{3}$$

The coefficients a_k in Eq. (1) can be obtained from the orthogonality property as

$$a_k = \int_0^\infty e^{-t} f(t) \lambda_k(t) dt \tag{4}$$

On the other hand, the Laguerre polynomials are defined by

$$\lambda_k(t) = \frac{e^t}{i!} \frac{d^k}{dt^k}(t^k e^{-t}) \quad ,k=0,1,2,.... \tag{5}$$

It is obvious from Eq. (5) that

$$\lambda_0(t) = 1$$

$$\lambda_1(t) = 1 - t$$

$$\lambda_2(t) = 1 - 2t + 0.5t^2$$

$$\lambda_3(t) = 1 - 3t + (3/2)t^2 - (1/6)t^3$$

$$\lambda_4(t) = 1 - 4t + 3t^2 - (2/3)t^3 + (1/24)t^4$$

. . . .

. . . .

Therefore, the relationship between the $(i+1)_{th}$ and i_{th} polynomials can be given as

$$(i+1)\lambda_{i+1}(t) = (1+2i-t)\lambda_i(t) - i\lambda_{i-1}(t) \quad ,i=1,2,..... \tag{6}$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 12 , December 2017

Integrating the polynomial vector $\lambda(t)$ on the interval t∈[0,∞) gives

$$\int_0^t \lambda(\tau)d\tau = \mathbf{P}\lambda(\tau) \qquad (7)$$

where

$$\mathbf{P} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & -1 \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

The matrix **P** of size $r_x r$ in Eq. (7) is called the integration operation matrix.

III. THE PROPOSED ESTIMATION ALGORITHMS

The proposed estimation algorithm can be considered as state observer and its simulation diagram is given in Fig. 1.State and error equations for the state observer shown in Figure 1 are

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{M}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{y}(t) \qquad (8)$$

$$\dot{\mathbf{e}}(t) = \mathbf{M}\mathbf{e}(t),\ \mathbf{e}(0) = \mathbf{x}(0) - \hat{\mathbf{x}}(0) \qquad (9)$$

Where

$$\mathbf{M} = (\mathbf{A} - \mathbf{GC}) \qquad (10)$$

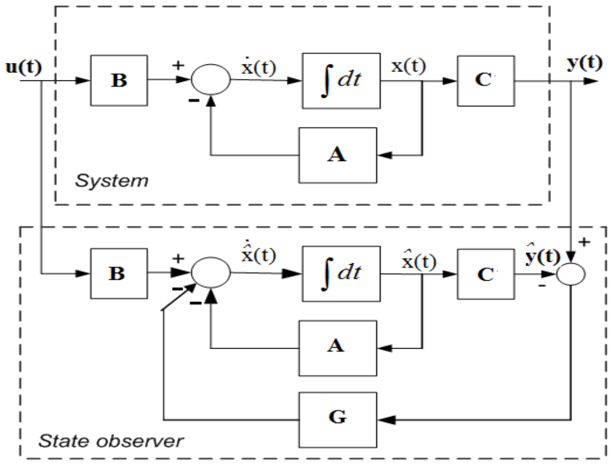

Figure 3.1 Simulation diagram representation of a stateestimator.

In Eq. (8) and (9) , $\hat{\mathbf{x}}(t)$, $\mathbf{u}(t)$, $\mathbf{e}(t)$ and y(t) are the nx1 state vector, m_x1 input vector nx1 error vector and px1 output vector , respectively. **A**,**B**, **C** and **G** are n_xn state matrix, n_xm input matrix, p_xn output matrix and n_xp gain matrix, respectively. The gain matrix **G** is effective only if $\mathbf{x}(0) \neq \hat{\mathbf{x}}(0)$ and it should be chosen such that the estimation error goes to zero in a short period of time. The elements of the gain matrix G can be determined form the characteristic equation given in (11) by using arbitrary eigenvalues of (**A-GC**) denoted $\alpha_1, \alpha_2, \dots, \alpha_n$ (note that eigenvalues are chosen such that $\mathbf{e}(t)$ goes to zero as quickly as possible) (Kailath, T., 1983):

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 12 , December 2017

$$\left|\alpha\mathbf{I}-(\mathbf{A}-\mathbf{GC})\right|=0 \quad (11)$$

In Eq. (17), **I** is $n_x n$ identity matrix.

In order to calculate the unknown estimation vector $\hat{\mathbf{x}}(t)$ in Eq. (8), the estimation error vector $\mathbf{e}(t)$ should be known. If we integrate both side of Eq. (9), we obtain

$$\mathbf{e}(t)-\mathbf{e}(0)=\int_0^t \mathbf{Me}(\tau)d\tau \quad (12)$$

The Laguerre series approximation of the estimation error vector $\mathbf{e}(t)$ can be given by

$$\mathbf{e}(t)\cong\begin{bmatrix} e_{10} & e_{11} & \cdots & e_{1,r-1} \\ e_{20} & e_{20} & \cdots & e_{2,r-1} \\ . & . & & . \\ & & \cdots & \\ . & . & & . \\ e_{n0} & e_{n1} & \cdots & e_{n,r-1} \end{bmatrix}\boldsymbol{\lambda}(\mathrm{t})=\mathbf{E}\boldsymbol{\lambda}(t) \quad (13)$$

where

$$\mathbf{E}=[\mathbf{e}_0 \quad \mathbf{e}_1 \dots \quad \mathbf{e}_{r-1}], \quad (14)$$

$$\mathbf{e}_k=\begin{bmatrix} e_{1k} & e_{2k} & \cdots & e_{nk} \end{bmatrix}^{\mathrm{T}}, \ k=0,1,2,\dots,r-1.$$

In Eq. (14), **E** is the Laguerre series coefficients matrix of the error vector. Similarly, the Laguerre series approximation of the initial value of the estimation error vector $\mathbf{e}(t)$ can be written as

$$\mathbf{e}(0)\cong\begin{bmatrix} e_1(0) & 0 & \cdots & 0 \\ e_2(0) & 0 & \cdots & 0 \\ . & . & & . \\ & & \cdots & \\ . & . & & . \\ e_n(0) & 0 & \cdots & 0 \end{bmatrix}\boldsymbol{\lambda}(t)=\mathbf{E}_0\boldsymbol{\lambda}(t) \quad (15)$$

where

$$\mathbf{E}_0=[\mathbf{e}(0) \quad \mathbf{0}\dots \quad \mathbf{0}], \quad (16)$$

$$\mathbf{e}(0)=\begin{bmatrix} e_1(0) & e_2(0) & \cdots & e_n(0) \end{bmatrix}^{\mathrm{T}}, \ \mathbf{0}=\begin{bmatrix} 0 & 0 & \dots & 0 \end{bmatrix}^{\mathrm{T}}. \quad (17)$$

In Eq. (16), $\boldsymbol{E}_0$ is the Laguerre series coefficients matrix of the $\boldsymbol{e}(0)$. If we substitute Eq. (13) and (15) in Eq. (12), we have

$$\mathbf{E}\boldsymbol{\lambda}(t)-\mathbf{E}_0\boldsymbol{\lambda}(\tau)=\int_0^t \mathbf{ME}\boldsymbol{\lambda}(\tau)d\tau \quad (18)$$

From Eq. (7), the right hand side of Eq. (18) can be written as

$$\mathbf{E}\boldsymbol{\lambda}(t)-\mathbf{E}_0\boldsymbol{\lambda}(\tau)=\mathbf{MEP}\,\boldsymbol{\lambda}(t) \quad (19)$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 12 , December 2017

If we remove the time-dependent terms in Eq. (19), we obtain the following the set of equations with constant coefficients

$$\mathbf{E} - \mathbf{E}_0 = \mathbf{MEP} \tag{20}$$

The only unknown in Eq. (20) is the matrix **E**. However, as shown in Appendix **A,** Eq. (20) can be put into a recursive form in terms of $\mathbf{e}_i$, i=1,2,...,r-1as follows

$$\mathbf{e}_0 = (\mathbf{I} - \mathbf{M})^{-1}\mathbf{e}(0) \tag{21.a}$$

$$\mathbf{e}_k = -(\mathbf{I} - \mathbf{M})^{-1}\mathbf{M}\mathbf{e}_{k\text{-}1}, k=1,2,\ldots\text{-}r\text{-}1 \tag{21.b}$$

The recursive equation given in Eq. (21) can easily be implemented by using a computer program and the unknown vectors e_i , i=1,2,...,n can be obtained. Now, let us consider the estimation of $\hat{\mathbf{x}}(t)$. If we integrate both side of Eq. (8) we obtain

$$\hat{\mathbf{x}}(t)\text{-}\hat{\mathbf{x}}(0)=\int_0^t \mathbf{M}\hat{\mathbf{x}}(t)d\tau+\int_0^t \mathbf{Bu}(\tau)d\tau+\int_0^t \mathbf{Gy}(\tau)d\tau \tag{22}$$

Let us assume that $\hat{\mathbf{x}}(t)$, $\mathbf{u}(t)$, $\hat{\mathbf{x}}(0)$ and y(t) are continuous functions for [0,t]. Then, their Laguerre series approximations can be given as

$$\hat{x}(t) \cong \begin{bmatrix} x_{10} & x_{11} & \cdots & x_{1,r-1} \\ x_{20} & x_{20} & \cdots & x_{2,r-1} \\ . & . & & . \\ & & \cdots & \\ . & . & & . \\ x_{n0} & x_{n1} & \cdots & x_{n,r-1} \end{bmatrix} \lambda(t) = X\lambda(t) \tag{23}$$

where

$$\mathbf{X} = [\mathbf{x}_0 \quad \mathbf{x}_1 \ldots \quad \mathbf{x}_{r-1}] \tag{24}$$

$$\mathbf{x}_k = \begin{bmatrix} x_{1k} & x_{2k} & \ldots & x_{nk} \end{bmatrix}^T, \text{ k=0,1,2,\ldots,r-1 ,}$$

$$\hat{\mathbf{x}}(0) \cong \begin{bmatrix} x_1(0) & 0 & \ldots & 0 \\ x_1(0) & 0 & \ldots & 0 \\ . & . & & . \\ & & \ldots & \\ . & . & & . \\ x_1(0) & 0 & \ldots & 0 \end{bmatrix} \lambda(t) = \mathbf{X}_0\lambda(t) \tag{25}$$

where

$$\mathbf{X}_0 = [\mathbf{x}(0) \quad \mathbf{0} \ldots \quad \mathbf{0}] \tag{26}$$

$$\mathbf{x}(0) = \begin{bmatrix} x_1(0) & x_2(0) & \ldots & x_n(0) \end{bmatrix}^T$$

$$\mathbf{u}(t)=\begin{bmatrix} u_{10} & u_{11} & \cdots & u_{1,r-1} \\ u_{20} & x_{21} & \cdots & u_{2,r-1} \\ . & . & & . \\ & & \cdots & \\ . & . & & . \\ u_{m0} & u_{m1} & \cdots & u_{m,r-1} \end{bmatrix}\lambda(t)=\mathbf{U}\lambda(t) \qquad (27)$$

where

$$\mathbf{U}=[\mathbf{u}_0 \quad \mathbf{u}_1 \cdots \quad \mathbf{u}_{r-1}],$$

$$\mathbf{u}_k=\begin{bmatrix} u_{1k} & u_{2k} & \cdots & u_{mk} \end{bmatrix}^T, \quad k=0,1,2,\ldots,r-1$$

$$\mathbf{y}(t)\cong\begin{bmatrix}\mathbf{y}_0 & \mathbf{y}_1 & \cdots & \mathbf{y}_{r-1}\end{bmatrix}\lambda(t)=\mathbf{Y}\lambda(t) \qquad (28)$$

where

$$\mathbf{y}_k=\begin{bmatrix} y_{1k} & y_{2k} & \cdots & y_{pk} \end{bmatrix}^T, \quad k=0,1,2,\ldots,r-1.$$

Output vector, $\mathbf{y}(t)$ can be obtain from output measurements by using curve fitting methods (Steven and Raymond,2009). If we substitute Eq. (23), (25), (27) and (28) in Eq. (22), we have

$$\mathbf{X}\lambda(t)-\mathbf{X}_0\lambda(t)=\int_0^t \mathbf{MX}\lambda(\tau)d\tau+\int_0^t \mathbf{BU}\lambda(\tau)d\tau+\int_0^t \mathbf{GY}\lambda(\tau)d\tau \qquad (29)$$

From Eq. (12), Eq. (25) can be written as

$$(\mathbf{X}-\mathbf{X_0})\lambda(t)=\mathbf{MXP}\lambda(t)+\mathbf{BUP}\lambda(t)+\mathbf{GYP}\lambda(t) \qquad (30)$$

If we remove the time-dependent terms in Eq. (30), the following set of constant coefficient algebraic equations is obtained

$$(\mathbf{X}-\mathbf{X_0})=\mathbf{MXP}+\mathbf{BUP}+\mathbf{GYP} \qquad (31)$$

The unknown coefficient vectors $\mathbf{x}_k$,k=0,1,2,…-r-1in Eq. (31) are in a complex form. Hence, we need to rearrange Eq. (31). After applying the steps given in Appendix **A**, the following recursive equations are obtained

$$\mathbf{x}_0=(\mathbf{I}-\mathbf{M})^{-1}(\mathbf{x}(0)+\mathbf{f}_0) \qquad (32.a)$$

$$\mathbf{x}_k=(\mathbf{I}-\mathbf{M})^{-1}(\mathbf{f}_k-\mathbf{M}\mathbf{x}_{k-1}), \quad k=1,2,\ldots-r-1 \qquad (32.b)$$

The unknown estimation vectors $\mathbf{x}_k$,k=0,1,2,…-r-1 are obtained by using a computer program. Then, these vectors are substituted in Eq. (23), and the Laguerre series approximation of the estimation vector $\hat{\mathbf{x}}(t)$ is obtained. For the special case in which **G** is zero matrix, the algorithm behaves as an open-loop observer. Since the observer error dynamics in open-loop observers is determined by eigenvalues of the system, the system must be asymptotically stable for convergence (Kailath, T., 1983). The proposed estimation algorithm was implemented in MATLAB™ and was applied to different examples.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 4, Issue 12 , December 2017

IV. NUMERICAL APPLICATIONS

Example 1. Consider a system having state equations given by

$$\dot{\mathbf{x}}(t)=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}0\\ 1\end{bmatrix}u(t) \quad y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\mathbf{x}(t), \quad \mathbf{x}(0)=\begin{bmatrix}0.5\\ -0.5\end{bmatrix}$$

This system has an analytical solution, which can be obtained from the unit-step input response as

$$\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}=\begin{bmatrix}2e^{-t}-e^{-2t} & e^{-t}-e^{-2t}\\ 2(-e^{-t}+e^{-2t}) & -e^{-t}+2e^{-2t}\end{bmatrix}\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}+\begin{bmatrix}(1/2)-e^{-t}+(1/2)e^{-2t}\\ e^{-t}-e^{-2t}\end{bmatrix}$$

From Eq. (11), the gain vectors [-1 2]T, [1 -1]T and [17 47]T are computed to place eigenvalues of the observer error at -1 , -2 and -10, with multiplicity 2, respectively. The analytical solution as well as solution obtained by the proposed algorithm for each gain vectors is given in Figure 2.

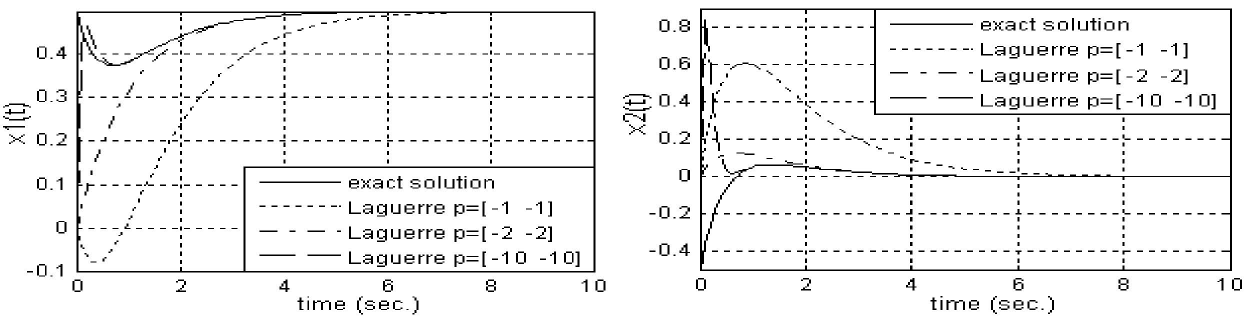

Figure 4.1 Analytical solution and estimation curves of state variables obtained by the proposed algorithm for example 1 a) Results for x1(t) b) Results for x2(t)

As can be see from Figure 2, as eigenvalue of the observer move to the left in the left half s-plane, the estimation curves converge to the true solution in a shorter time. Estimation error values of Laguerre series approximations for several values of r and the observer are given in Table 1. The Runge-Kutta numerical solution method was used for the observer andh=0.01 was chosen. It is obvious from Table 1 that as r increases the estimation error decreases.

Table 1. e1(t) estimation error results for example 1.

	observer	Laguerre		
t	Runge kutta	r=36	Runge kutta	r=144
0.0	0.5000	0.4415	0.4967	0.5000
0.2	-0.0406	-0.0495	-0.0411	-0.0406
0.4	-0.0201	-0.0220	-0.0199	-0.0201
0.6	-0.0047	-0.0065	-0.0053	-0.0047
0.8	-0.0009	-0.0014	-0.0003	-0.0009
1.0	-0.0002	-0.0097	-0.0001	-0.0002
2.0	-0.0000	-0.0015	-0.0004	-0.0000
3.0	-0.0000	-0.0090	-0.0001	-0.0000
4.0	-0.0000	-0.0127	-0.0011	-0.0000
5.0	-0.0000	-0.0123	-0.0013	-0.0000
10.0	-0.0000	-0.1455	-0.0102	-0.0000

Table 2.e2(t) estimation error results for example 1.

	observer	Laguerre		
t	Runge kutta	r=36	r=72	r=144
0.0	-0.5000	-0.7803	-0.5187	-0.5000
0.2	-0.0406	-0.8704	-0.8281	-0.0406
0.4	-0.0201	-0.2191	-0.2129	-0.0201
0.6	-0.0047	-0.0071	-0.0460	-0.0047
0.8	-0.0009	-0.0130	-0.0044	-0.0009
1.0	-0.0002	-0.0441	-0.0008	-0.0002
2.0	-0.0000	-0.0090	-0.0025	-0.0002
3.0	-0.0000	-0.0402	-0.0004	-0.0000
4.0	0.0000	0.0558	0.0061	0.0000
5.0	0.0000	0.0557	0.0071	0.0000
10.0	0.0000	0.6048	0.0562	0.0001

Estimation error of the proposed algorithm as a function of time for several values of r is illustrated in Figure 3 from which one can see that estimation error decreases as r increases.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 12 , December 2017

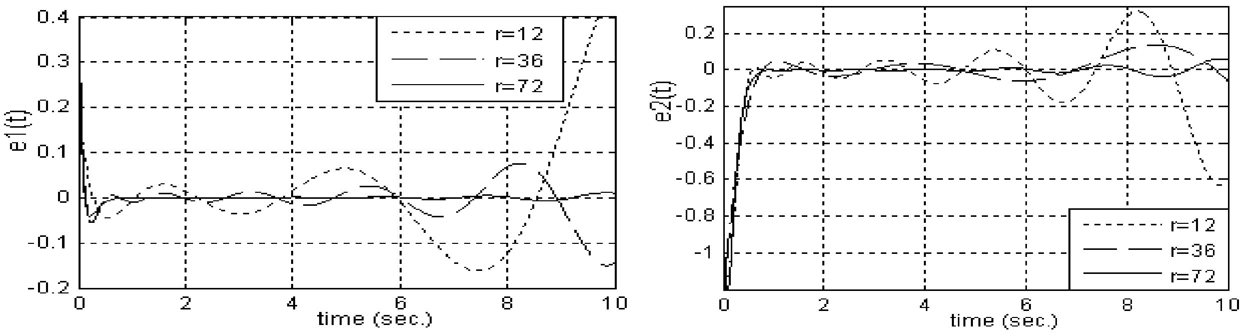

Figure 4.2 Estimation error curves for Example 1. a) Results for e1(t) b) Results for e2(t)

Example 2. Consider a 2 input/2 output dynamic system described by the following state equation.

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & -1 \\ 2 & 0 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}u(t)$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{x}(t), \quad \mathbf{x}(0) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

By using Eq. (11), feedback gain matrices corresponding to the eigenvalues of the observer error dynamic (at -1, -2 and -5 , with multiplicity 2, respectively) are computed as

$$\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1.5 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 8 & 1 \\ -6 & 1 \end{bmatrix}$$

Simulation results of the observer and the proposed algorithm for each gain matrix and for unit step input function are given in Figures 4 and 5. The Runge-Kutta numerical solution method with h=0.01 was used for the observer simulation. One can deduce from Figure 4 that the proposed Laguerre series approximation based estimation algorithm gives state curves that are very close to those of the exact solution when the number of terms in the Laguerre series is sufficiently large. Estimation error curves as a function of time for different values of r are given in Figure 5. Like the results obtained in Example 1, the estimation error decreases with increasing r.

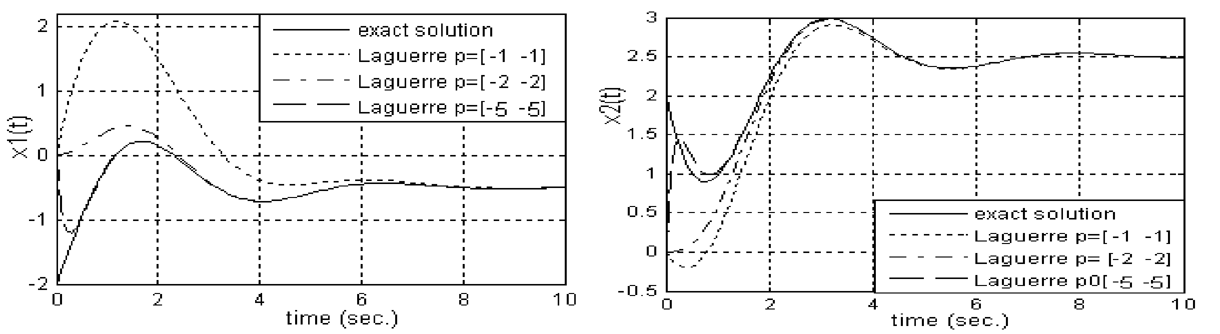

Figure4.3 State curves for Example 2 obtained by the proposed algorithm with r=72 a) Results for x1(t) b) Results for x2(t)

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 12 , December 2017

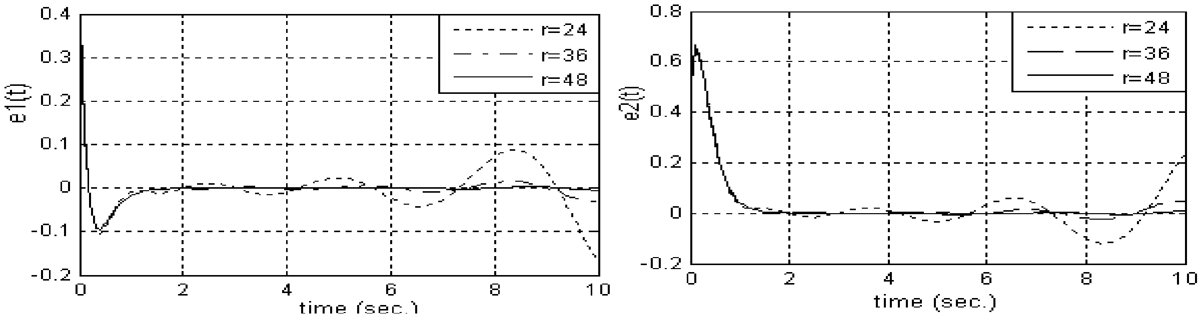

Figure 4.4 Estimation error curves for Example 2 obtained by the proposed algorithm with r=24, 36 and 48. a) Results for x1(t) b) Results for x2(t)

V. CONCLUSION

In this study, a direct algorithm based on the truncated Laguerre polynomials and the operational matrix of integration has been suggested to estimate the elements of the state vector for a linear time-invariant system. The algorithm uses the Laguerre series approximations of the observer and estimation error integration equations in terms of the unknown coefficients of the state and error vectors. Thus, integration problem is reduced to solving a set of nonlinear equations with constant coefficients. The equations for the solution of unknown coefficients are in a complex form. The algorithm we propose transforms these complex equations to a form, which is solved easily in a recursive manner. Furthermore, the algorithm can be a sily implemented by using a computer program.

Even though orthogonal series approximations like the Taylor, Chebyshev and Walsh series are valid for $t\in[0,1]$, the Laguerre series are valid for $t\in[0,\infty)$. In addition, the proposed technique does not suffer from the issues such as choice of the step-size and the rounding errors that are present in the numerical integration algorithms. Proposed algorithm gives the solution of $\hat{x}(t)$ as a analytical function of time in the interval $t\in[0,\infty)$. As a result, numerical computation of the CGM for each t in the interval (0,t) is eliminated.

For the special case in which the gain matrix **G**is chosen as zero matrix, very small number of operations will be sufficient for the solution since the algorithm behaves an open-loop estimator. However, the system has to be asymptotically stable for convergence since the eigenvalues of the matrix **A** determine the observer error dynamics.
Numerical examples illustrate the fact that exact and approximated values of the state variables are in good agreement.

REFERENCES

[1]. D. B. Miron, " Design of Feedback Control Systems." USA: Harcourt Brace Jovanovic Inc, 1989.
[2]. C.-T. Chen, " Linear System Theory and Design. " USA: CBS Publishing, 1984.
[3]. F. M. Brasch and J. B. Pearson, "Pole Placement Using Dynamic Compensators, IEEE Transactions on Automatic Control." vol. AC-15, 1970., pp. 34-43.
[4]. Y. A. Phillips, "Controller Design of Systems with Multicate Noise, IEEE Transactions On Automatic Control." vol. AC-30, no. 2, 1985.
[5]. K. J. Aström and B. Wittermark, Adaptive Control. USA: Addison Wesley Publication Inc, 1989.
[6]. T. Kailath,. "Linear Systems." Tokyo, Japan: Prentice-Hall, 1980.
[7]. O. Reilly, "Observers for Linear Systems." London, UK: Academic Press, 1983.
[8]. F. B. Hildebrand, "Introduction to Numerical Analysis." 2nd ed. New York: McGraw-Hill, 1937.
[9]. A. Roltson, " A First Course In Numerical Analysis." 2nd ed. New York, USA: McGraw-Hill, 1978.
[10]. F. J. Sheid, "Schaum's Outline of Theory and Problems of Numerical Analysis." New York: McGraw-Hill, 1968.
[11]. Kendall E. Atkinson, "An Introduction to Numerical Analysis. Canada: John Wiley&Sons, 1978.
[12]. G. A. Evans, "Practical Numerical Analysis." England: John Wiley&Sons Ltd, 1995.
[13]. G. Sansone, "Orthogonal Functions." New York: Interscience Publishers, 1991.
[14]. Stavroulakis P., and Tzafestas,S., "Walsh Series approach to observer an fitler design in optimal control systes" , Int. Journal. Control,

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 12 , December 2017

Vol.26,No.5, 1977, pp. 721-736.

[15]. Cheng-Chilan Liu and Y. P. Shih, "Analysis and optimal control of time-varying systems via Chebyshev polynomials" ,International Journal. Control, Vol.38, No.5, 1983, pp.1003-012.

[16]. Hwang, C., and Shih, Y.P., "Laguerre Operational Matrices for Fractional Calculus and Applications", Int. J. Control, Vol. 34, No. 3, 1981, pp. 577-584.

[17]. Chyi Hwang and Y. P. Shih, "Parameter identification via Laguerre polynomial"s, Int. J. Systems Sci., Vol.13,No.2, 1982, pp. 209-217.

[18]. M. H. Perng, "Laguerre polynoialsanalysis in optimal control systems incorporating observers", Int. J. Control, Vol. 44, No. 1, 1986, pp. 43-48.