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A New Formulation And Numerical Simulations of Immiscible Compressible Two-Phase Flow in Porous Media

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ABSTRACT:A new formulation is presented to describe immiscible compressible two-phase flow in porous media by the concept of Global pressure. The main feature of this formulation is the introduction of new global pressure and it is fully equivalent to the original equations. The resulting equations are written in a fractional flow formulation and lead to a coupled system which consists of a nonlinear parabolic (the global pressure equation) and a non linear-convention equation (the saturation equation) which can be efficiently solved numerically.

1. INTRODUCTION

Two-phase flow in porous media is important to many practical problems, including those in petroleum reservoir engineering, unsaturated zone hydrology, and soil sciences. Most recently, modeling multiphase flow received an increasing attention in connection with the disposal of radioactive waste and sequestration of CO2.

This paper focuses on the modeling and analysis of immiscible compressible twophase flow through porous media, in the framework of the geological disposal of radioactive waste. As a matter of fact, one of the solutions envisaged for managing waste produced by nuclear industry is to dispose of it in deep geological formations chosen for their ability to prevent and attenuate possible releases of radionuclides in the geosphere. In the frame of designing nuclear waste geological repositories appears a problem of possible two-phase flow of water and gas, mainly hydrogen [7,8].

The mathematical analysis of two-phase flow in porous media has been a problem of interest for many years and many methods have been developed. There is an extensive literature on this subject. We will not attempt a literature review here, but merely mention a few references. Here we restrict ourselves to two-phase flow in porous media.

Here, the models are based on phase formulations, i.e. the main unknowns are the phase pressures and the saturation of one phase, and the feature of the global pressure as introduced in [3,5] for incompressible immiscible flows is used to establish a priori estimates. The obtained results are established under the assumption that the capillary pressure is bounded and no discontinuity of the porosity and the permeability is permitted, which is too restrictive for some realistic problems, such as gas migration through engineered and geological barriers for a deep repository for radioactive waste.

For modeling such flow problems, there are two main approaches known as the phase and the global pressure formulations. The phase formulation is based on individual balance equations for each of the fluids. For such formulation, in regions without the wetting fluid, the wetting pressure is physically not well-defined. So the pressures are not mathematically well defined globally. Also, as a consequence of the degeneracy of the relative permeability functions is that no uniform bounds for the pressure gradients in L2-spaces are available. To overcome these difficulties, the global pressure formulation of the original flow equations was introduced for incompressible two-phase flows in [3,5], and generalized recently to compressible two-phase flows in [1, 2, 9] and for three-phase flows in [4].

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II. PRELIMINARIES FOR THE REGULARIZED PROBLEM

- The model of water-gas flow to be presented here is formulated in terms of the non- wetting (gas) phase saturation and the global pressure and it is developed in [2,3].
- The saturations of the wetting and the non – wetting phases are denoted by S_w and $S_g = 1- S_w$
  Relative mobility is $\lambda_j = \lambda_j(s), j \in\{w,g\}$.
- The pressures and the mass densities of the wetting and the non- wetting phases are

  denoted by P_w , P_g and ρ_w, ρ_g.

- The wetting phase (water) is assumed incompressible
  (ρ_w = const) and the non – wetting (gas) phase is compressible
  $\rho_g = \rho_g(P_g)$
- The fully equivalent global pressure formulation of immiscible, compressible two – phase flow in [2,3] is defined in terms of the global pressure P.
- The saturation potential θ defined by
  $$\theta = \beta(S) = \int_o^s \sqrt{\lambda_w(S)\lambda_g(S)}\; P_c'(s)\, ds \longrightarrow \quad 2.1$$
  Where $P_c(S) = P_g, P_w$ is the capillary pressure function .
- The global pressure P is a pressure like variable which allows to express the phase pressures P_w, P_g and the global pressures
  $P_w = P_w(S, P)$, $P_g = P_g(S, P)$.
- The non – wetting phase mass density will be expressed as a function of S and P
  $$\rho_g = \rho_g(P_g(S, P)) =: \rho_g(S, P).$$
- The phase pressures are given by [2]
  $$P_g(S, P) = P + P_c(0) + \int_o^S f_w(S,P)\, P_c'(s)\, ds \longrightarrow \quad 2.2$$
  $$P_w(S, P) = P_g(S, P) - P_c(s) \longrightarrow \quad 2.3$$
  where the fractional flow functions are defined by,
  $$f_w(S,P) = \frac{\rho_w \lambda_w(S)}{\lambda(S,P)}$$
  $$f_g(S,P) = \frac{\rho_g(S,P)\lambda_g(S)}{\lambda(S,P)}$$
- The total mobility $\lambda(S, P)$ defined by

  $$\lambda(S, P) = \rho_w \lambda_w(S) + \rho_g(S, P)\lambda_g(S)$$

- The water – gas flow equations fully equivalent global pressure formulation are described by the following equations [2].

  $$-\rho w\, \Phi\frac{\partial S}{\partial t} - \mathrm{div}(\Lambda_w(S, P)K\nabla P) + \mathrm{div}(A(S, P)K\nabla\theta) + \rho_w^{\,2}\, \mathrm{div}(\lambda_w(S)\,Kg) = F_w \longrightarrow \quad 2.4$$

  $$\Phi\frac{\partial}{\partial t}(\rho_g(S,P)S) - \mathrm{div}(\Lambda_g(S,P)K\nabla P) - \mathrm{div}(A(S,P)K\nabla\theta) + \mathrm{div}(\lambda_g(S)\rho_g(S, P)^2\, Kg) = F_g \longrightarrow \quad 2.5$$

  Where $\Phi(x)$ is porosity, $K(x)$ is the absolute permeability tensor of the porous medium, F_w ,F_g are source terms and g is gravity vector.

- The coefficient A (S, P) is given by

$A(S, P) = \rho_w\ \rho_g (S, P)\ \frac{\sqrt{\lambda_w\ (S)\lambda_g\ (S)}}{\lambda\ (S,P)}$ ⟶ 2.6

- The mobility functions Λ_w, Λ_g are given by

$$\Lambda_w (S, P) = \rho_w\ \lambda_w(S)\ \omega(S, P)$$

$$\Lambda_g (S, P) = \rho_g\ (S, P)\ \ \lambda_g(S)\ \omega(S, P),$$

where the function ω(S, P) is defined by ([2], [3])

$\omega\ (S,P)\ \ = \frac{\partial P_w\ (S,P)}{\partial P} = \ \frac{\partial P_g\ (S,P)}{\partial P}$

- Let a Porous domain $\Omega\ C\ R^d$, d = 1, 2, 3 be bounded, connected, Lipschitz domain.

The domain boundary is considered to be decomposed as

$$\partial\Omega = \Gamma D \cup\ \Gamma N$$

The time interval is (0, T) of $Q = \Omega$ x (0, T), $\Gamma i^T\ \ = \Gamma i$ x (0, T), $i \in \{ D, N\}$.

- The boundary conditions for the system as follows,

$\theta = \theta_D$, $P = P_D$ on ΓD^T ⟶ 2.7

$Q_w.n = G_w$, $Q_n.\ n = G_g$ on ΓN^T ⟶ 2.8

Here P_D, θ_D, G_w and G_g are given functions, n is the outward unit normal to $\partial\Omega$ and

$Q_w = \rho_w q_w\ \ = -\ \Lambda_w(S, P)K\nabla P + A(S, P)K\nabla\theta + \rho^2\ \lambda_w(S)\ Kg$

$Q_g = \rho_g\ (P_g)\ q_g\ \ = -\ \Lambda_g (S, P)K\nabla P - A(S, P)K\nabla\theta + \rho_g (S, P)^2\ \lambda_g (s)\ Kg$

are the phase mass fluxes with q_j being the volumetric velocity of the j – phase, $j \in \{ w, g\}$

- The Dirichlet boundary data P_D, θ_D are assume to extended to the whole domain Q.

The space $W = \{\phi \in L^2\ (0, T, H^1\ (\Omega))\ \phi \in L^\infty\ (0, T, L^1 (\Omega)), \partial_t\ \phi \in L^1 (Q)\ \}$

With the norm

$|||\phi||| \ = ||\phi||_{L^2\ (O,T,H^1(\Omega))} + ||\phi||_{L^\infty\ (O,T,H^1(\Omega))} + \ ||\partial_t\phi||_{L^1\ (Q)}$

- Define $S_D = S\ (\theta_D)$, where $S = \beta^{-1}$ and $P_{wD} = P_w (S_{D,}\ P_D\ )$, $P_{gD} = P_g (S_{D,}\ P_D)$

The initial conditions are,

$\theta\ (x, 0) = \theta_0\ (x)$, $P(x, 0) = p_o (x)$ in Ω ⟶ 2.9

(A_1) The porosity $\Phi \in L^\infty\ (\Omega)$ and there exist constants

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$0 < \phi_m \leq \phi_M < +\infty$, such that $0 < \phi_m \leq \Phi(x) \leq \phi_M$ a.e. in Ω

(A$_2$) The permeability tensor $K \in (L^{\infty}(\Omega))^{dxd}$, and there exist constants

$0 < k_m \leq k_M < +\infty$, such that for almost all $x \in \Omega$ and all $\xi \in R^d$ it holds:

$k_m |\xi|^2 \leq K(x)\xi.\xi \leq k_M |\xi|^2$

(A$_3$) Relative mobilities $\lambda_w, \lambda_g \in C([0,1]; R^+)$, $\lambda_w(S_w = 0) = 0$

and $\lambda_g(S_g = 0) = 0$. λ_j is an increasing function of S_j. There exist a constants $\lambda_M \geq \lambda_m > 0$ such that for all $S \in [0, 1]$

$$0 < \lambda_m \leq \lambda_w(S) + \lambda_g(S) \leq \lambda_M$$

(A$_4$) There exist a constants $p_{c,\ min} > 0$ and $M > 0$ such that the capillary pressure function

$S \longrightarrow P_c(s)$, $P_c \in C([0,1); R^+) \cap C^1((0,1); R^+)$,

for all $S \in (0, 1)$ satisfy $P_c'(S) \geq p_{c},\ min > 0$ $\longrightarrow$ 2.10

$\int_0^1 Pc'(S)\, ds + \sqrt{\lambda_w(S)\lambda_{g(S)}}\ Pc'(S) \leq M$ $\longrightarrow$ 2.11

There exist $S^{\#} \in (0, 1)$ and $\gamma > 0$ such that for all $S \in (0, S^{\#})$

$S^{2-\gamma} P_c(S) \leq M,$ $\longrightarrow$ 2.12

$P_c'(S) - P_c(0) \leq M\, S\, P_c'(S)$ $\longrightarrow$ 2.13

(A5) There exist $0 < \tau < 1$ and $C > 0$ such that for all $S_1, S_2 \in [0, 1]$

$$C\left|\int_{S_1}^{S_2} \sqrt{\lambda_w(S)\lambda_g(S)\, ds}\right|^{\tau} \geq |S_1 - S_2|$$

(A6) $\rho_w > 0$, ρ_g is a $C^1(R)$ increasing function and there exist

$\rho_m, \rho_M > 0$ such that for all $p \in R$ it holds

$\rho_m \leq \rho_g(p) \leq \rho_M$, $0 < \rho_g'(p) \leq \rho_M$

(A7) $F_w, F_g \in L^2(Q)$, $F_w \geq 0$ a. e. in Q

(A8) The boundary and initial data satisfy

$P_D, P_C(S_D) \in W$, $0 \leq S_D \leq 1$ a.e in Q;

$G_w, G_g \in L^2(\Gamma_N)$, $G_w \leq 0$;

$p_0, \theta_0 \in L^2(\Omega)$, $0 \leq \theta_0 \leq \beta(1)$ a.e in Ω

Remark 2.1

A consequence of incompressibility of the wetting phase, the restrictions on the capillary pressure Pc in **(A_4)** are given only at S = 0, which is less strict compared to the corresponding assumptions in [4], where both phase are compressible.

From assumptions,

P_D , $P_C(S_D) \in W$ in **(A_8)** if follows that the functions

$P_{wD} = P_w(S_D, P_D)$ and $P_{gD} = P_g(S_D, P_D)$ are also in the space W.

That is $S_D \in W$.

Remark 2.2

- In [2] that ω is smooth function for which there is a constant C such that

$$e^{-CS} \leq \omega(s, p) \leq 1 \text{ in } [0, 1] \times R \longrightarrow 2.14$$

It follows from (3.10) and **(A5)** that $S = \beta^{-1}$ is Holder Continuous with exponent τ.

$$\frac{{p_c}^{\tau},min}{C} \quad |S_2 - S_1| \leq |\beta(S_2) - \beta(S_1)|^{\tau} \longrightarrow 2.15$$

- The Dirichlet boundary condition, introduce the space

$$V = \{u \in H^1(\Omega),\ u\,|\,\Gamma_D = 0\}$$

III.REGULARIZED GLOBAL FORMULATION

REGULARIZED PROBLEM

Introducing a regularized capillary pressure derivative, a regularized capillary pressure and regularized phase pressure as follows:

$$R_\eta(P_c'(S)) = \begin{cases} 2(1-S/\eta)\dfrac{P_C(\eta)-P_C(0)}{\eta} + [2\dfrac{S}{\eta}-1]\,P'_c(\eta) & \text{for } S \leq \eta \\ P'_c(S) & \text{for } \eta \leq S \leq 1-\eta \\ P'_c(1-\eta) & \text{for } 1-\eta \leq S \leq 1 \end{cases} \longrightarrow 3.1$$

$$P_c^{\eta}(S) = P_c(0) + \int_0^S R_\eta\, p_c'(s))\, ds \longrightarrow 3.2$$

$$P_g^{\eta}(S,P) = P + P_c(0) + \int_0^S f_w(S, P)\, R_\eta\, p_c'(s))\, ds \longrightarrow 3.3$$

$$P_w^{\eta}(S,P) = P - \int_0^S f_g(S, P)\, R_\eta\, p_c'(s))\, ds \longrightarrow 3.4$$

Clear that $P_g^{\eta}(S,P) - P_{w_\eta}^{\eta}(S,P) = P_c^{\eta}(S)$.

In [4], for any $\eta > 0$, $P_c(S)$ is bounded, monotone, $C^1[0,1])$ function

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P_c^{η} (S) = P_c (S) for S ∈ [η, 1−η].

For Small η it holds

$\frac{d}{dS} P_c^{\eta}$ (S) ≥ $P_{C,\min}$ /2> 0 ⟶ 3.5

| R_η P_c^{η} (S)) | $P_{C,\max}$ < +∞

For some constant $P_{c,max}^{\eta}$ and there is a constant M ≥ 1 such that

R_η ($P_{c'}$ (S) ≤ M $P_{c'}$ (S), for S ∈ (0,1) ⟶ 3.6

The derivatives of the regularized phase pressures are equal in the non regularized case can be written as

$\frac{\partial P_g^{\eta}}{\partial P} = \frac{\partial P_w}{\partial P} = \omega^{\eta}$ (S,P).

∇P_g^{η} = ω^{η} (S,P) ∇P + f_w (S,P) R_η (P_c' (S) ∇ S ⟶ 3.7

∇P_w^{η} = ω^{η} (S,P) ∇P – f_g (S,P) R_η (P_c' (S) ∇ S ⟶ 3.8

P_g^{η}, P_w^{η}, ∈ L^2 (0,T; H^1Ω)) for P,S ∈ L^2 (0,T; H^1Ω)).

Consider the regularized system (2.4) (2.5) in which replace ρ_g (S, P) by

P_g^{η}(S,P) : = P_g^{η}(P_g^{η}(S,P)) ⟶ 3.9

and the function A(S, P) by A^{η}(S, P) for η > 0, defined by

A^{η} (S,P) = $\frac{\rho_w \rho_g (S,P)}{\lambda(S,P)}$ λ_w (S) λ_g (S) R_η (P_c' (S)) + η> 0 ⟶ 3.10

- Regularized system as

$-\rho_w \Phi \frac{\partial S^{\eta}}{\partial t}$ – div (Λ_w^{η} (S^{η}, P^{η}) K∇ P^{η}) + div A^{η} (S^{η}, P^{η}) K∇ S^{η})+ρ_w^2 div (λ_w (S^{η}) K_g) = F_w ⟶ 3.11

Φ $\frac{\partial}{\partial t}$ (ρ_g^{η} (S^{η}, P^{η})S^{η})-div (Λ_g^{η} (S^{η}, P^{η})K∇P^{η})-divA^{η}($S^{\eta}$$P^{\eta}$)K∇$S^{\eta}$)+ div ($\lambda_g$ (S^{η})ρ_g^{η}(S^{η},P^{η})^2 K_g)=F_g ⟶ 3.12

Where Λ_w^{η} (S,P) = ρ_w λ_w (S) ω^{η} (S,P)

Λ_g^{η} (S,P) = ρ_g (S,P) λ_g (S) ω^{η} (S,P) ⟶ 3.13

- The regularized total mobility

Λ^{η} (S,P) = Λ_w^{η} (S,P) + Λ_g^{η} (S,P) ⟶ 3.14

and the regularized function β

$$\beta^{\eta}(S) = \int_0^S \sqrt{\lambda_w(s)\lambda_g(s)} \quad R_\eta P_c'(s))\, ds \longrightarrow \quad 3.15$$

Where $S^{\eta} = (\beta^{\eta}) - 1$

Some uniform estimates and limits for regularized co-efficient proved in [4].

Lemma (3.1)

Assume **(A4)** and **(A6).** Then three exists a constant C > 0. independent of η, such that

$$\left.\begin{array}{l} |P_g^{\eta}(S,P)| \leq C(|P| +1), \\ P_w^{\eta}(S,P) \leq P, \\ |\lambda_W(S) P_w^{\eta}(S,P)| \leq C(|P| +1) \\ e^{-cs} \leq \omega^{\eta}(S,P) \leq 1 \end{array}\right\} \longrightarrow \quad 3.16$$

and the following sequences converge uniformly in [0,1] x R as $\eta \to 0$.

$$\left.\begin{array}{l} P_g^{\eta}(S,P) \longrightarrow P_g(S,P), \\ \omega^{\eta}(S,P) \longrightarrow \omega(S,P) \\ \Lambda_j^{\eta}(S,P) \longrightarrow \Lambda_j(S,P), \quad j \in \{w, g\} \\ \beta(S) \longrightarrow \beta^{\eta}(S) \text{ uniformly in } [0,1] \end{array}\right\} \longrightarrow \quad 3.17$$

Remark : 3.2

The assumption on the boundary data P_D, $P_c(S_D) \in W$ in **(A8).**

P_{wD}, P_{gD}, $\beta(S_D) \in W$. Define $P_{w_D}^{\eta} = P_w^{\eta}(S_D, P_D)$ and $P_{g_D}^{\eta} = P_g^{\eta}(S_D, P_D)$ and use (2.21), show that the norms $|||P_{wD}|||$, $|||P_{gD}|||$ and $|||\beta(S_D)|||$ are uniformly bounded with respect to the parameter η.

For example,

$$\nabla P_w^{\eta} D = \omega^{\eta}(S_D,P_D) \nabla P_D + f_g(S_D,P_D) R_\eta(P_c'(S_D) \nabla S_D \text{ by the estimate}$$

$$|\nabla P_w^{\eta} D| \leq \nabla P_D + M P_c'(S_D) |\nabla(S_D)|$$

leading to $|\nabla P_w^{\eta} D|_{L^2(0,T;H^1\Omega))} \leq C(1 + ||\nabla P_D||_{L^2(0,T;H^1\Omega))} + ||P_c(S_D)||_{L^2(0,T;H^1\Omega)})$

Then we have

$$\beta(S_D) \longrightarrow \theta_D \text{ weakly in } L^2(0,T;H^1\Omega)) \text{ as } \eta \longrightarrow 0.$$

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Theorem : 3.3 REGULARIZED GLOBAL FORMULATION

Let **(A1)** – **(A8)**. Denote $S = S(\theta)$. Then there exists (P, θ)

such that

$P \in L^2(0, T; V) + P_D,\ \theta \in L^2(0, T; V) + \theta_D,\quad 0 \le \theta \le \beta(1)$ a.e in Q

$\partial_t (\Phi S) \in L^2(0, T; V'),\quad \partial_t (\Phi\, \rho_g (S, P) S) \in L^2(0, T; V');$

for all $\phi, \psi \in L^2(0, T; V)$

$$-\rho_w \int_0^T < \partial_t (\Phi S), \phi > dt + \int_Q [\, \Lambda_w (S,P) K\nabla P.\nabla\phi]dxdt - A(S,P) K \nabla\theta.\nabla\phi]\, dx\, dt - \int_Q \lambda_w (S)\, \rho_w^2 Kg.\nabla\phi\, dx\, dt$$

$$= \int_Q F_w\, \phi\, dx\, dt - \int_{\Gamma_N^T} G_w\, \phi\, d\sigma\, dt \qquad \longrightarrow \qquad 3.18$$

$$\int_0^T < \partial_t(\Phi\rho_g(S,P)\, S),\Psi> dt + \int_Q [\Lambda_g(S,P) K\, \nabla P \nabla\Psi] dxdt + A(S,P) K \nabla\theta.\nabla\Psi]\, dx\, dt - \int_Q \lambda_g (S)\, \rho_g^2 Kg.\nabla\Psi dxdt$$

$$= F_g\, \Psi\, dx \int_Q dt - G_g\, \Psi\, d\sigma \int_{\Gamma_N^T} dt \qquad \longrightarrow \qquad 3.19$$

Furthermore, for all $\psi \in V$ the functions

$$t \rightarrow \int_\Omega \Phi\, S\Psi dx,\quad t \rightarrow \int_\Omega \Phi\rho_g (P_g(S,P))\ S\Psi dx$$

are continuous in [0, T] and the initial conditions all satisfied in the following sense.

$$\left[\int_\Omega \Phi\, S\Psi dx\right](0) = \int_\Omega \Phi\, S_0\Psi dx$$

$$\left[\int_\Omega \Phi\rho_g (P_g(S,P))\ S\Psi dx\right](0) = \int_\Omega \Phi\rho_g\ (P_g\,(s_0, \rho_0)\, s_0\, \psi\, dx \qquad \text{where } s_0 = S(\theta_0)$$

IV. A NEW GLOBAL FORMULATION

In this section, we extend the global pressure formulation in the general case and the purpose of this computation is to define a global pressure p such that (2.2) and (2.3) are exactly equivalent to a set of two coupled equations.

Setting,

$P_g^\eta(S, P) \rightarrow P_g(S, P),$

$\omega^\eta(S, P) \rightarrow \omega(S, P)$

$\Lambda_j^\eta(S, P) \rightarrow \Lambda_j(S, P),\qquad j \in \{w, g\}$

$\Lambda_g(S, P) = \rho_g(S, P)\ \lambda_g(S)\ \omega(S, P)$

Now, it is easy to obtain an analytic solution of this problem, then $A^n(S,P)$ is well determined.

The rest of the computations to obtain the fractional flow formulations is the same as in section 3 with the difference that A(S,P) should be replaced as $A^n(S,P)$ in all coefficients. Then we obtain a new global pressure saturation formulation of the problem given by:

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$$P_w^{\eta} \int_0^T < \partial_\tau(\phi s), \phi > dt + \int_\Omega [\Lambda_w^{\eta} (S,P)\, k\, \nabla p_o \nabla Q - A^n (S,P) \nabla Q]\, dx\, dt -$$

$$\int_Q \lambda_w^{\eta} (S)\, \rho_w^2\, kg.\, \nabla\phi\, dx = \int_Q F_w^{\eta}\, \phi\, dx\, dt - \int_{\Gamma_N^T} G_w^{\eta} \phi\, d\, \sigma\, dt \longrightarrow (4.1)$$

$$\int_0^T < \partial_t (\phi \rho_g^{\eta} (s,p)\, s, \Psi > dt + \int_Q [\Lambda_g^{\eta} (s,p)\, k\, \nabla p_o \nabla\Psi + A^{\eta} (s,p)\, k\, \nabla\theta \nabla\Psi]\, dx\, dt$$

$$- \int_Q \lambda_g (s)\, \rho_g^{\eta} (s,p)^2\, k\, g_0 \nabla\Psi\, dx = \int_Q F_g \Psi\, dx = \int_{\Gamma^T{}_N} G_g \Psi d\sigma\, dt \longrightarrow (4.2)$$

Note that the expression of ω in the regularized formulation (3.18) and (3.19) are not simply related. It should be noted that this new formulation require to solve a family of ordinary differential equations which could numerically be done by using standard libraries existing in the literature.

NUMERICAL COMPARISION

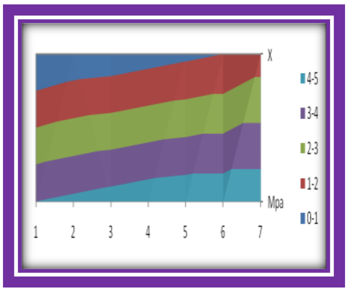
In this section we compare the coefficients numerically in regularized and new global pressure formulations.

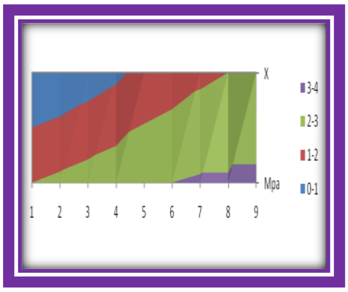
Data are chosen from the benchmark problem Couplex-Gaz [6] proposed by the ANDRA (the French National Radioactive Waste Management Agency), we consider the van Genuchten function with parameters n = 1.54 and P_r = 2 MPa.

Differences between corresponding coefficients in PDEs (3.18)–(3.19) and (4.1)–(4.2) behave in a consistent way when varying the global pressure p, so that it is sufficient to represent one of them. We choose to present the diffusion coefficient A(S.P) from (3.18)–(3.19), and the corresponding diffusion coefficient A^n(S,P) from (4.1)–(4.2).

The two diffusion coefficients are presented in the right colon of Figure 1 to 4. In the left colon we plot gas pressure $\Lambda_w{}^n$(S,P) from the new global formulation and the gas pressure Λ_w(S,P) given from the regularized global formulation, where hat denotes again that the water saturation is replaced by the capillary pressure. We present functions for three different fixed values of the global pressure p, namely, 2 MPa & 4 MPa.

For p = 2 MPa,





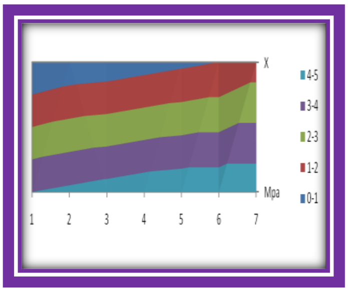
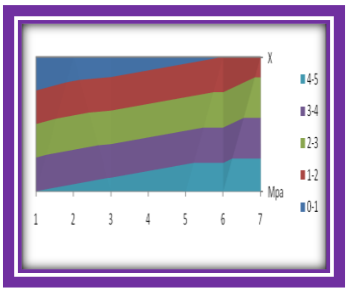
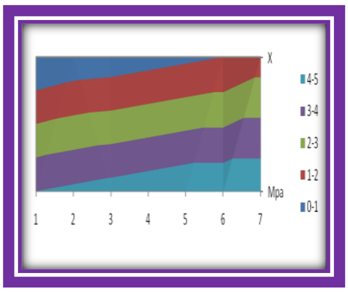


Figure 1 Figure 2 Figure 3





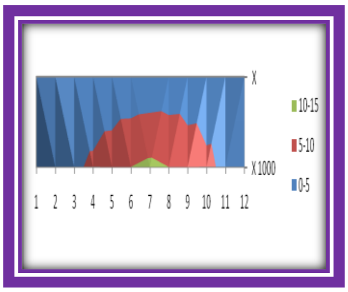


Figure 4 Figure 5 Figure 6

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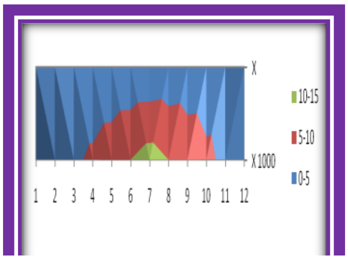

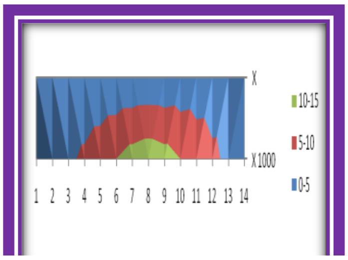

Figure 7 Figure 8

From the behavior of coefficients we can draw several conclusions. If we look only at the error committed by calculating the coefficients in global pressure instead of gas pressure then we see that this error can be significant only when typical capillary pressure in the system is comparable to global or gas pressure. That may be the case for small operating pressures, for example in hydro-geological applications of water-air system. In the other hand that difference can be safely ignored in typical oil field conditions. Contrary to that the error committed in calculating gas density in (3.18) by replacing the gas pressure by the global pressure stays significant and leads to unacceptable loss of mass balance which is an important property of the physical solution. This can clearly be seen from the left colon of Figure 1 to 4.

V.CONCLUSION

A new global formulation is formulated for the immiscible compressible two-phase flow in porous media by the concept of global pressure and the formulations between the regularized and the new global one are compared numerically.

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