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Regular Algorithms of Synthesis of Parametrically Invariant Control Systems of Uncertainty Objects

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ABSTRACT: Computational regularized procedures for the synthesis of parametrically invariant control systems for undefined objects are presented based on the concepts of modal control. To solve this problem, we use a statistical regularizing algorithm based on the singular matrix decomposition. The expediency of choosing a regularization parameter based on the L-curve is shown. These computational procedures make it possible to regularize the problem of synthesizing invariant control systems and to improve the quality indicators of the processes of controlling dynamic objects under conditions of uncertainty.

KEYWORDS: indeterminate objects, parametrically invariant control systems, pseudo-inversion, singular expansion, regularization, regularization parameter.

I. INTRODUCTION

Synthesis of automatic control systems is usually carried out based on the known design model of the object, when there is complete information about the static and dynamic properties of the control object and the effects on the object of the external environment. Under the completeness of the information here is meant also the knowledge of possible changes of certain dynamic properties of the object and the impacts, i.e. These changes are controlled and can be measured. If they are uncontrollable, then under certain assumptions and certain boundaries of uncontrolled changes in the properties of the object and its effects on it, in principle one can ensure a lower sensitivity to these changes of the desired properties of the control system. As a rule, these changes relate to small changes in respect of the design properties specified by the object-to-environment model [1-4].

However, there are many important objects and processes in virtually all industries, characterized by the fact that the static and dynamic properties of objects and environmental influences change in an uncontrolled manner, and a priori information about these changes and about the physical, mathematical and calculation models «object-environment» only to a certain extent correspond to reality. This is due to the fact that the a priori incompleteness of the multidimensional model is due to the unavoidable assumptions that are made when compiling the mathematical model of the controlled object, the complexity of the topology and the operators linking the variables at the input and output of the object model, as well as the increased sensitivity of the output variables of the object to uncontrollable changes in perturbations [5-8].

Recently, a very constructive trend is developing, when minimization of the undesirable effect of parametric uncertainty of the model representation of the initial objects is achieved in the class of control laws that ensure the invariance of the «parametric input-output» relation. Basic concepts of reducing the sensitivity problem to the problem of analyzing the system properties of the synthesized system of the relation «parametric input - system variables» are described in [4,9-12].

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II. FORMULATION OF THE PROBLEM

Consider a discrete control object that is represented in such a basis that the variation of the parameters leads only to perturbation of the state matrix of the control object so that it obtains a model representation

$$\begin{cases} x_{k+1} = (A + \Delta A)x_k + Bu_k, \\ y_k = Cx_k. \end{cases} \quad (1)$$

We represent the variation ΔA of the state matrix of the control object in the additive form

$$\Delta A = \sum_{j=1}^{p} \Delta A_j , \quad (2)$$

where each *j*-th matrix component ΔA_j of the total variation satisfies condition

$$rang \Delta A_j = 1, \ \ j = \overline{1, p}, \ \ \Delta A_j = d_j h_j^T , \quad (3)$$

where $d_j, h_j \in R^n$.

Then, taking into account (2), (3), expression (1) can be written in the form

$$\begin{cases} x_{k+1} = Ax_k + \sum_{j=1}^{p} d_j h_j^T x_k + Bu_k, \\ y_k = Cx_k, \end{cases} \quad (4)$$

in which $x \in R^n$, $u \in R^m$, $y \in R^l$ – state, control and output vectors, $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{l \times n}$ – control and output state matrices.

Equation (4) on the basis of representations

$$\zeta_k = col\{\zeta_{j,k} = h_j^T x_k; \ j = \overline{1, p}\},$$

$$D = row\{D_j = d_j; j = \overline{1, p}\},$$

can be represented by a vector-matrix model of the form [9-11]

$$x_{k+1} = Ax_k + D\zeta_k + Bu_k; \quad y_k = Cx_k,$$

where the vector ζ_k is called the external parametric action.

Assuming the control law u_k in the form

$$u_k = K_g g_k - Kx_k \quad (5)$$

and combining (1) and (5) we write the equation of the system as follows

$$x_{k+1} = Fx_k + Gg_k + D\zeta_k; \quad y_k = Cx_k,$$

where $F = A - BK$; $G = BK_g$, g_k – the setting influence.

$$K = HM^{-1} : M = \arg\{MS - AM = -BH\}.$$

where $(S, H) = \arg\{S = diag\{s_i; \ i = \overline{1, n}\} \ \& \ observ(S, H)\}$.

$$\left[D \mid \tilde{M}\right] \begin{bmatrix} S_p & 0 \\ 0 & \tilde{S} \end{bmatrix} - A\left[D \mid \tilde{M}\right] = -B\left[H_p \mid \tilde{H}\right].$$

To calculate the matrices K and K_g, we usually use the Sylvester matrix equation [9] of the form

$$\begin{aligned} DS_p - AD &= -BH_p, \\ \tilde{M}\tilde{S} - A\tilde{M} &= -B\tilde{H}, \end{aligned} \quad (6)$$

where

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$$\begin{gathered}\left(S_p,\tilde{S}\right): S = diag\left\{S_p = diag\left(s_i;\ i=\overline{1,p}\right);\right.\\ \left.\tilde{S} = diag\left(s_i;\ i=\overline{p+1,n}\right)\right\},\\ \tilde{M} = \arg\{\tilde{M}S - A\tilde{M} = -B\tilde{H}\},\\ \tilde{H} = \arg\left\{observ\left(\tilde{S},H\right)\right\},\end{gathered} \tag{7}$$

where $\arg\{\tilde{M}S - A\tilde{M} = -B\tilde{H}\}$ – argument of performance of a condition $\tilde{M}S - A\tilde{M} = -B\tilde{H}$, $observ\left(\tilde{S},H\right)$ – predicate of the existence of complete observability of a pair of matrices $\tilde{S},H$.
Taking (6.7) into account, we can write

$$H_p = B^+(AD - DS_p), \tag{8}$$

$$K = \left[B^+(AD-DS_p)\middle|\tilde{H}\right]\left[D\middle|\tilde{M}\right]^{-1}, \tag{9}$$

$$K_g = -\left(CF^{-1}B\right)^{-1} = -\left\{C\left[D\middle|\tilde{M}\right]S^{-1}\left[D\middle|\tilde{M}\right]^{-1}\right\}^{-1}. \tag{10}$$

The direct use of relations (8), (9) can lead to a decrease in the accuracy of the estimation of matrices H_p and K, since they use the pseudo inversion operation. It is known [13,14] that if the matrix B has full rank ($k = \min(m,n)$), then

$$B^+ = (B^T B)^{-1} B^T.$$

In the case where the matrix B is a matrix of incomplete rank, then the problem under consideration is ill-posed. To give numerical stability to the pseudo inversion procedure for the matrix B, it is advisable to use the concepts of regular methods [14-16].

III. SOLUTION OF THE TASK

For convenience of further calculations, we introduce the following notation:

$$BH_p = L, \qquad L = (AD - DS_p), \tag{11}$$

$$BK = P, \qquad P = \left[(AD-DS_p)\middle|\tilde{H}\right]\left[D\middle|\tilde{M}\right]^{-1}. \tag{12}$$

We write equation (11) in the next coordinate form

$$Bh_{p,j} = l_j, \quad j = 1,2,\ldots,p, \tag{13}$$

where $h_{p,j}$ and l_j – j– th vectors are the columns of the matrices H_p and L, respectively.

Assume that the system (13) for the «exact» right-hand side l_j has an «exact» normal pseudo-solution $h^+_{p,j}$. Assuming a realistic point of view, we assume that instead of the exact right-hand side l_j, we know the vector

$$\tilde{l}_j = l_j + \eta,$$

where η is the random error vector of the right-hand side.
The vector η has a $M[\eta] = 0_N$ and covariance matrix $V_\eta = M[\eta\eta^T]$ that can be represented

$$V_\eta = \sigma^2_\eta \cdot C_\eta,$$

where σ^2_η – is the variance, and the elements of the matrix C_η characterize the relative values of the variances or the correlation relationships between the projections of the vector η.
Let us consider some of the most constructive algorithms for determining pseudoinverse matrices [14, 17-19]. Suppose that the matrix B is a matrix of incomplete rank $k < m$. Then there exist an orthogonal $n \times n$ -matrix U, an orthogonal $m \times m$ -matrix V, and a diagonal $n \times m$ -matrix Λ such that

$$U^T BV = \Lambda, \quad B = U\Lambda V^T. \tag{14}$$

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The matrix Λ can be chosen so that its diagonal elements constitute a nonincreasing sequence; all these elements are nonnegative and exactly *k* of them are strictly positive. Moreover, the diagonal elements Λ are the singular numbers of the matrix *B*.
Let

$$Q^T BW = R\,,\; B = QRW^T\,, \tag{15}$$

where

$$R = \left[\begin{array}{c|c} R_{11} & 0 \\ \hline 0 & 0 \end{array}\right], \tag{16}$$

and R_{11} – nondegenerate triangular $k \times k$ – a matrix.
The choice H and K in the equation (15), it can be achieved that R_{11} in (16) is upper or lower triangular [13,17]. Since the $k \times k$ -matrix R_{11} of (16) is non-degenerate, we can write

$$R_{11} = \tilde{U}\tilde{\Lambda}\tilde{V}^T\,. \tag{17}$$

Here $\tilde{U}$ and $\tilde{V}$ – are orthogonal $k \times k$ – matrices, and $\tilde{\Lambda}$ – is a nondegenerate diagonal matrix whose diagonal elements are positive and do not increase.
It follows from (17) that the matrix *R* of equation (16) can be written in the form

$$R = \hat{U}\Lambda\hat{V}^T\,, \tag{18}$$

where $\hat{U}$ – the orthogonal $n \times n$ – matrix:

$$\hat{U} = \left[\begin{array}{c|c} \tilde{U} & 0 \\ \hline 0 & I_{m-k} \end{array}\right], \tag{19}$$

$\hat{V}$ – the orthogonal $m \times m$ – matrix:

$$\hat{V} = \left[\begin{array}{c|c} \tilde{V} & 0 \\ \hline 0 & I_{n-k} \end{array}\right] \tag{20}$$

and Λ – diagonal $n \times m$ – matrix:

$$\Lambda = \left[\begin{array}{c|c} \tilde{\Lambda} & 0 \\ \hline 0 & 0 \end{array}\right]. \tag{21}$$

Now, defining *U* and *V* by formulas

$$U = Q\hat{U}\,, \tag{22}$$

$$V = W\hat{V}\,, \tag{23}$$

we conclude from (15), (17) - (23) that

$$B = U\Lambda V^T\,, \tag{24}$$

Where U , Λ and V have the properties given in (14).
The singular numbers of the matrix *B* are uniquely determined in spite of the fact that in the choice of the orthogonal matrices *U* and *V* of (24) there is an arbitrariness.
Taking into account the orthogonal decomposition $B = QRW^T$, the unique solution of the minimum length of the problem (13) is expressed by the formula [17]:

$$h_{p,j} = W\left[\begin{array}{c|c} R_{11}^{-1} & 0 \\ \hline 0 & 0 \end{array}\right]Q^T l_j\,, \qquad j = 1,2,\ldots,p\,.$$

Thus, for the orthogonal decomposition $B = QRW^T$, the pseudo-inverse matrix is defined by the expression $B^+ = WR^+Q^T$, where R^+ is given by the formula.

$$R^+ = \left[\begin{array}{c|c} R_{11}^{-1} & 0 \\ \hline 0 & 0 \end{array}\right].$$

If for *B* there is an orthogonal decomposition (15) and R_{11} in (16) is a non-degenerate triangular matrix, then

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$$B^{+} = W\begin{bmatrix} R_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} Q^{T}.$$

To solve equation (13), we will use regularizing algorithms based on the singular matrix expansion [17-19]. The same algorithm can also be used to estimate the matrix K in equation (12).

Using the singular expansion, we consider a statistical regularizing algorithm that minimizes the mean square error of finding the normal pseudosolution of system (13) in the following form [20]:

$$h_{p,j}^{\alpha} = \sum_{j=1}^{p} \left[\frac{\lambda_j}{\lambda_j^2 + \alpha m(\lambda_j)} \cdot \left\langle u_j, \tilde{l}_j \right\rangle \right] v_j \ , \tag{25}$$

where the v_j, u_j – j – th column of the matrices V, U, entering the singular expansion $B = U\Lambda V^{T}$; $m(\lambda) = 1/\lambda^{\gamma}$, $\gamma > 1$; $\alpha > 0$ – is the regularization parameter.

There are various ways of choosing the regularization parameter [15, 16, 20]. We will use the L–curve method [20,21]. A L – curve is a parametric curve with coordinates $(\rho(\alpha), \gamma(\alpha))$, where $\rho(\alpha) = \left\| \tilde{l}_j - Bh_{p,j}^{\alpha} \right\|^2$, $\gamma(\alpha) = \left\| h_{p,j}^{\alpha} \right\|^2$ – are functionals in the A.N.Tikhonov regularization method [15].

Consider the algorithm for calculating the regularization parameter α_L at the corner point of the L – curve. For the regularized solution (25) using the singular expansion of $B = U\Lambda V^{T}$ one can write the following coordinates of the L – curve [20]:

$$\rho(\alpha) = \left\| \tilde{l}_j - Bh_{p,j}^{\alpha} \right\|^2 = \sum_{j=1}^{p} \left(\frac{\alpha m(\lambda_j)}{\lambda_j^2 + \alpha m(\lambda_j)} \right)^2 \tilde{y}_j^2 + \sum_{j=p+1}^{N} \tilde{y}_j^2 \ ,$$

$$\gamma(\alpha) = \left\| h_{p,j}^{\alpha} \right\|_{W_{h_{p,j}}}^2 = \sum_{j=1}^{p} m(\lambda_j) \cdot \left(\frac{\lambda_j}{\lambda_j^2 + \alpha m(\lambda_j)} \right)^2 \tilde{y}_j^2 \ ,$$

where $\tilde{y}_j$ – the projection of vector $\tilde{y} = U^{T} \tilde{l}_j$.

In logarithmic coordinates

$$\hat{\rho}(\alpha) = \ln \rho(\alpha), \quad \hat{\gamma}(\alpha) = \ln \gamma(\alpha).$$

Curvature $k_L(\alpha)$ L – the curve of the regularized solution $h_{p,j}^{\alpha}$ can be defined by the expression [20]:

$$k_L(\alpha) = \frac{2\gamma(\alpha)\rho(\alpha)\left[\alpha \cdot \rho(\alpha) \cdot \gamma'(\alpha) + \rho(\alpha) \cdot \gamma(\alpha) + \alpha^2 \cdot \gamma(\alpha) \cdot \gamma'(\alpha)\right]}{\gamma'(\alpha)\left[\alpha^2 \cdot \gamma^2(\alpha) + \rho^2(\alpha)\right]^{3/2}}.$$

As the regularization parameter α_L, we take the solution of the following variational problem:

$$\max_{\alpha > 0} k_L(\alpha). \tag{26}$$

The use of singular decomposition significantly reduces computational costs both for calculating the curvature of the L – curve and for solving the variational problem (26) as a whole. The choice of α by the L – curve method at $m(\lambda) = 1/\lambda^{\gamma}$, $\gamma > 1$, leads to an overestimation compared to α_{opt} values. Therefore, it is recommended to select α by the L – curve method for $m(\lambda) = const$.

IV. CONCLUSION

These computational procedures make it possible to regularize the problem of synthesizing parametrically invariant control systems and improve the quality of control processes under conditions of parametric uncertainty of the state matrix of the controlled object.

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