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Peristaltic Transport of a Williamson Fluid between Porous Walls with Suction and Injection

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ABSTRACT: In this paper, we investigated The Peristaltic flow of a Williamson fluid between two porous walls with suction and injection l, under the assumptions of low Reynolds number and long wavelength. The flow is investigated in a wave frame of reference moving with velocity of the wave. The perturbation series in the Weissenberg number We (less than one) was used to obtain explicit forms for velocity field, pressure gradient and friction force per one wavelength. The effects of Weissenberg number We, Darcy number Da and amplitude ratio φ on the pumping characteristics, friction force and heat transfer are discussed with the help of graphs in detail.

KEYWORDS: Williamson Fluid, Porous walls, Exact solutions.

I. INTRODUCTION

Peristalsis is a well-known mechanism for pumping biological and industrial fluids. This mechanism generally occurs in the gastrointestinal, urinary and re-productive tracts in the living body. Sreenadh and Arunachalam [1] studied theCouette flow between two permeable beds with suction and injection. Mishra and RamachandraRao [2] made a detailed analysis on the peristaltic transport with permeable walls. The first study of peristaltic flow through a porous medium is presented by Elsehawey et al. [3]. Elshehawey et al. [4] have studied the peristaltic motion of a Carreau fluid through a porous medium in a channel. Haroun [5] have studied the non-linear peristaltic flowof a fourth grade fluid in an inclined asymmetric channel. Recently, Kavitha et al. [6] and Hemadri Reddy et al. [7] discussed the peristaltic pumping of non- Newtonian Jeffrey and Carreau fluids between porous walls with suction and injection In view this, it will be interesting to study the peristaltic transport of a non-Newtonian Williamson fluid in a channel between two porous beds with suction and injection is investigated, under long wavelength and low Reynolds number assumptions. The velocity, the stream function, the pressure rise and friction force are obtained. The results are deduced and discussed.

II. MATHEMATICAL FORMULATION

Consider the peristaltic flow of an incompressible Williamson fluid of half width a. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower permeable walls of the channel. The fluid is injected into the channel perpendicular to the lower permeable wall with a constant velocity v0 and is sucked out of the upper permeable wall with the same velocity v0 as shown in figure 1. For simplicity, we restrict our discussion to the half width of the channel.

The wall deformation is given by

$$Y = H(X,t) = a + b \sin 2(X - ct) \quad (1)$$

where b is the amplitude, λ is the wave length and c is the wave speed. We introduce a wave frame of reference (x, y) moving with the velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wave length and the pressure difference at the ends of the channel is a constant. The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$$x = X - ct,\ y = Y,\ u = U - c,\ v = V,\ p'(x) = P'(X, t) \quad (2)$$

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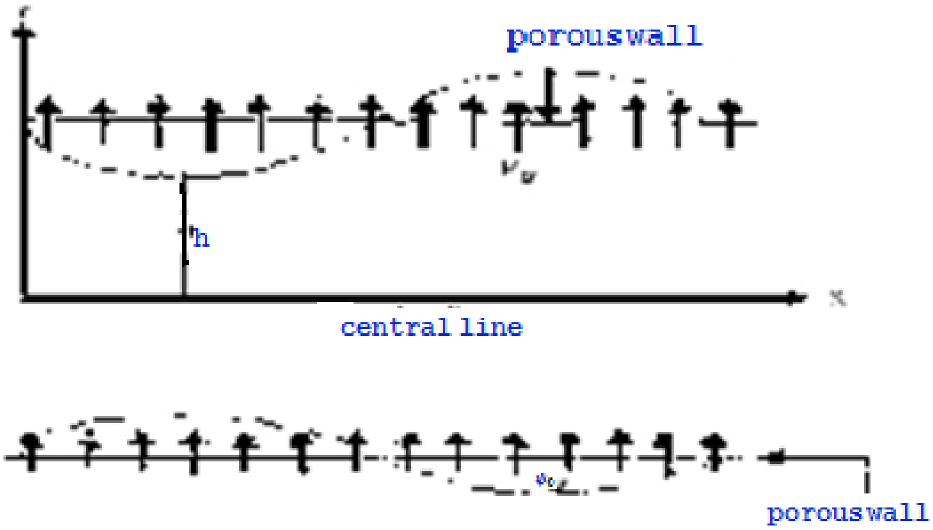

Figure 1: Physical Model

where (u, v) and (U, V) are the velocity components, p and P are the pressures in the wave and fixed frames of reference respectively. The equations governing the flow field, in the wave frame of reference are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial P'}{\partial x}+\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y} \tag{4}$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P'}{\partial y}+\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y} \tag{5}$$

Due to symmetry, the problem is studied only for upper half of the channel. The boundary conditions for the velocity are

$$\frac{\partial u}{\partial y}=0 \text{ at } y = 0 \tag{6}$$

$$u = -c \text{ at } y = H \tag{7}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\bar{x}=\frac{x}{\lambda},\hat{y}=\frac{y}{a},\bar{u}=\frac{u}{c},\bar{v}=\frac{v}{c},h=\frac{H}{a},\overline{P}'=\frac{P'a^2}{\lambda\mu c},\overline{S}=\frac{aS}{\mu c}$$

$$\bar{t}=\frac{ct}{\lambda}, deltta=\frac{a}{\lambda}, \phi=\frac{b}{a}, \mathrm{Re}=\frac{\rho ca}{\mu}, \lambda_i=\frac{\alpha_i c}{\mu a}(i=1,2)$$

$$\xi_j=\frac{\beta_j c^2}{\mu a^2}(j=1,2,3), \eta_k=\frac{\gamma_k c^3}{\mu a^3}(k=1-8) \qquad (8)$$

where Re and δ represents the Reynolds number and wave number respectively. In view of dimensionless quantities (8), the Equations (3) - (5), after dropping bars, reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (9)$$

$$\mathrm{Re}\,\delta(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial P'}{\partial x}+\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y} \qquad (10)$$

$$\mathrm{Re}\,\delta^3(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P'}{\partial y}+\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y} \qquad (11)$$

Under lubrication approach, neglecting the terms of order, from Equations(10) we get.

$$\frac{\partial P}{\partial x}=\frac{\partial}{\partial y}[1+W_e(\frac{\partial u}{\partial y})]\frac{\partial u}{\partial y}-k\frac{\partial u}{\partial y} \qquad (12)$$

where k = Rev0, v0is suction/injection velocity and $w_e=\xi_2+\xi_3$ is the Deborah number. The corresponding dimensionless boundary conditions in thewave frame of reference are given by

$$\frac{\partial u}{\partial y}=0 \text{ at } y=0 \qquad (13)$$

$$u=-1 \qquad \text{at } y=h=1+\phi\sin(2\pi x) \qquad (14)$$

The volume flow rate q in a wave frame of reference is given by

$$q=\int_0^{h(x)} u\,dy \qquad (15)$$

The instantaneous flux Q(X, t) in a fixed frame is

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$$Q(X,t)=\int_0^h UdY=\int_0^h udy=q+h \quad (16)$$

The time average flux $\overline{Q}$ over one period T (= $\frac{\lambda}{c}$) of the peristaltic wave is

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt=\int_0^1 (q+h)dx=q+1 \quad (17)$$

III. PERTURBATION SOLUTION

The equation (12) is non-linear and its closed form solution is not possible. So,we expand u, p and q in terms of G as

$$u=u_0+W_e u_1+O(W_e^2)$$

$$\frac{\partial P}{\partial x}=\frac{\partial p_0}{\partial x}+W_e\frac{\partial P_1}{\partial x}+O(W_e^2)$$

$$q+q_0+W_e q_1+o(W_e^2) \quad (18)$$

Substituting the equations of (18) in (12) and solving the resulting systems, we get

$$u_0--1+\frac{P_0 h}{k}-\frac{p_0}{k^2}e^{kh}+\frac{p_0}{k^2}e^{ky}-\frac{yp_0}{k} \quad (19)$$

$$\frac{dP_0}{dx}=\frac{(q+h)}{k_1} \quad (21)$$

$$\frac{dP_1}{dx}=\frac{q_1}{k_1}-P_0^3\frac{k_2}{k_1} \quad (22)$$

$$q_0=\int_0^h u_0 dy=P_0 k_1-h \quad (23)$$

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$$q_1 = \int_0^h u_1 dy = P_1 k_1 + P_0^3 k_2 \tag{24}$$

in which $P_0 = \frac{\partial P_0}{\partial x}$ and $P_1 = \frac{\partial P_1}{\partial x}$ 1

$$k_1 = \frac{h^2}{k} - \frac{he^{kh}}{k^2} + \frac{e^{kh}}{k^3} - \frac{1}{k^3} - \frac{h^2}{2k}$$

$$k_2 = \frac{1}{k^3}(he^{2kh} + 2he^{kh} - 2h^2 ke^{kh} - \frac{e^{2kh}}{2k} + \frac{1}{2k} \tag{25}$$

Substituting equations (21) and (22) into the equation (18) and using the relation $\frac{dp_0}{dx} = \frac{dP}{dx} - w_e \frac{dp_1}{dx}$ and neglecting terms greater than O(w_e^2) we get

$$\frac{dP}{dx} = \frac{(q+h)}{k_1} - W_e \frac{(q+h)^3 k_2}{k_1^3} \tag{26}$$

The dimensionless pressure rise and frictional force per one wavelength inthe wave frame are defined, respectively as

$$\Delta P = \int_0^1 \frac{dp}{dx} dx \tag{27}$$

$$F = \int_0^1 h(-\frac{dp}{dx})dx \tag{28}$$

IV. RESULTS AND DISCUSSION

The variation of pressure difference as a function of ¯Q for different values of k is shown in Fig.2. We observe that the larger the parameter ∅, thesmaller the pressure rise against which the pump works. For a given flux ¯Q,the pressure rise increases with increasing k. The variation of pressure difference as a function of ¯Q for different values of ∅. The variation of pressure difference as a function of ¯Q for different values of kis shown in Figure3. We observe that the larger the suction parameter k, the smaller the pressure rise against which the pump works. For a given flux ¯Q, the pressure rise decreases with increasing k.

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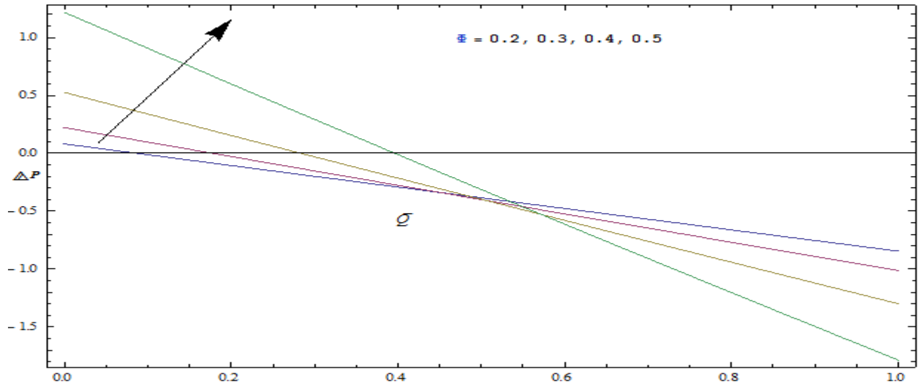

Fig2: The variation of pressure difference as a function of $\bar{Q}$ for different values of k

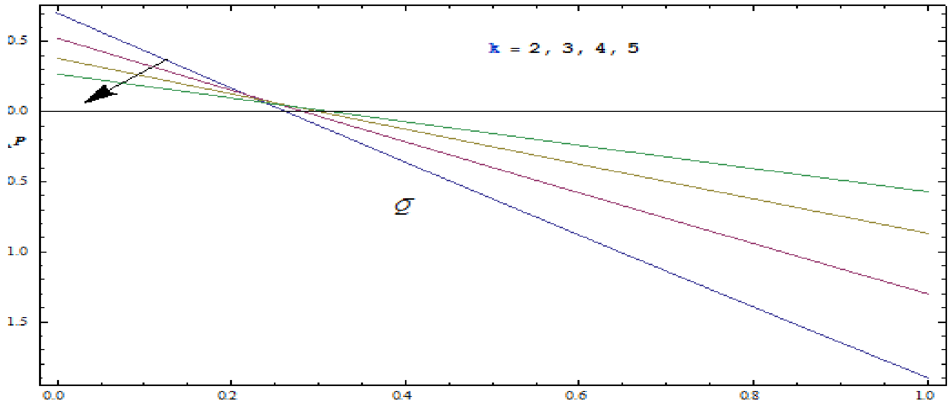

Fig3: The variation of pressure difference as a function of $\bar{Q}$ for different values of k.

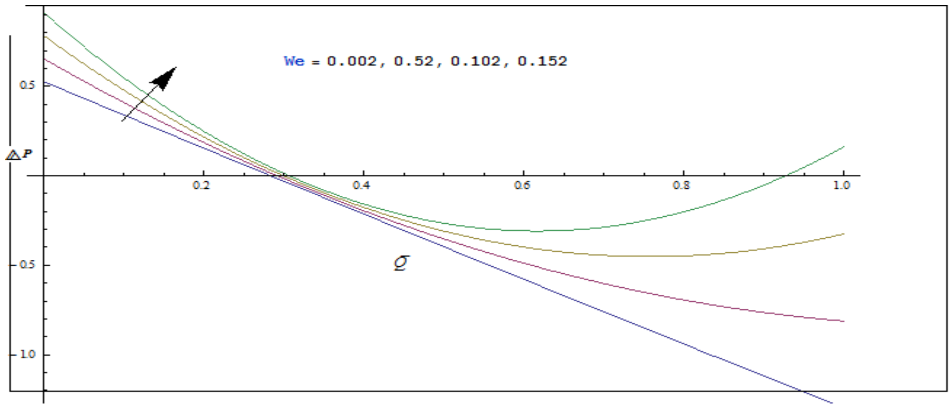

Fig4: The variation of pressure difference as a function of $\bar{Q}$ for different values of k.

We observe that the larger the parameter w_e, thesmaller the pressure rise against which the pump works. For a given flux $\bar{Q}$,the pressure rise decreases with increasing k

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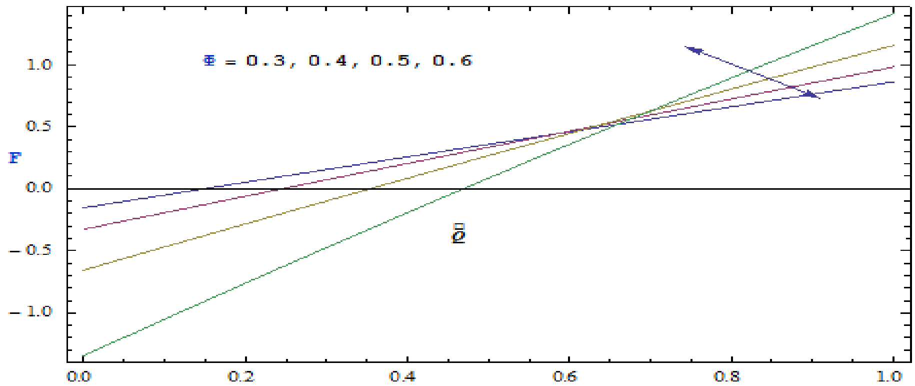

Fig5: The variation of F with ¯Q for different values of ∅ with k =3 we = 0.002.

We see that the friction force first decreases and then increases with an increase in ∅.

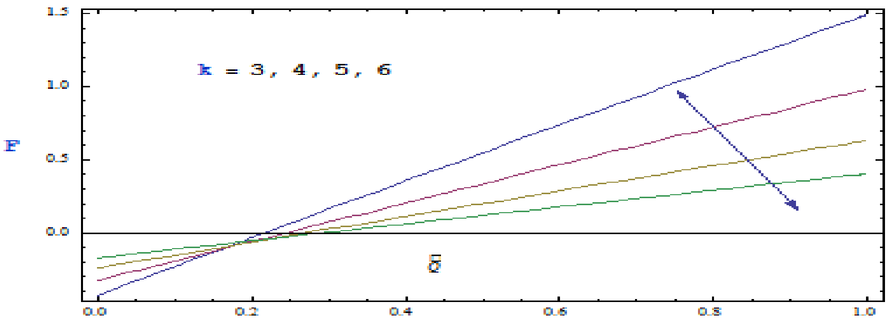

Fig6: The variation of F with ¯Q for different values of ∅ with ∅ =0.4, we = 0.002. We see that the friction force first increases and then decreases with an increase in *k*

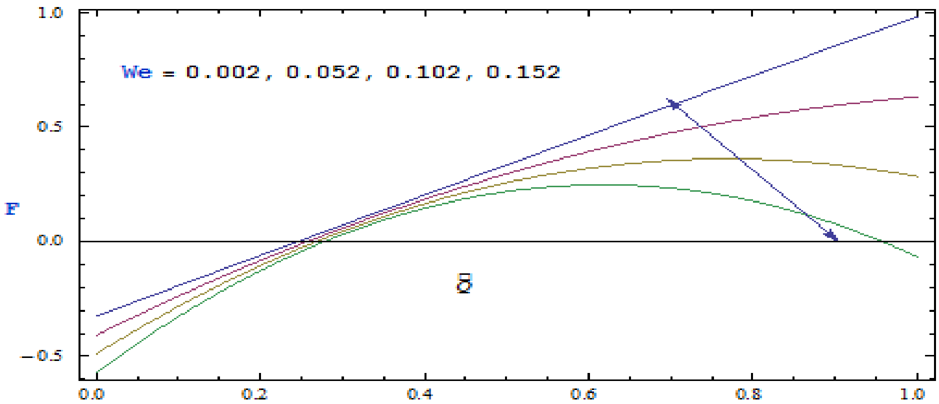

Fig7: The variation of F with ¯Q for different values of ∅ with k =3 ∅ = 0.4.

We see that the friction force first decreases and then decreases with an increase in *we*

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