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Investigating the Hot Spot Temperature Evolution at the Presence of a Degenerate DT Plasma

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ABSTRACT: A degenerate plasma can effectively lessen the possibility of ion-electron collisions since its electron temperature is smaller than the Fermi temperature. This leads to a suppression in most of the energy transfer processes. However, the Bremsstrahlung emission is reckoned as one of the most serious ones. In this case, it is expected that degeneracy plays a prominent role in enhancing the ignition condition. For this respect, the ignition behaviour of a degenerate DT plasma is analysed by benefiting from the laser-accelerated deuteron fast ignition concept and two nodes energy equation model. The analytical results are shown that as the deuteron beam propagates its energy into a degenerate DT plasma, the hot spot ignition situation is appeared approximately 15 ps sooner than the similar mechanism existed in a classical plasma. Therefore, it can significantly take more efficacious step to maximize the fast ignition efficiency.

KEYWORDS: nuclear fusion; deuteron-fast ignition; degenerate plasma; hot spot ignition; two nodal concept.

I. INTRODUCTION

Today, one of the most important methods to produce energy by nuclear fusion is inertial confinement fusion (ICF), consisting different approaches that " hot spark " can be regarded as one of them [1]. In this technique, first a high temperature region is generated in the target, namely, hot spot, during the compression phase; then the fusion burning wave is propagated in the low temperature plasma [1]. Although the hot spark technique can perfectly offer us the prospect of an ultimate solution to the energy issue, the creation of instabilities during the compression phase results in obtaining low gains [2].

In order to overcome instability difficulties and enhancing the fusion efficiency, fast ignition scheme was proposed in which a small region of a high density pre-compressed fuel will be directly ignited by an external heating source, such as electrons, light / heavy ions or intensive lasers. However, the stiffer transfer of ions within the hot spot and their higher energy propagation, made these ignitors as a significant choice in fast ignition analyses rather than the lasers or energetic electrons [3]. Following the related consideration, in recent years, deuteron beam has been known as the conquest of other ion ignition triggers in DT fast ignition scenario due to its beam fusions which can cause a bonus energy in the compressed target. This can considerably lead to a reduction in the ignition energy [4]. Following the related claim, an efficacious method for the deuteron fast ignition was proposed. This concept relies on the arrangement of a spherical ultra-thin deuterated foil inside a cone guide that can beneficially produce the laser-accelerated deuteron trigger with quasi-Maxwellian spectrum by converting a high percentage of the laser energy into the deuterons acceleration energy [4].

In the case with the conditions of high density and low temperature, the plasma electrons can be degenerate which can perfectly minimize both the reaction rate and ion-electron Coulomb collisions. When the plasma is degenerate, the Pauli exclusion leads to a decrease in ions and electrons collisions. Therefore, the Bremsstrahlung radiations are minimized compared with the classical values. In this way, the conservation equations that demonstrate the behaviour of fuel plasma and hot spot are slightly different from the classical one [5].

Over the past years, taking the advantage of degenerate fuels, in order to solve the conservation equations, the program " FINE " (fast ignition nodal energy) was proposed that evaluated the fast ignition concept in two different nodes. The first node, describing a small portion of the fuel which is heated by the ignition trigger and the second node defining the volume surrounded the ignitor [1]. This leads to two different plasma temperatures that can affect the

behaviour of hot region and thermal energy dissipations. Figure 1 shows the related nodal separation for a deuteron fast ignition scheme.

To demonstrate the assertions, in this paper, taking into account the laser-accelerated deuteron beam, by solving the FINE equations, first the thermal Bremsstrahlung radiation will be analysed by having regard for the motion of ions in a degenerate plasma, in sect. II. Moreover, the role of degeneracy in the power dissipation will be discussed. Then, in sect. III, the hot spot behaviour will be compared in two degenerate and classical plasmas. Finally, sect. IV will present the conclusions and results.

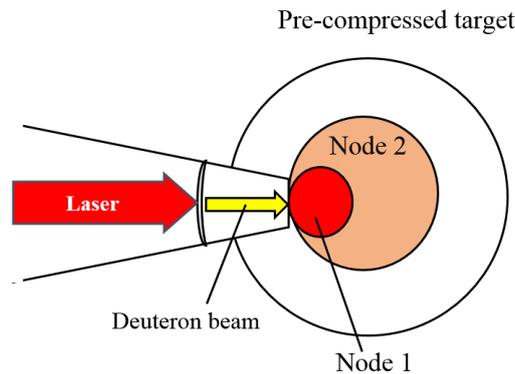


Fig.1. Fast ignition by the laser-accelerated deuteron trigger based on the two nodal technique.

II.FAST IGNITION BASED ON THE NODAL ENERGY CODE

A. The Nodal Energy Equations

As shown in fig.1, in a two node energy model, the first node demonstrates the ignitor that here is considered as the mixture of deuteriums and tritiums with equal density numbers. Moreover, the size of the ignitor portion is determined by the external deuteron beam range. However, the range of the 3.5 MeV alpha particles produced by DT fusion reactions play a role in defining the size of node 2.

As the physical model, in this paper, a conically guided degenerate pre-compressed fuel with the uniform density, $\rho = 2000 \text{ g/cm}^3$ has been considered which is heated by a laser-accelerated deuteron beam. The nodal energy equations for the ions and electrons of the separated volumes can be written as [6],

Node 1:

$$\left(\frac{dE_e}{dt}\right)_{node1} = \frac{3}{2} \cdot \frac{d(n_e k_B T_e)}{dt} = P_{ie} - P_b + f \cdot \eta \cdot P_{f\alpha} + \eta_d \cdot P_{ign} - P_{he} - P_{me}, \quad (1)$$

$$\left(\frac{dE_i}{dt}\right)_{node1} = \frac{3}{2} \cdot \frac{d(n_i k_B T_i)}{dt} = -P_{ie} - P_b + f \cdot (1-\eta) \cdot P_{f\alpha} + (1-\eta_d) \cdot P_{ign} - P_{mi}, \quad (2)$$

Node 2:

$$\left(\frac{dE_e}{dt}\right)_{node2} = \frac{3}{2} \cdot \frac{d(n_e k_B T_e)}{dt} = P_{ie} - P_b + (1-f) \cdot \eta \cdot P_{f\alpha} - P_{he} - P_{me} + [f \cdot P_b + P_{he} + P_{me}]_{ignitor}, \quad (3)$$

$$\left(\frac{dE_i}{dt}\right)_{node2} = \frac{3}{2} \cdot \frac{d(n_i k_B T_i)}{dt} = -P_{ie} - P_b + (1-f) \cdot (1-\eta) \cdot P_{f\alpha} - P_{mi} + [f \cdot P_b + P_{mi}]_{ignitor}, \quad (4)$$

where k_B defines the Boltzmann constant, P_{ign} is the power density deposited by the external deuteron driver (the term $\eta_d = \lambda_i / (\lambda_i + \lambda_e)$, is the amount of the total energy of the driver deposited in the electrons, the rest deposited in the ions, all the energy is deposited in the ignitor regime by definition of ignitor). $\lambda_{i,e}$ expresses the mean free path of ions / electrons of the plasma region [6]. P_{ie} is the electron-ion exchange power density. P_{fa} is the DT fusion power density, where f determines the fraction of alpha particles energy deposited in node 1 and $(1-f)$ is the fraction deposited in node 2. Moreover, the term $\eta = 32 / (32+T_e)$ and $(1-\eta)$ effectively rely on the plasma temperature and density and define the energy fraction that the alpha particles deposit into the electrons and ions, respectively [7]. P_{he} and P_m respectively show the electron heat conduction and the mechanical expansion dissipation [7]. (see the appendix for details).

This paper only discuss the node 1 behaviour. In the following, benefiting from eqs.(1) and (2), we will elaborate the Bremsstrahlung radiation, P_b , as one of the most important energy loss mechanisms in achieving ignition of degenerate plasma.

B. Bremsstrahlung Radiation in Degenerate Fuel

Plasma tends to turn into a low temperature one due to the outflowing of energy. The high percentage of this energy dissipation comes from the Bremsstrahlung radiations. When the plasma is classical, the higher charge to mass ratio of the ions compared with the electrons, leads to a decrease in their thermal velocity. However, when the plasma is degenerate, by having regard for two-temperature nodal technique, the ions are comparable in thermal motions to the electrons. In other word, the electrons temperature is lower than the Fermi temperature in degenerate plasma. This condition makes the electrons to obey the Fermi-Dirac distribution function [8]. Taking account of degeneracy, the electron density, n_e , and the Fermi temperature, kT_F , can be given as,

$$n_e = \frac{2}{\lambda_e^3} \cdot e^{\beta_e \mu_e} \left[1 - \frac{e^{\beta_e \mu_e}}{\sqrt{8}} \right], \tag{5}$$

$$kT_F = \frac{\hbar^2}{2m_e} (3\pi^2 n_e)^{2/3}, \tag{6}$$

where λ_e defines the electron thermal wavelength, $\beta_e = 1/kT_e$ and μ_e is the electron chemical potential. For the high value of $\beta_e \mu_e$, the Fermi-Dirac distribution is limited to the Maxwellian velocity distribution [8]. In this case, the specific emissivity can be expressed as a function of DT fuel frequency, ν , by the following equation [9],

$$j(\nu) = \frac{256\pi^3 Z^2 e^6}{3c^3 m_e^2} n_e n_i \left(\frac{m_e}{2\pi k_B T_e} \right)^{3/2} \left(\frac{m_i}{2\pi k_B T_i} \right)^{3/2} \int_0^\infty \exp\left(-\frac{m_e v_e^2}{2k_B T_e}\right) v_e^2 dv_e \int_0^\infty \exp\left(-\frac{m_i v_i^2}{2k_B T_i}\right) v_i^2 dv_i \int_0^\pi \ln \frac{m_e v^3}{2\pi Z e^2 \nu} \cdot \frac{1}{v} \sin \theta d\theta, \tag{7}$$

where n_i , Z and T_i are the density, atomic number and the temperature of the ions, respectively. ν is the relative velocity. v_i and v_e define the thermal velocity of the ions and electrons, respectively. Benefiting from eq.(7), one can calculate the Bremsstrahlung radiation loss through the integration over ν parameter [9],

$$P_b (keV \cdot \text{cm}^{-3} / s) = \int_0^\infty j(\nu) \nu d\nu. \tag{8}$$

Taking account of two-node temperature concept for degenerate DT fuel, fig.2a shows the specific emissivity of the Bremsstrahlung radiation by considering three different electron temperatures and $T_i = 700T_e$. Moreover, fig.2b exhibits the Bremsstrahlung radiation loss as a function of electron temperature for two different plasma densities.

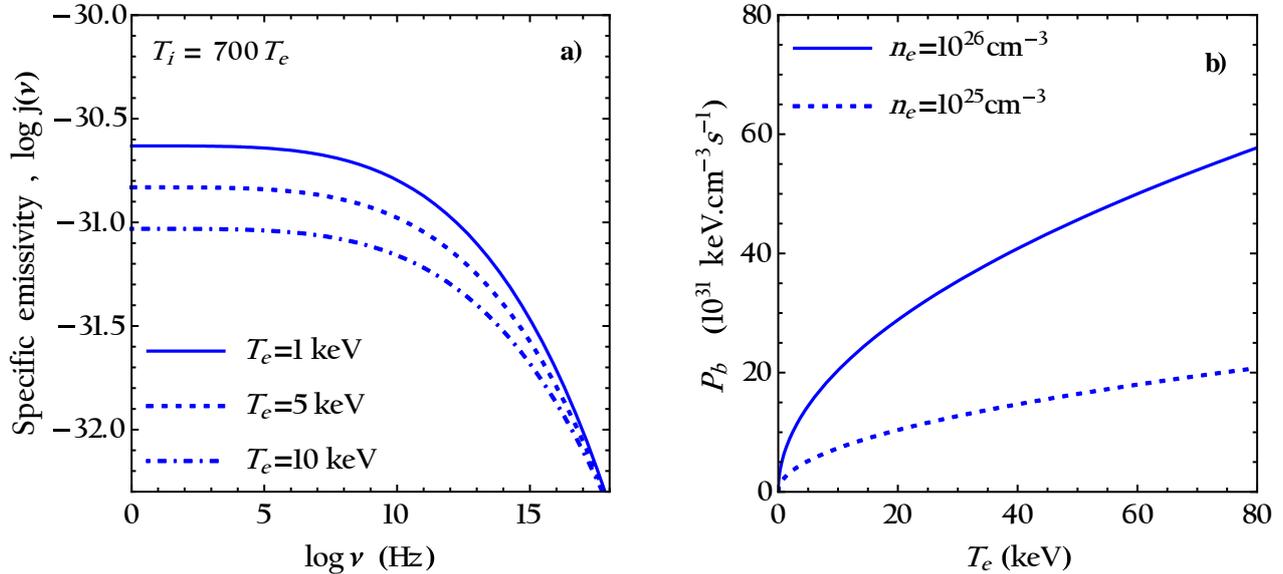


Fig.2. a) Specific emissivity of the Bremsstrahlung radiation in a degenerate DT plasma at, $T_i = 700 T_e$, as the ion temperature value and three different electron temperatures: $T_e = 1$ keV (solid line), $T_e = 5$ keV (dashed line) and $T_e = 10$ keV (dashed-dotted line). b) The Bremsstrahlung radiation loss as a function of electron temperature and density, n_e .

As the electrons temperature increases, the energy exchange value among the electrons and ions of the plasma maximizes. In this case, the plasma has a tendency to turn into non-degenerate (classical) form. This leads to a reduction in specific emissivity. Moreover, from fig.2a one can obtain a slight decrease in emissivity at low DT plasma frequencies (< 10 Hz). However, this is followed by a sharp fall over the next frequency values. This can prominently express the inverse relation between the emissivity and the plasma frequency. Figure 2b, obviously shows that for a given plasma density, as the electrons contain higher temperature values, a gradual decrease in degeneracy, causes increase in Bremsstrahlung radiation loss due to the more ion-electron collisions.

III. HOT SPOT BEHAVIOUR IN DEGENERATE PLASMA

In order to find the degeneracy effect on the improvement in the ignition situation, discussing the evolution of the hot spot behaviour can be regarded as a significant point. For the ignition regime, the fusion energy deposited into the hot spot must be greater than the power losses. However, while the spark DT fuel undergoes a thermonuclear ignition, the initial energy content of the plasma is comparable to the total energy released during the confinement time. In this case, the equation of power balance at the spark centre can be approximately calculated as [10],

$$\frac{P_{ign}}{V_{hs}} - (P'_b + P_m + P_{he} - P_{fa}) = \frac{3}{2} \cdot \frac{d}{dt} ((n_i + n_e) \cdot k_B T_{hs}), \quad (9)$$

where V_{hs} is the volume of hot spot, P'_b is the Bremsstrahlung radiation loss in a classical plasma (see the appendix), n_e is the electron density and $n_i = n_d + n_t$ is the sum of deuterium and tritium densities of DT plasma as a function of time,

$$\frac{dn_d}{dt} = \frac{dn_t}{dt} = -n_d n_t \langle \sigma v \rangle_{DT}, \quad (10)$$

in which $\langle \sigma v \rangle_{DT} = 1.1 \times 10^{-18} T^2$ cm 3 /s expresses the Maxwell-averaged DT fusion reactivity [10]. Equation 9 gives the hot spot evolution for a normal one-temperature plasma. However, the hot spot evolution of a two-temperature degenerate plasma is calculated by taking into account the nodal energy model (eqs. (1) and (2)). Following the claims, fig.3 exhibits the temporal evolution of the hot spot in two degenerate and classical plasmas by benefiting from the data in table.1. (see ref.[11] for details).

Table.1. The required data to calculate the hot spot behaviour in degenerate and classical plasma.

Plasma	R_{hs} (μm)	ρ_{hs} (g/cm^3)	d (mm)	T_{ign} (MeV)	N_{ign}	T_{ini} (keV)
Degenerate	20	2000	0.16	3	1.5×10^{16}	1
Classical	20	300	1.02	5	1.5×10^{16}	1

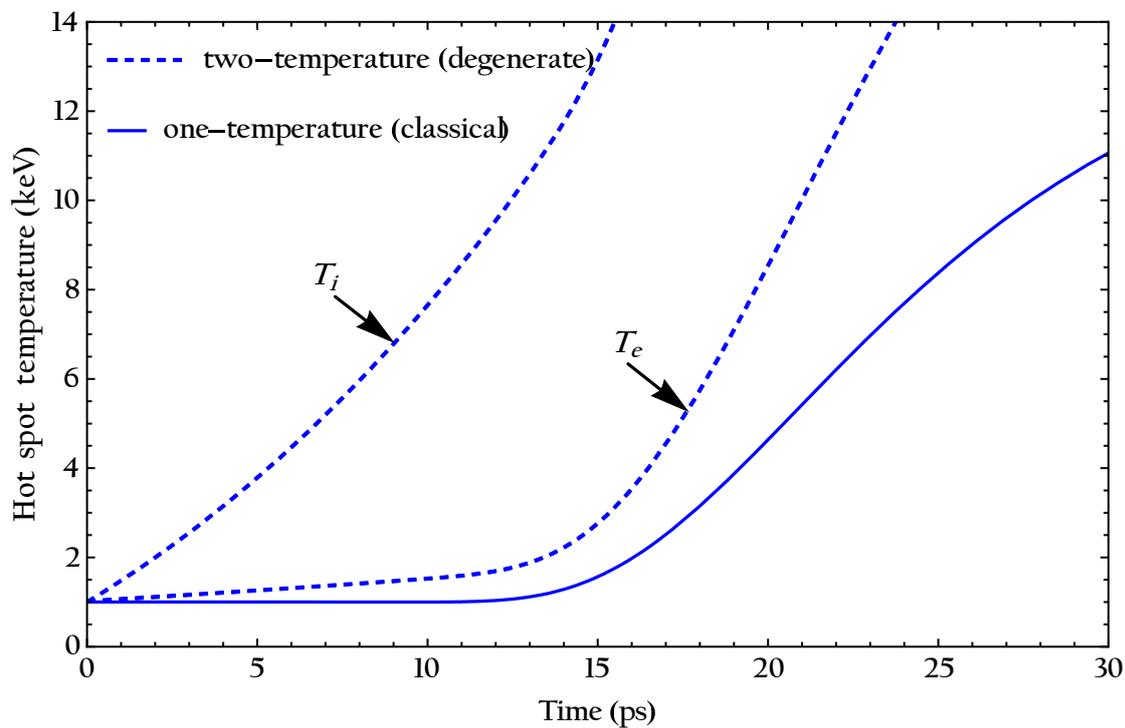


Fig.3. Time evolution of the hot spot temperature for two different plasma conditions: degenerate (dashed line) based on the nodal energy model and classical (solid line) based on the normal equation of power balance.

In table.1, R_{hs} , ρ_{hs} and d refer to the hot spot radius, hot spot density and the foil-target distance, respectively. However, T_{ign} and N_{ign} are the external beam parameters that define the Maxwellian distribution temperature and density of the accelerated deuteron driver, respectively. Based on the conservative ignition criterion, $R_{hs}\rho_{hs} T_{hs} > 6 \text{ g}\cdot\text{cm}^{-2}\cdot\text{keV}$ [10], we can obtain that for both classical and degenerate plasmas, the temperature of the hot spot, T_{hs} , should be ($> 10 \text{ keV}$) in order to achieve the ignition situation. By analysing fig.3, one can find that degeneracy by reducing the ignition time can play an important role in enhancing the fast ignition condition. As shown in fig.3, the ignition time interval is limited to ($< 30 \text{ ps}$) corresponds to $t = R_{hs} / C_s$, where $C_s = 2.8 \times 10^7 (T / \text{keV}) \text{ cm/s}$ defines the sound speed. Furthermore, the initial temperature, T_{ini} , has been considered 1 keV for both degenerate and classical types.

When the accelerated deuteron beam strikes the classical DT plasma, the fuel takes about 15 ps to involve in increasing the hot spot temperature. However, this temperature maximizes from the beginning when the DT plasma is degenerate (see fig.3). It should be asserted that in degenerate plasma both the ions and electrons involve in the spark ignition. However, the role of ions is more efficacious. On the other hand, based on the nodal energy model, during the burning of node 1, the energy of the external driver tries to heat the spark and provide the ignition situation. For this respect, deposit its energy to the electrons and ions of the plasma. However, ions are more participant in the related energy distribution since the electrons are degenerate and their temperature is smaller than the Fermi temperature. For

this reason, the electrons are expected to undergo less reactions during the node 1 burning time interval. Figure 4, expresses the temperature evolution of the ions and electrons as a function of time.

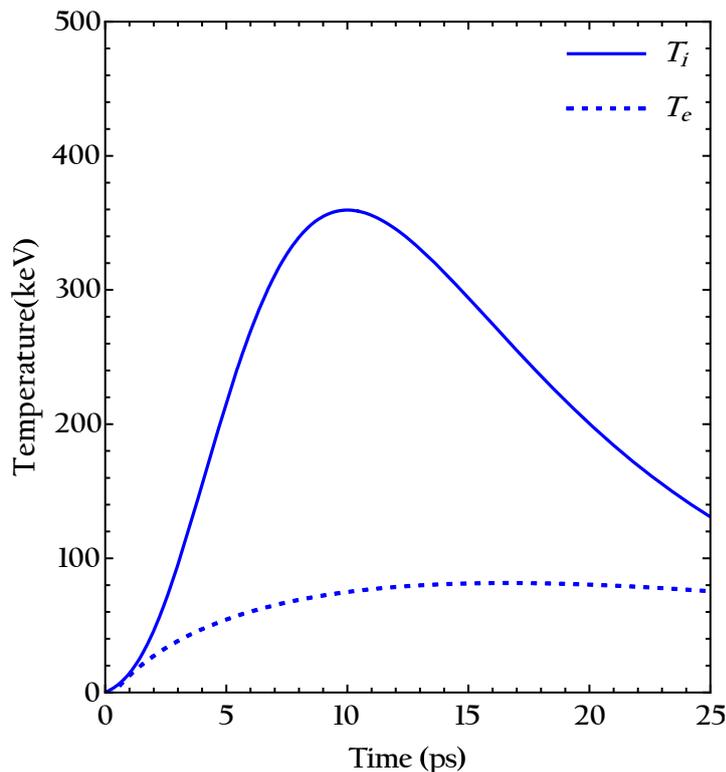


Fig.4. Temperature evolution of the ions (solid line) and electrons (dashed line) of a degenerate plasma vs. time. The calculations has been done based on the two nodal concept.

Figure 4 has been depicted by the numerical calculations of eqs.(1), (2) and (10). From fig.4, one can see that from the beginning to $t \sim 1$ ps, the electrons and ions remaining towards a common temperature, which is a direct result of degeneracy. Just after this time, by developing the fusion burning wave, the energy exchanges between the ions and electrons. The temperature of electrons increases gradually and following the plasma tendency to turn classical form, the burning wave propagates into node 2.

IV.CONCLUSION

In this paper, fast ignition of a laser-accelerated deuteron beam was investigated at the presence of degenerate DT plasma. When a plasma is degenerate, the Pauli exclusion principle can prominently decrease the ion-electron collisions since the electron temperature is smaller than the Fermi temperature. This leads to a reduction in the Bremsstrahlung radiation emissions, which can effectively minimize the ignition temperature. In this case, the energy equations will be different compared with the classical plasma. For this respect, the two nodes energy equations was proposed as a means to analyse the ignition behaviour in degenerate plasma. This consideration was demonstrated in this paper. The ignition situation was compared in two various degenerate and classical DT plasmas. The results showed that, as the external deuteron beam propagates into the classical plasma, the fuel takes about 15 ps to involve in maximizing the hot spot temperature. However, the related time interval decreases significantly in a degenerate condition. Furthermore, degeneracy by appearing the hot spot ignition condition in shorter time, can be reckoned as a positive point in fast ignition scenario.

V.APPENDIX. THE RADIATION PROCESS IN DT PLASMA

A. Power of External Ion Beam

Taking into account the Maxwellian energy distribution for the laser-accelerated ion beam, the beam deposition power can be given as [3],

$$P_{ign}(t) = \frac{8}{3\sqrt{\pi}} \frac{E_{tot}}{\tau} \left(\frac{\tau}{t}\right)^6 \exp^{-(\tau/t)^2}, \tau = \sqrt{\frac{m_{ign} d^2}{2T_{ign}}} \tag{A1}$$

where d , m_{ign} and T_{ign} are the foil-target distance, the mass and Maxwellian distribution temperature of the ion beam, respectively. Moreover, by considering N_{ign} as the number of generated ion triggers, the total beam energy, E_{tot} , can be regarded as, $E_{tot}=3/2 \times N_{ign} \times T_{ign}$.

B. Ion-Electron Collision Power

In degenerate plasma, the stopping power of a characteristic particle per unit length can be calculated by Fermi and Lindhard formula as [12-13],

$$\frac{dk}{dl} = \frac{q^2}{2\pi^2} \int d^3k \left[\frac{k \nu \text{Im} D^l(k, k \nu)}{k^2 |D^l(k, k \nu)|^2} \right], \tag{A2}$$

where k , q and D are the energy of ions, the charge of ions and the electron dielectric function, respectively. In addition, at the presence of full degeneracy, we can obtain the dielectric function, D^l , as,

$$D^l = 1 + \frac{3\omega_{pe}^2}{k^2 v_F^2} (f_r + f_i), \tag{A3}$$

in which v_F defines the Fermi velocity and f_r and f_i are the real part and the imaginary part of dielectric function, respectively. As $v \ll v_F$, the energy loss depends on the electron density and ion velocity. In this case, one can recalculate eq.(A2) as,

$$\frac{dk}{dl} = \frac{4Z^2 e^4 m_e}{3\pi \hbar^3} v_{ie} \int_0^1 \frac{z^3 dz}{(z^2 + \chi^2 f_r(0, z))^2}, \quad z = \frac{k}{2k_F}, \quad \chi = \sqrt{\frac{e^2}{\pi \hbar v_F}}. \tag{A4}$$

By defining, $v_{ie} = v (dk / dl)/k$, as the ion-electron frequency collision, the ion-electron collision power is gotten as,

$$P_{ie} (keV cm^{-3} / s) = \frac{3}{2} v_{ie} N_i k_B T_i. \tag{A5}$$

C. DT Fusion Power Density

As the 3.5 MeV alpha particles produced by DT fusion reactions, deposit their energy into the pellet, one can find its heating rate by [3],

$$P_{f\alpha} [keV cm^{-3} s^{-1}] = 5.1 \times 10^{49} \langle \sigma v \rangle_{DT} f \rho_{hs}^2, \tag{A6}$$

where f is the fraction of alpha particles deposit their energy in the hot spot volume,

$$f = \begin{cases} 1 - 1/4\tau + 1/160\tau^3 & \tau \geq 0.5 \\ 1.5\tau - 0.8\tau^2 & \tau < 0.5 \end{cases}, \quad \tau = 9 \frac{\rho_{hs} R_{hs} \ln \Lambda_{ae}}{(k_B T_{hs})^{3/2}}. \tag{A7}$$

D. Electron Heat Conduction

The electron heat conduction is reckoned as a significant means in propagating the hot spot energy to surrounding cold fuel. The parameter can be expressed as [3],

(A8)

$$P_{ne} [keV \cdot cm^{-3} \cdot s^{-1}] = \left(\frac{17.8 \times 10^{27}}{\ln \Lambda} \right) \frac{(k_B T_{hs})^{7/2}}{R_{hs}^2}.$$

E. Mechanical Expansion Dissipation

Benefiting from, $C_s = (n_i k_B T_F / \rho_{hs})^{1/2}$, as the speed of sound in degenerate plasma, one can obtain the mechanical expansion dissipation by [5],

$$P_{mi,e} (keV \cdot cm^{-3} / s) = \frac{3}{R_{hs}} N_{i,e} k_B T_{i,e} C_s. \quad (A9)$$

F. Bremsstrahlung Radiation in Classical Plasma

By assuming the non-relativistic Maxwellian distribution velocity for the electrons existed in a classical DT plasma, the specific emissivity formula is [9],

$$j(\nu) = \frac{128\pi^2 Z^2 e^6}{3c^3 m_e^2} n_e n_i \left(\frac{m_e}{2\pi k_B T_e} \right)^{3/2} \int_0^\infty \exp\left(-\frac{m_e v_e^2}{2k_B T_e}\right) v_e \ln \frac{m_e v_e^3}{2\pi Z v e^2} dv_e. \quad (A10)$$

In this case, we can calculate the Bremsstrahlung radiation loss in classical DT plasma through the integration over ν parameter,

$$P_b'(keV \cdot cm^{-3} / s) = \int_0^\infty j(\nu) d\nu. \quad (A11)$$

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