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A Model of Suspension Filtration in Porous Media with Multistage Accumulation Kinetics

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ABSTRACT: In the paper a problem of suspension filtration in a porous medium with modified multistage accumulation kinetics is considered. The model includes dynamic factors and accounts for changes in media porosity. It is suggested that suspended particles are forming depositions of two types, passive and active, at any location inside the filter. Irreversible depositions are formed in passive zones, reversible ones are in active zones. The process of deposition forming continues until the deposition reaches its locally maximum value. Then, filter breakthrough takes place. On the basis of numerical solutions of the problem influences of dynamic factors on solid particles transport and deposition characteristics are analyzed.

KEYWORDS: filtration, porous media, suspension, accumulation, mathematical model.

I. INTRODUCTION

The processes of suspension filtration in a porous medium are widely used in the study of wastewater treatment technology and the preparation of drinking water, oil recovery using water flooding, etc. Various theoretical and experimental approaches are used to study the filtration of suspensions in a porous medium: the mechanisms of transport, attachment and detachment of particles, changes in the permeability and structure of pore space. A phenomenological approach is widely used to describe the process: attachment kinetics of suspended particles onto filter grains and detachment of particles already deposited onto the filter grains [1]. Several varieties of phenomenological relationships have been obtained to describe the flow of various types of disperse systems. However, the applicability range of the obtained models and their uniqueness remains open.

In the paper we formulate a problem of suspension filtration in a porous medium. We describe a one-dimensional model with two types of depositions, active and passive. We used modified kinetics of depositions that take into account smooth changes of kinetic coefficients between different stages of deposition. The problem is numerically solved. Influences of different model parameter on filtration and transport characteristics are studied.

II. FORMULATION OF THE PROBLEM

Previous studies show that the kinetics of particle deposition in suspensions filtration in porous media occurs in several stages [1-5]. At the initial stage, if the filter is clean (no deposited particle), suspended particles attach directly on the surface of the filter grains. When the initial stage has been completed, the transition stage begins. At this stage, the deposition of particles occurs simultaneously on the walls of the filter and on previously deposited particles. After the transition stage, a breakthrough occurs, accumulation of particles stops after reaching, locally, a certain amount of deposition [1-5].

Deposition in the porous space of the deep filter structures has two forms - washable and non-washable. Accordingly, the filter zones are called active and passive [6, 7]. Washable deposition with concentration ρ_a is formed in active zones, washed by the liquid stream, form a washable deposition with concentration ρ_a, passive zones that are stagnant, form an non washable deposition with ρ_p concentration. We denote the total capacity of the filter for deposition by ρ_0. It

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follows from the foregoing $\rho_0 = \rho_{a0} + \rho_{n0}$, where ρ_{a0} and ρ_{p0} are the capacities of the active and passive zones, respectively.

The present paper deals with the problems of suspension filtration in porous media, with taking into account the attachment of solid particles of the suspension in the pore volume and their release. We use the well-known model [6-9] and its modification proposed here. We know that after the "charging" stage of the filter, the main working stage begins, where both attachment and detachment of previously deposited particles occur with simultaneous "aging" (compaction) of the deposition in the stationary(passive) zone.

In the known models [1, 6], it is considered that the effect of "aging" ends abruptly when the deposition concentration reaches the limit values, which is not entirely physically correct. Here we propose a generalization of the model, which eliminates such an abrupt change in the kinetics of deposition in the passive zone. Based on the solutions of problems using the known [6] model and the modification of the kinetic equation proposed here, characteristic differences are estimated.

We consider a semi-infinite homogeneous porous medium with initial porosity m_0, filled with homogeneous liquid (with liquid without dispersed particles). At a point $x = 0$ beginning from $t > 0$ a suspension with constant concentration c_0 and filtration velocity $v(t) = v_0 = const$ is injected.

The system of equations for suspension filtration with a given velocity regime consists of the mass balance equation and kinetics, which in the one-dimensional form with neglecting of diffusion effects is represented in the following form [6]

$$\begin{cases} m_0 \frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} + \frac{\partial \rho_a}{\partial t} + \frac{\partial \rho_p}{\partial t} = 0, \\ \frac{\partial \rho_a}{\partial t} = \beta_a (c - \frac{\rho_a}{\rho_{a0}} c_0), \\ \frac{\partial \rho_p}{\partial t} = \beta_p (\rho_p) c, \end{cases} \tag{1}$$

where c is the volume concentration of the suspension (m^3/m^3), v - filtration velocity (m/s), m - the porosity of the media, β_a - the coefficient characterizing the kinetics in the active zone (1/s), β_p - the coefficient (1/s) associated with the compaction effect (aging) of the deposition and $\beta_p(\rho_p) = \alpha(\rho_p)\beta_{p0}, \beta_{p0} = const$.

In [1] it was proposed

$$\alpha(\rho_p) = \begin{cases} 1 & for \quad \rho_p \le \rho_{p1}, \\ \rho_{p1} / \rho_p & for \quad \rho_{p1} < \rho_p < \rho_{p0}, \\ 0 & for \quad \rho_p = \rho_{p0}, \end{cases} \tag{2}$$

where ρ_{p1} - the value of ρ_p at which "aging" begins.

It can be seen from (2) that starting from $\rho_p = \rho_{p0}$ the process of attachment stops in the passive zone. This, in general, contradicts the physical picture of the process. Therefore, we are to change the kinetics of sedimentation in order to suply continuity in its dynamics. For this purpose, the following choice of coefficient is proposed:

$$\alpha(\rho_p) = \begin{cases} 1 & for \quad \rho_p \le \rho_{p1}, \\ f(\rho) & for \quad \rho_{p1} < \rho_p < \rho_{p0}, \\ 0 & for \quad \rho_p = \rho_{p0}, \end{cases} \tag{3}$$

where $f(\rho_p)$ monotonically decreasing function, characterizes the effect of "aging".

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This function is selected in one of the following types:

$$f(\rho_p) = \frac{\rho_{p1}}{\rho_p}, \tag{4}$$

$$f(\rho_p) = \frac{\rho_{p1}}{\rho_{p0} - \rho_{p1}} \left(\frac{\rho_{p0}}{\rho_p} - 1 \right), \tag{5}$$

$$f(\rho_p) = \frac{exp(-\lambda\rho_p) - exp(-\lambda\rho_{p0})}{exp(-\lambda\rho_{p1}) - exp(-\lambda\rho_{p0})}, \tag{6}$$

where $\lambda = const$ characterizes the intensity of "aging". In the above expressions $f(\rho_p)$ in (4) corresponds to the case [1], and (5), (6) are proposed here.

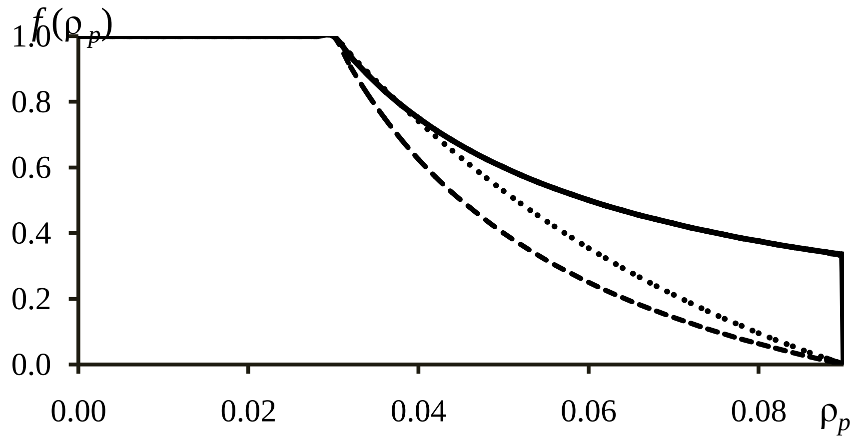

Fig1. Characteristic curves of $f(\rho_p)$ according to (4) (), (5) () and (6) (• •) with $\lambda = 20$.

According to formulation of the problem the initial and boundary conditions are taken in the form

$$c(x,0) = 0, \ \rho_a(x,0) = \rho_p(x,0) = 0, \ \ c(0,t) = c_0 = const. \tag{7}$$

So, (1), (3), (5), (6), (7) represent the modified model of the suspension filtration in porous media with multistage deposition kinetics.

III. NUMERICAL SOLUTION OF THE PROBLEM

To solve the problem (1) - (7), we apply the finite difference method [10].In the domain $D = \{0 \le x < \infty, \ 0 \le t \le T\}$ we introduce the following net,

$$\omega_{h\tau} = \{(x_i, t_j), x_i = ih, \ i = 0,1,..., \ t_j = j\tau, \ j = 0,1,...,J, \ \ \tau = T/J\}$$

Now, instead of functions $c(t,x)$, $\rho_a(t,x)$, $\rho_p(t,x)$ we use net functions c_i^j, $\rho_{a,i}^j$, $\rho_{p,i}^j$, determined on $\omega_{h\tau}$.

On this net, we approximate Eqs. (1)by the following implicit finite difference schemes:

$$m_0 \frac{c_i^{j+1} - c_i^j}{\tau} + v \frac{c_i^{j+1} - c_{i-1}^{j+1}}{h} + \frac{\rho_{a,i}^{j+1} - \rho_{a,i}^j}{\tau} + \frac{\rho_{p,i}^{j+1} - \rho_{p,i}^j}{\tau} = 0. \tag{8}$$

$$\frac{\rho_{a,i}^{j+1} - \rho_{a,i}^j}{\tau} = \beta_a \left(c_i^j - \frac{\rho_{a,i}^j}{\rho_{a0}} c_0 \right). \tag{9}$$

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$$\frac{\rho_{n,i}^{j+1}-\rho_{n,i}^{j}}{\tau}=\alpha\left(\rho_{n,i}^{j}\right)\beta_{n0}c_i^j. \quad (10)$$

Initialand boundary conditions (7) approximated as

$$\rho_{a,i}^0=0,\ \ i=\overline{0,I},$$
$$\rho_{p,i}^0=0,\ \ i=\overline{0,I},$$
$$c_i^0=0,\ \ i=\overline{0,I}, \quad (11)$$
$$c_0^j=c_0,\ \ j=\overline{0,J},$$

where I - enough large number, for which approximately $c_I^j=0$.

Algebraic equations (8) – (10) can be reduced to the following form:

$$c_i^{j+1}=\frac{h}{v\tau+hm_0}\left(\frac{v\tau}{h}c_{i-1}^{j+1}+m_0c_i^j-\left(\rho_{a,i}^{j+1}-\rho_{a,i}^{j}+\rho_{p,i}^{j+1}-\rho_{p,i}^{j}\right)\right), \quad (12)$$

$$\rho_{a,i}^{j+1}=\rho_{a,i}^{j}+\tau\beta_a\left(c_i^j-\frac{\rho_{a,i}^j}{\rho_{a0}}c_0\right), \quad (13)$$

$$\rho_{p,i}^{j+1}=\rho_{p,i}^{j}+\tau\alpha\left(\rho_{p,i}^{j}\right)\beta_{p0}c_i^j. \quad (14)$$

Equations (12) – (14) explicitly express solutions on $j+1$ time layer by solutions on j time layer. We solve them with known initial and boundary conditions (11). The computing algorithm is the following. According to(13) and (14) we find $\rho_{a,i}^{j+1}$ and $\rho_{p,i}^{j+1}$ from known values of $\rho_{a,i}^j$, $\rho_{n,i}^j$ and c_i^j in bottom time layer, then from(12) we find c_i^{j+1}.

IV. RESULTS

As initial data, we take the following values: $c_0=0{,}05$ m^3/m^3, $m_0=0{,}3$, $v_0=10^{-4}$ m/s, $\rho_0=0{,}1$ m^3/m^3, $\rho_{p0}=0{,}09$ m^3/m^3, $\rho_{a0}=0{,}01$m^3/m^3, $\beta_a=0{,}001$s^{-1}. The values of some other parameters are indicated in the figures.

Figure 2, 3 shows the results of calculations for the dependence (5) for different values of parameters. Figure 2 indicated that three types of concentrations are formed moving in the medium. Concentration profiles have more values for larger times that are explained with filling of the pore space of the medium by solid particles, injected together with suspension into the medium.

Increasing of ρ_{p1} leads to the total increasing of ρ_p up to the certain x. One can observe that for some x values of ρ_p for larger ρ_{p1} can be less than for smaller ρ_{p1}. It can be explained by the character of the first stage of deposition. As a whole, increasing of ρ_{p1} intensifies the dynamics of ρ_p. As a result, in c/c_0 and ρ_a we have inverse behaviour (Figure 3). As the intensification of ρ_p leads to the greater charge of the passive zone by solid particles, so in mobile liquid and in active zone comparatively less amount of solid particles remains.

Comparative graphs, for two dependences - (4), (6) are shown in Figure4. As can be seen from the presented data, dependence (6) leads to the smoothing of the profiles. At small values of x for the entire range of the time considered, the profiles for (6) are behind those corresponding to (4). This is accordingly reflected in the curves of c/c_0 and ρ_a in Figure 4.

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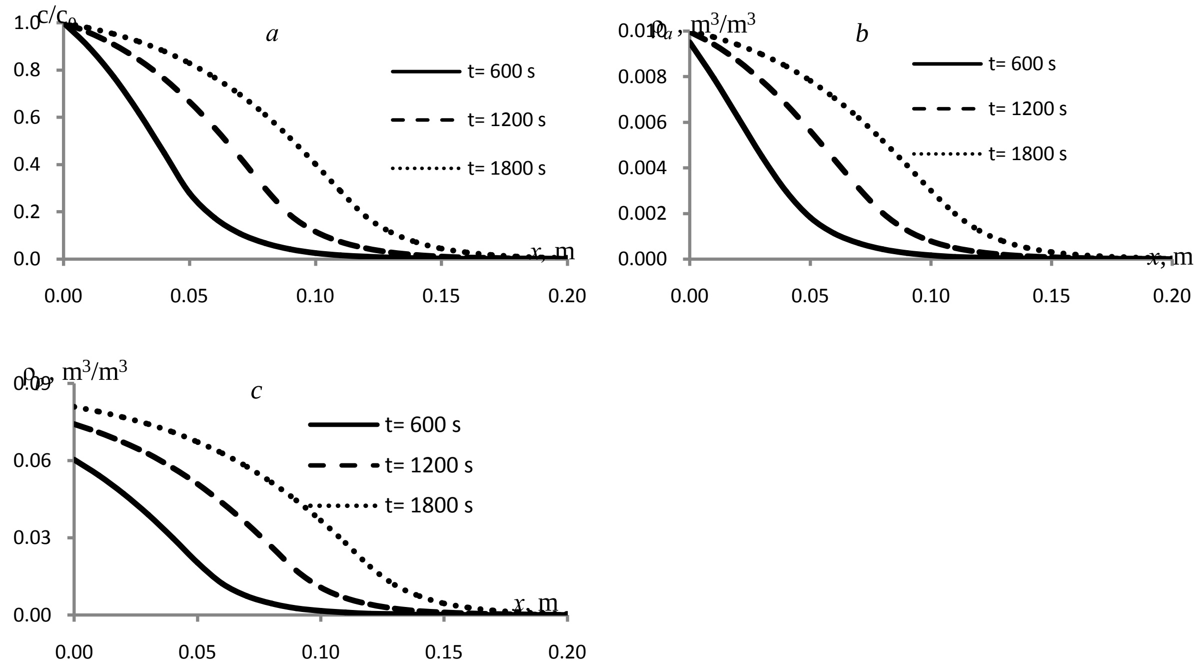

Fig 2. Profiles of concentrations c/c_0 (*a*), ρ_a (*б*), ρ_p (*в*), with $\rho_{p1} = 0{,}02$ m^3/m^3, $\beta_{p0} = 0{,}005 s^{-1}$.

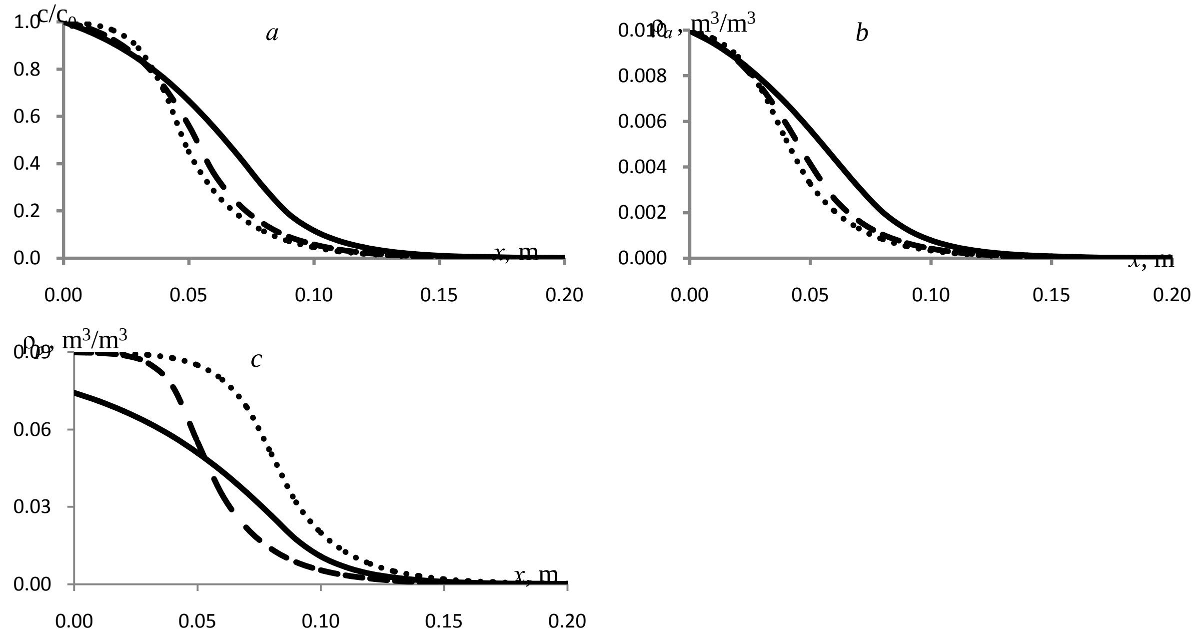

Fig 3. Profiles of concentrations c/c_0 (*a*), ρ_a (*b*), ρ_p (*c*), where $t = 1200s$ $\beta_{p0} = 0{,}005 c^{-1}$, $\rho_{p1} = 0{,}02$ (———), 0,04 (–––), 0,06 (·········) m^3/m^3.

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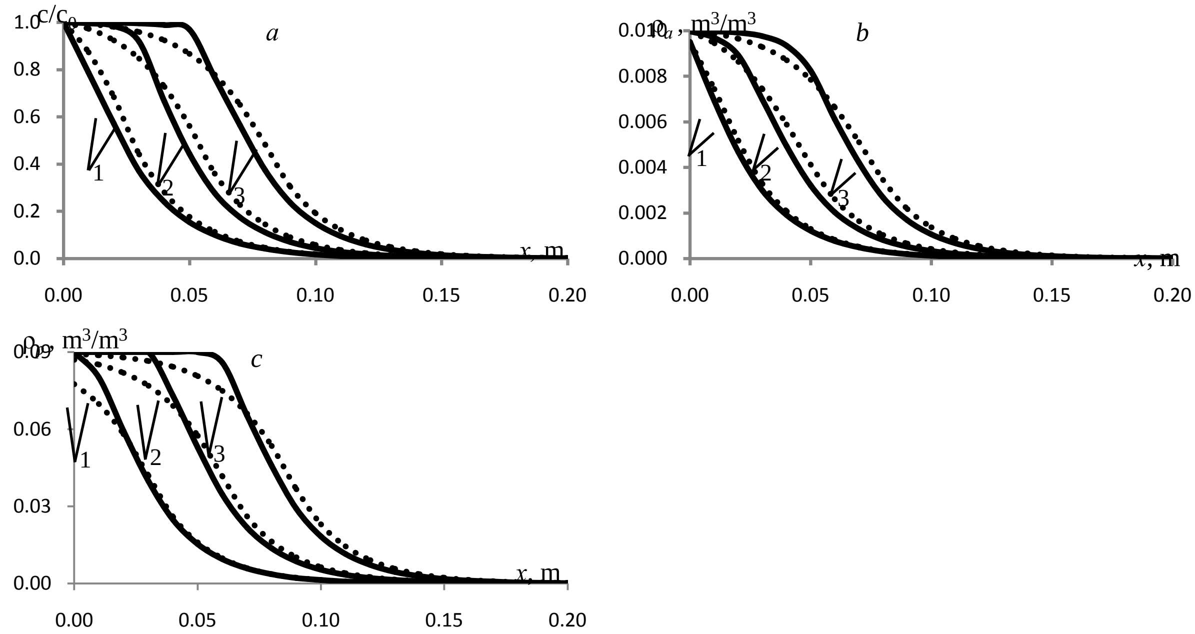

Fig 4. Profiles of concentrations c/c_0 (*a*), ρ_a (*b*), ρ_p (*c*), at $\rho_{p1} = 0{,}04\,\mathrm{m^3/m^3}$, $\beta_{p0} = 0{,}005\,\mathrm{s^{-1}}$ (4) (———) and (6) (·········), *t*=900 (1), 1200 (2), 1800 (3) s.

V. CONCLUSION

A modified suspension filtration model with multistage deposition kinetics is offered. It is shown that the modification of dependences $f(\rho_p)$ leads to the smoothing of transport characteristics, such as concentration of solid particles in mobile liquid, concentrations of depositions in active and passive zones. It can be noted that modified model describes transport processes in more physically correct form.

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