ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 5, Issue 4 , April 2018

On Fuzzy Normed Algebra over Fuzzy Field

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ABSTRACT:This paper studies the concepts of fuzzy normed algebra over fuzzy field, introduces definitions to the fuzzy algebra, fuzzy normed space over fuzzy field, fuzzy normed algebra. It also proves some theorems in this subject.

KEY WORDS:fuzzy algebra, fuzzy normed space over fuzzy field, fuzzy normed algebra over fuzzy field .

I.INTRODUCTION

Many mathematicians have studied fuzzy normed spaces from several angles. The concept of fuzzy fields and fuzzy linear spaces was defined first by S.Nanda [5] and redefined by R.Biswas [4]. Also introduced the concept of fuzzy algebra over fuzzy field was defined first S,Nanda [6] and redefined by Gu and Lu [3]. Gu Wenxiang and Lu Tu [2] introduced the notions of fuzzy linear spaces. In this paper we proceed as follows. Section 2 gives a brief summary of fuzzy algebra. In section 3, we introduces the concepts of fuzzy normed space over fuzzy field. In section 4, we introduce the idea of fuzzy normed algebra over fuzzy field and some of their properties.

II. PRELIMINARIES

This section deals with the basic concepts of fuzzy algebra and some of their properties.

A. Definition:[3]

Let (S,F) be a fuzzy field in F. A fuzzy set A in algebra Xover F is called a fuzzy algebra(A,X)over fuzzy field (S,F) . If the following conditions hold :

1) $A(x+y) \geq \min\{A(x),A(y)\}$, $\forall x,y \in X$.
2) $A(\lambda x) \geq \min\{S(\lambda),A(x)\}$, $\forall \lambda \in F$ and $x \in X$.
3) $A(xy) \geq \min\{A(x),A(y)\}$, $\forall x,y \in X$.
4) $S(1) \geq A(x)$, $\forall x \in X$.

B. Proposition:[3]

Let (S,F) be a fuzzy field of the F. X an algebra over F and A a fuzzy set of X .Then (A,X) is a fuzzy algebra over a fuzzy field (S,F) iff,

1) For any $\lambda,\beta \in F$ and $x,y \in X$.
$A(\lambda x+\beta y) \geq \min\{\min\{S(\lambda),A(x)\},\min\{S(\beta),A(y)\}\}$.
2) For any $x,y \in X$, $A(xy) \geq \min\{A(x),A(y)\}$.
3) $S(1) \geq A(x)$, $\forall x \in X$.

C. Proposition:[3]

Let Y and X be algebras over the field F and f an algebraic homomorphism of Y into X and (A,X) a fuzzy algebra over (S,F).Then $(f^{-1}(A),Y)$is a fuzzy algebra over (S,F).

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D. Proposition:[3]

Let Y and X be linear space over the field F and f an algebraic homomorphism of Y into X and (A, Y) a fuzzy algebra over (S, F).Then $(f(A), X)$is a fuzzy algebra over(S, F).

E. Proposition

Let (S, F) be a fuzzy field of F and let $(A_1, X_1), (A_2, X_2), \dots, (A_n, X_n)$ be a fuzzy algebra over (S, F), then $(A_1 \times A_2 \times \dots \times An, X1 \times X2 \times \dots \times Xn$ is a fuzzy algebra over S, F.

Proof:

Let $A = A_1 \times A_2 \times \dots \times A_n$.
Let $x = (x_1 \times x_2 \times \dots \times x_n)$, $y = (y_1 \times y_2 \times \dots \times y_n) \in X_1 \times X_2 \times \dots \times X_n$ and $\alpha, \beta \in F$.
1) $A(\alpha x + \beta y) = A_1 \times A_2 \times \dots \times A_n(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \dots, \alpha x_n + \beta y_n)$

$$= \min_{j=1,2,\dots,n} A_j(\alpha x_j + \beta y_j)$$

$$\geq \min_{j=1,2,\dots,n} \{\min\{S(\alpha), A_j(x_j), S(\beta), A_j(y_j)\}\}$$

$$\geq \min\{S(\alpha), \min_{j=1,2,\dots,n} A_j(x_j), S(\beta), \min_{j=1,2,\dots,n} A_j(y_j)\}$$

$$= \min\{S(\alpha), A(x), S(\beta), A(y)\}.$$

2) $A(xy) = A_1 \times A_2 \times \dots \times A_n(x_1 y_1, x_2 y_2, \dots, x_n y_n)$

$$= \min_{j=1,2,\dots,n} A_j(x_j y_j)$$

$$\geq \min_{j=1,2,\dots,n} \{\min\{A_j(x_j), A_j(y_j)\}\}$$

$$\geq \min\{\min_{j=1,2,\dots,n} A_j(x_j), \min_{j=1,2,\dots,n} A_j(y_j)\}$$

$$= \min\{A(x), A(y)\}.$$

3) $S(1) \geq A_j(x_j)$ for all $j = 1,2, \dots, n$
So $S(1) \geq \min_{j=1,2,\dots,n} A_j(x_j) = A(x)$
for all $x \in X_1 \times X_2 \times \dots \times X_n$.
Hence then $(A_1 \times A_2 \times \dots \times A_n, X_1 \times X_2 \times \dots \times X_n)$ is a fuzzy algebra over (S, F).

III. FUZZY NORMED SPACE OVER FUZZY FIELD

This section deals with the basic concepts of fuzzy normed space over fuzzy field and some of their properties.

A. Definition:[1]

Let (S, F) be a fuzzy field in F, X be linear space over F , and let (A, X) be a fuzzy linear space over (S, F) . A norm on (A, X) is a function , $\|\cdot\|: X \longrightarrow \mathbb{R}$ satisfies the following conditions :
1)$S(\| x \|) \geq A(x)$, for all $x \in X$.
2) $\| x \| \geq 0$ for all $x \in X$.
3)$\| x \| = 0$ if and only if $x = 0$.
4) $\| \lambda x \| = |\lambda| \, \| x \|$, for all $\lambda \in F$ and $x \in X$.
5) $\| x + y \| \leq \| x \| + \| y \|$ for all $x, y \in X$.
The tuple $(A, X, \|\cdot\|)$ is called a fuzzy normed linear space.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

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B. Example:[1]

Let (S, F) be a fuzzy field of F. We define the function $\|\cdot\|: X \longrightarrow\Leftrightarrow \mathbb{R}$ by $\| x \| = |x|$ for all $x \in X$ is a fuzzy norm on (S, F).

C. Proposition:[1]

Let X be a linear space over F, (B, Y) be a fuzzy linear space over a fuzzy field $(S, F)$and $f: X \longrightarrow Y$ be an injective linear function . If $(B, Y)$is a fuzzy normed space over fuzzy field (S, F), then $(f^{-1}(B), X)$ is a fuzzy normed space over fuzzy field(S, F).

D. Proposition:[1]

Let (A, X) be a fuzzy linear space over a fuzzy field (S, F), Y be a linear space over $F$and f be an isomorphism of X onto Y.$(A, X)$is a fuzzy normed space over fuzzy field (S, F), if and only if $(f(A), Y)$ is a fuzzy normed space over fuzzy field (S, F).

E. Proposition:[1]

Let $(A_i, X_i, \|\cdot\|_i)$be a fuzzy normed space over fuzzy field$(S, F)$for $i = 1, 2, \ldots, n$ .The norms $\| \|$ define by $\| x \| = \| x_1 \|_1 + \| x_2 \|_2 + \cdots + \| x_n \|_n$, $(x = (x_1, x_2, \ldots, x_n) \in X_1 \times X_2 \ldots \times X_n)$ is norm on the fuzzy linear space $(A_1 \times A_2 \times \ldots \times Ax_n, X_1 \times X_2 \times \ldots \times X_n)$.

IV. FUZZY NORMED ALGEBRA

In this section we defined the fuzzy normed algebra space over fuzzy field. Also we proves some proposition in this subject.

A. Definition

Let (S, F) be a fuzzy field in F. And A a fuzzy set in algebra Xover F . $(A, X, \|\cdot\|)$ is called a fuzzy normed algebra space over fuzzy field (S, F), if:
1) X is a fuzzy algebra.
2) $(A, X, \|\cdot\|)$ is a fuzzy norm on (A, X).
3) $\| xy \| \leq \| x \| \| y \|$ for all $x, y \in X$.

B. Proposition

Let X be an algebra over F, (B, Y) be a fuzzy linear space over a fuzzy field $(S, F)$and $f: X \longrightarrow Y$ an algebraic homomorphism. If (B, Y) is a fuzzy normed algebra over fuzzy field (S, F), then $(f^{-1}(B), X)$ is a fuzzy normed algebra over fuzzy field (S, F).

Proof:

1) Since$(f^{-1}(B), X)$is a fuzzy algebra by proposition (C) in II.
2) Since $(f^{-1}(B), X)$is a fuzzy normed space by proposition (C) in III.
3) Assume that $\| \|_Y$ be a fuzzy norm on (B, Y) . Consider the fuzzy norm $\| \|_X$ on X define by

$$\| x \|_X = \| f(x) \|_Y .$$

Let $x_1, x_2, y_1, y_2 \in X$ $\| x_1 x_2 \|_Y = \| f(x_1) f(x_2) \|_Y$

$$\leq \| f(x_1) \|_Y \| f(x_2) \|_Y = \| x_1 \|_X \| x_2 \|_X .$$

Therefore $\| \|_X$ is a norm algebra on $(f^{-1}(B), X)$.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
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C. Proposition

Let (A, X) be a fuzzy linear space over a fuzzy field (S, F), Y be a algebra over $F$and f be an algebra of X onto Y .$(A, X)$is a fuzzy normed algebra over fuzzy field (S, F), if and only if $(f(A), Y)$ is a fuzzy normed algebra over fuzzy field (S, F).

Proof :

1) Since $(f(A), Y)$ is a fuzzy algebra by proposition (D) in II.
2) Since $(f(A), Y)$ is a fuzzy normed space by proposition (D) in III.
3) Let $\| \|_X$ be fuzzy norm on (A, X) . Consider the fuzzy norm $\| \|_Y$ on Y define by $\| y \|_Y = \| x \|_X$, where $y = f(x)$.
Let $x_1, x_2, y_1, y_2 \in Y$ $\| y_1 y_2 \|_Y = \| x_1 x_2 \|_X \le \| x_1 \|_X \| x_2 \|_X$
$= \| y_1 \|_Y \| y_2 \|_Y$.
Therefore $\| \|_Y$ is a norm algebra on $(f(A), Y)$.
Conversely,
1) Since (A, X) is a fuzzy algebra by proposition (D) inII.
2) Since (A, X) is a fuzzy normed space by proposition (D) in III.
3) Assume that $\| \|_Y$ is fuzzy norm on $(f(A), Y)$. Consider the fuzzy norm $\| \|_X$ on defined by

$$\| x \|_X = \| f(x) \|_Y$$

Let $x_1, x_2, y_1, y_2 \in X$ $\| x_1 x_2 \|_Y = \| f(x_1) f(x_2) \|_Y$

$$\le \| f(x_1) \|_Y \| f(x_2) \|_Y = \| x_1 \|_X \| x_2 \|_X .$$

Therefore $\| \|_X$ is a norm algebra on (A, X).

D. Proposition

Let $(A_i, X_i, \|\cdot\|_i)$ be a fuzzy normed algebra over fuzzy field (S, F) for $i = 1, 2, \dots, n$ .The norms $\| \|$ define by $\| x \| = \| x_1 \|_1 + \| x_2 \|_2 + \cdots + \| x_n \|_n$,$(x = (x_1, x_2, \dots, x_n) \in X_1 \times X_2 \dots \times X_n)$ is norm algebra on the fuzzy linear space $(A_1 \times A_2 \times \dots \times A_n, X_1 \times X_2 \times \dots \times X_n)$.

Proof:

Suppose$(A_i, X_i, \|\cdot\|_i)$ is a fuzzy normed algebra over fuzzy field (S, F) for $i = 1, 2, \dots, n$
1) Since $(A_1 \times A_2 \times \dots \times A_n, X_1 \times X_2 \times \dots \times X_n)$ is a fuzzy algebra over the fuzzy field (S, F) by proposition (E) inII.
2) Since The norm $\| \|$ define by $\| x \| = \| x_1 \|_1 + \| x_2 \|_2 + \cdots + \| x_n \|_n$,$(x = (x_1, x_2, \dots, x_n) \in X_1 \times X_2 \dots \times X_n)$ is a norm on the fuzzy linear space $(A_1 \times A_2 \times \dots \times A_n, X_1 \times X_2 \times \dots \times X_n)$ by proposition (E) in III.
3) $\| xy \| = \| x_1 y_1 \|_1 + \| x_2 y_2 \|_2 + \cdots + \| x_n y_n \|_n$ for i = 1, 2,…, n

$$\le \| x_1 \|_1 \| y_1 \|_1 + \| x_2 \|_2 \| y_2 \|_2 + \cdots + \| x_n \|_n \| y_n \|_n$$

$$\le (\| x_1 \|_1 + \| x_2 \|_2 + \cdots + \| x_n \|_n) \times (\| y_1 \|_1 + \| y_2 \|_2 + \cdots + \| y_n \|_n)$$

$\le \| x \| \| y \|$.
Therefore $\| \|$is a norm algebra on$(A_1 \times A_2 \times \dots \times A_n, X_1 \times X_2 \times \dots \times X_n)$.

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