ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 5, Issue 1 , January 2018

Numerical solution of relaxation filtration equations with forming a consolidating cake layer

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ABSTRACT: In the paper on the basis of conservation laws suspensions filtration equations with forming a relaxing cake layer are derived. The equations are numerically solved. To solve the equation for cake layer growth a Stefan's problem is posed. The problem is solved with using the method of catching a moving front. On the basis of numerical results influence of relaxation phenomena on filtration characteristics is established.

KEYWORDS: cake filtration, filter, relaxation, relaxing cake layer, Stefan is problem, suspension

I. INTRODUCTION

Filtration of suspensions through porous media is of great practical importance. The regime with formation of a cake layer on the surface of the filter is of special interest [1,2,3,4,5]. If the dispersion phase of the suspensions consists of polymer solutions or other highly viscous liquids, the suspension may have non-Newtonian rheological properties [6]. In particular, the suspensions exhibit relaxation properties. Then they are not considered as viscous, but viscoelastic liquids. In principle, we can consider filtering models with regard to the rheological models of relaxing suspensions. However, it is more convenient to use relaxation filtration laws, implying that the relaxation effects in the filtration laws are a direct consequence of the relaxation properties of the suspension [7,8,9]. Classical Darcy`s law establishes an equilibrium relationship between pressure gradient ∇p and filtration velocity $\vec{v}$ that sometimes leads to the discrepancy of real end theoretical data. Probably, the non-equilibrium character of the dependences of $\vec{v}$ on ∇p depends on numerous factors such as rheological non-equilibrium properties of the liquid (in particular, visco-elastic behaviors), interaction the liquid with matrix of porous media, adsorption of some components of oil on the surface of the matrix etc. At filtration of polymer solutions in porous media this phenomenon can by explained through filling and releasing of pores by polymers macromolecules [10]. In these conditions the equilibrium character of the Darcy's law is usually violated, it takes relaxing character[11, 12,13,14]).

Many researchers have attempted to generalize the Darcy's law with using different approaches[15,16]. Iaffaldano et al. [17] proposed a memory model for advection of water in porous media. The proposed model fits well the flux rate observed in experiments of water flux through sands. Giuseppe et al. [18] modified constitutive equations by introducing a memory formalism operating on both the pressure gradient – flux and the pressure – density variations. The memory formalism is represented with fractional order derivatives. Experimental results show that memory effects lead to the delaying of the flux rate and its asymptotic values will be reached later.

In this paper we attempt to adopt the non-Darcyan filtration theory to derive relaxation equations of consolidating cake filtration. Principal differences of cake filtration from deep bed filtration are the formation of moving unknown front – the thickness of cake layer. We are to derive an additional equation to determine this parameter. As a consequence, we are to spend additional efforts to numerically solve the governing equations. Firstly, we formulate the problem and then give its numerical solution. On the basis of computing experiments we describe some results.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 1 , January 2018

II. FORMULATION OF THE PROBLEM

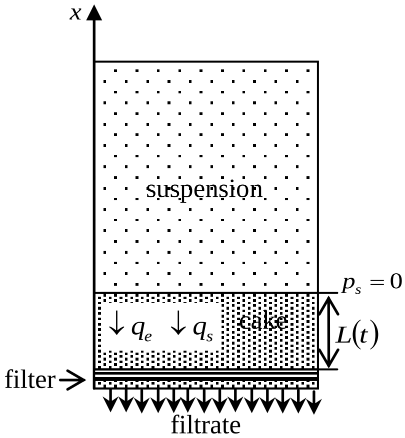

Fig.1. A schematic of cake filtration

A schematic diagram depicting cake filtration is shown in Fig. 1. A suspension with a particle size under pressure flows toward a medium. It is assumed that the suspended particles cannot penetrate into the medium and are retained on the upstream side of the medium to form a cake. The suspending fluid passes through the medium as filtrate. The thickness of the cake $L(t)$ increases with time as filtration proceeds.

Let us suppose, what the filtration velocity of the liquid phase relative to the pressure gradient has a nonequilibrium nature. The nonequilibrium relationship is assumed to be in linear differential form

$$q_\ell = -\frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_\ell}{\partial x}, \tag{1}$$

where q_ℓ - liquid phase velocity, k - permeability coefficient, μ - viscosity, p_ℓ - pressure in the liquid phase, $\lambda_{p\ell}$ - relaxation time of filtration velocities, t - time, x - distance away from the medium.

Since the rates of phase filtration can have different scales of variation, the relaxation effects can also occur with different characteristic times. In this problem, we can neglect the relaxation effects of the filtration rate of the solid phase in comparison with the liquid phase [19,20]. Then from (1) on the basis of conservation laws we obtain the following equation with respect to the compressive stress of the cake phase p_s

$$\frac{\partial p_s}{\partial t} = \frac{k^0 p_A}{\mu\beta}\left(1+\frac{p_s}{p_A}\right)^{1-\beta}\frac{\partial}{\partial x}\left[\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\left(\frac{\partial p_s}{\partial x}\right)\right] - q_{\ell m}\frac{\partial p_s}{\partial x}, \tag{2}$$

where p_A - characteristic pressure, k^0 - value of k at $p_s=0$, β, δ - constants.

The flow rate $q_{\ell m}$ is balanced by the flow through the filter, so we have:

$$q_{\ell m} = \frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\bigg|_{x=0}. \tag{3}$$

For a consolidating cake from the continuity equations, it follows [3] $\frac{\partial(q_\ell+q_s)}{\partial x}=0$, that for a given speed regime means $q_\ell+q_s=const$. In contrast to the regime with a given pressure, $p_\ell+p_s$ is not constant here, but is a function of time $p_\ell+p_s=r(t)$, which is determined in the process of solving the problem.

Here we consider a problem with a given speed regime $q_\ell+q_s=v_0=const$. For this regime, the initial and boundary conditions for (2) have the following form

$$p_s(0,x)=0,\ \frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\bigg|_{x=0} = -\frac{p_\ell}{\mu R_m}\bigg|_{x=0} = -v_0 = const<0,\ \ p_s(t,L(t))=0, \tag{4}$$

where R_m - relative resistance of the filtering element.

The equation of thickness growth for the cake layer $L(t)$ has the form

$$\frac{dL}{dt} = -\frac{\varepsilon_s^0}{\varepsilon_s^0-\varepsilon_{s_0}}\left[\frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\right]\bigg|_{L^-} + \left[\frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\right]\bigg|_{x=0}, \tag{5}$$

where ε_s^0 - solid content at zero pressure, ε_{s_0} - concentration of solid particles in suspension.

From equation (5) we can determine a mobile front $L(t)$ - the boundary between the suspension and the cake layer. This equation is solved together with the basic filtering equation (2) under the conditions (4) and $L(0)=0$.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 1 , January 2018

We introduce the following notations

$$a(p_s)=\left(1+\frac{p_s}{p_A}\right)^{1-\beta},\quad b(p_s)=\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta},\quad c(p_s)=\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)^{-\delta},\quad c^0(p_s)=\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)^{-\delta}\Bigg|_{x=0},\quad c_1=\frac{k^0 p_A}{\beta\mu},$$

$$c_2=\frac{\varepsilon_s^0}{\varepsilon_s^0-\varepsilon_{s_0}}.$$

With taking into account these notations equation (2) can be transformed into the following form

$$\frac{\partial p_s}{\partial t}=c_1 a(p_s)\frac{\partial}{\partial x}\left[b(p_s)\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\left(\frac{\partial p_s}{\partial x}\right)\right]-q_{\ell m}\frac{\partial p_s}{\partial x}. \quad (6)$$

The equation for the mobile boundary $L(t)$, (5), takes the form

$$\frac{dL}{dt}=-c_2\left[c(p_s)\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\right]\Bigg|_{L^-}+q_{\ell m}, \quad (7)$$

where

$$q_{\ell m}=c^0(p_s)\left[\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\right]\Bigg|_{x=0}.$$

To solve the problem (6), (7) with (4) and $L(0)=0$ we use the finite differences method [21, 22].

III. NUMERICAL SOLUTION OF THE PROBLEM

We introduce a uniform grid by t with the step τ $\overline{\omega}_\tau=\{t\,|\,t=t_j=j\tau,\ j=0,1,\ldots,N,\ \tau N=T\}$, and a non - uniform grid by coordinate x [21, 22] $\overline{\omega}_h=\{x\,|\,x=x_i=x_{i-1}+h_i,\ h_i=0,\ i=1,2,\ldots,N,N+1,N+1,\ldots,\ x_N=L\}$ with the variable steps $h_i>0$.

We are to choose the steps h_i from the interval $[x_i,x_{i+1}]$ so, that the mobile boundary moves exactly on one step along the time grid. This approach is known as the method of catching the front in a grid node. We denote by $p_{s,i}^{j+1}$ the grid function corresponding to p_s. We approximate equation (6) by an implicit difference scheme that is nonlinear with respect to the function $p_{s,i}^{j+1}$

$$\frac{p_{s,i}^{j+1}-p_{s,i}^{j}}{\tau}=c_1\frac{2a(p_{s,i}^{j})}{h_i+h_{i+1}}\left\{b(p_{s,i+1/2}^{j+1})\frac{p_{s,i+1}^{j+1}-p_{s,i-1}^{j+1}}{h_i+h_{i+1}}+\frac{\lambda_{pl}}{\tau}b(p_{s,i+1/2}^{j+1})\left[\frac{p_{s,i+1}^{j+1}-p_{s,i-1}^{j+1}}{h_i+h_{i+1}}-\frac{p_{s,i+1}^{j}-p_{s,i-1}^{j}}{h_i+h_{i+1}}\right]-b(p_{s,i-1/2}^{j+1})\frac{p_{s,i}^{j+1}-p_{s,i-1}^{j+1}}{h_i}-\right.$$

$$\left.-\frac{\lambda_{pl}}{\tau}b(p_{s,i-1/2}^{j+1})\left[\frac{p_{s,i}^{j+1}-p_{,si-1}^{j+1}}{h_i}-\frac{p_{s,i}^{j}-p_{s,i-1}^{j}}{h_i}\right]\right\}-(q_{\ell m})_0^{j+1}\frac{p_{s,i}^{j+1}-p_{s,i-1}^{j+1}}{h_i},\quad i=1,\ldots,N-1,\quad j=0,1,\ldots,N-1, \quad (8)$$

where

$$a(p_{s,i}^{j})=\left(1+\frac{p_{s,i}^{j}}{p_A}\right)^{1-\beta},\quad b(p_{s,i+1/2}^{j+1})=\frac{1}{2}\left[\left(1+\frac{p_{s,i+1}^{j+1}}{p_A}\right)^{\beta-\delta}+\left(1+\frac{p_{s,i}^{j+1}}{p_A}\right)^{\beta-\delta}\right],\quad c^0(p_{s,0}^{j+1})=\frac{k^0}{\mu}\left(1+\frac{p_{s,0}^{j+1}}{p_A}\right)^{-\delta},$$

$$(q_{\ell m})_0^{j+1}=c^0(p_{s,0}^{j+1})\left(\frac{p_{s,1}^{j+1}-p_{s,0}^{j+1}}{h_0}+\frac{\lambda_{p\ell}}{\tau}\left(\frac{p_{s,1}^{j+1}-p_{s,0}^{j+1}}{h_0}-\frac{p_{s,1}^{j}-p_{s,0}^{j}}{h_0}\right)\right).$$

Equation (7) when $\dfrac{dL}{dt}\approx\dfrac{h_{i+1}}{\tau}$ after the approximation can be written in the form

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 1 , January 2018

$$\frac{h_{i+1}}{\tau}=-c_2\left[c\left(p^{j}_{s,i-1/2}\right)\left(\frac{p^{j+1}_{s,i}-p^{j+1}_{s,i-1}}{h_{i+1}}+\frac{\lambda_{p\ell}}{\tau}\left(\frac{p^{j+1}_{s,i}-p^{j+1}_{s,i-1}}{h_{i+1}}-\frac{p^{j}_{s,i}-p^{j}_{s,i-1}}{h_{i+1}}\right)\right)\right]+\left(q_{\ell m}\right)^{j+1}_{0}, \tag{9}$$

where

$$c\left(p^{j}_{s,i-1/2}\right)=\frac{k^0}{2\mu}\left[\left(1+\frac{p^{j}_{s,i}}{p_A}\right)^{-\delta}+\left(1+\frac{p^{j}_{s,i-1}}{p_A}\right)^{-\delta}\right].$$

Approximation of initial and boundary conditions (4) gives

$$p^{j}_{s,i}=0,\ i=0,1,..,N,\ j=0,$$

$$-\mu c^0\left(p^{j}_{s,0}\right)\left(\frac{p^{j+1}_{s,1}-p^{j+1}_{s,0}}{h_1}+\frac{1}{\tau}\left(\frac{p^{j+1}_{s,1}-p^{j+1}_{s,0}}{h_1}-\frac{p^{j}_{s,1}-p^{j}_{s,0}}{h_1}\right)\right)=\frac{p_\ell}{R_m}=v_0,\ j=\overline{0,N}, \tag{10}$$

$$p^{j+1}_{s,i}=0,\ i=N+1,N+2,...,\quad j=0,1,....$$

The obtained set of equations (8) is nonlinear, so to solve it we use the method of simple iteration

$$\frac{\overset{(\sigma+1)}{p^{j+1}_{s,i}}-\phi^{j}_{i}}{\tau}=c_1\frac{2a\left(p^{j}_{s,i}\right)}{h_i+h_{i+1}}\left\{b(\overset{(\sigma)}{p^{j+1}_{s,i+1/2}})\frac{\overset{(\sigma+1)}{p^{j+1}_{s,i+1}}-\overset{(\sigma+1)}{p^{j+1}_{s,i-1}}}{h_i+h_{i+1}}+\frac{\lambda_{pl}}{\tau}b(\overset{(\sigma)}{p^{j+1}_{s,i+1/2}})\left[\frac{\overset{(\sigma+1)}{p^{j+1}_{s,i+1}}-\overset{(\sigma+1)}{p^{j+1}_{s,i-1}}}{h_i+h_{i+1}}-\frac{p^{j}_{s,i+1}-p^{j}_{s,i-1}}{h_i+h_{i+1}}\right]-b(\overset{(s)}{p^{j+1}_{s,i-1/2}})\frac{\overset{(\sigma+1)}{p^{j+1}_{s,i}}-\overset{(\sigma+1)}{p^{j+1}_{s,i-1}}}{h_i}-\right.$$

$$\left.-\frac{\lambda_{pl}}{\tau}b(\overset{(\sigma)}{p^{j+1}_{s,i-1/2}})\left[\frac{\overset{(\sigma+1)}{p^{j+1}_{s,i}}-\overset{(\sigma+1)}{p^{j+1}_{si-1}}}{h_i}-\frac{p^{j}_{s,i}-p^{j}_{s,i-1}}{h_i}\right]\right\}-\left(\overset{(\sigma)}{q_{\ell m}}\right)^{j+1}_{0}\frac{\overset{(\sigma+1)}{p^{j+1}_{s,i}}-\overset{(\sigma+1)}{p^{j+1}_{s,i-1}}}{h_i}, \tag{11}$$

where

$$b\left(\overset{(\sigma)}{p^{j+1}_{s,i+1/2}}\right)=\frac{1}{2}\left[\left(1+\frac{\overset{(\sigma)}{p^{j+1}_{s,i+1}}}{p_A}\right)^{\beta-\delta}+\left(1+\frac{\overset{(\sigma)}{p^{j+1}_{s,i}}}{p_A}\right)^{\beta-\delta}\right],\ \left(\overset{(\sigma)}{q_{\ell m}}\right)^{j+1}_{0}=c^0\left(\overset{(\sigma)}{p^{j+1}_{s,0}}\right)\left(\frac{\overset{(\sigma)}{p^{j+1}_{s,1}}-\overset{(\sigma)}{p^{j+1}_{s,0}}}{h_1}+\frac{\lambda_{p\ell}}{\tau}\left(\frac{\overset{(\sigma)}{p^{j+1}_{s,1}}-\overset{(\sigma)}{p^{j+1}_{s,0}}}{h_1}-\frac{p^{j}_{s,1}-p^{j}_{s,0}}{h_1}\right)\right),$$

σ is the number of iteration.

It can be seen that the system of equations (10) is now linear with respect to $\overset{(s+1)}{p}{}^{j+1}_{s,i}$, which allows us to use the Tomas's algorithm [21]. As a condition to stop iteration procedure on this time layer, the following relationship can be used:

$$\max_i\left|\overset{(s+1)}{p^{j+1}_{s,i}}-\overset{(s)}{p^{j+1}_{s,i}}\right|\le\varepsilon, \tag{12}$$

where ε is the given accuracy of calculations.

When condition (10) is satisfied then $\overset{(s+1)}{p}{}^{j+1}_{s,i}=p^{j+1}_{s,i}$. As an initial approach we can use $\overset{(s=0)}{p^{j+1}_{s,i}}=p^{j}_{s,i}$.

Equation (11) leads to the system of linear equations

$$\overset{(\sigma)}{A_i}\overset{(\sigma+1)}{p^{j+1}_{s,i-1}}-\overset{(\sigma)}{B_i}\overset{(\sigma+1)}{p^{j+1}_{s,i}}+\overset{(\sigma)}{C_i}\overset{(\sigma+1)}{p^{j+1}_{s,i+1}}=-\overset{(\sigma)}{F_i},\quad i=\overline{1,N-1}, \tag{13}$$

where

$$\overset{(\sigma)}{A_i}=-\frac{1}{h_i+h_{i+1}}\left(1+\frac{\lambda_{p\ell}}{\tau}\right)b\left(\overset{(\sigma)}{p^{j+1}_{s,i+1/2}}\right)+\frac{1}{h_i}\left(1+\frac{\lambda_{p\ell}}{\tau}\right)b\left(\overset{(\sigma)}{p^{j+1}_{s,i-1/2}}\right)+\frac{h_i+h_{i+1}}{2c_1h_ia\left(p^{j}_{s,i}\right)}q_{\ell m},$$

$$\overset{(\sigma)}{B_i}=\frac{1}{h_i}\left(1+\frac{\lambda_{p\ell}}{\tau}\right)b\left(\overset{(\sigma)}{p^{j+1}_{s,i-1/2}}\right)+\frac{h_i+h_{i+1}}{2\tau c_1a\left(p^{j}_{s,i}\right)}+\frac{h_i+h_{i+1}}{2c_1h_ia\left(p^{j}_{s,i}\right)}q_{\ell m},$$

$$\overset{(\sigma)}{C_i}=\frac{1}{h_i+h_{i+1}}\left(1+\frac{\lambda_{p\ell}}{\tau}\right)b\left(\overset{(\sigma)}{p^{j+1}_{s,i+1/2}}\right),$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 1 , January 2018

$$\overset{(\sigma)}{F_i} = \frac{h_i + h_{i+1}}{2\tau c_1 a\left(p_{s,i}^j\right)} p_{s,i}^j + \frac{\lambda_{p\ell}}{(h_i + h_{i+1})\tau} b\left(\overset{(\sigma)}{p_{s,i+1/2}^{j+1}}\right)\left(p_{s,i-1}^j - p_{s,i+1}^j\right) - \frac{\lambda_{p\ell}}{h_i\tau} b\left(\overset{(\sigma)}{p_{s,i-1/2}^{j+1}}\right)\left(p_{s,i-1}^j - p_{s,i}^j\right).$$

The equation (9) is used to determine the step h_{i+1} and it can be written in the form

$$(h_{i+1})^2 - \tau(q_{\ell m})_0^{j+1} h_{i+1} + \tau c_2\, c\left(p_{s,i-1/2}^j\right)\left(p_{s,i}^{j+1} - p_{s,i-1}^{j+1} + \frac{\lambda_{p\ell}}{\tau}\left(p_{s,i}^{j+1} - p_{s,i-1}^{j+1} - p_{s,i}^j + p_{s,i-1}^j\right)\right) = 0. \quad (14)$$

By solving this nonlinear equation for each temporal layer we can determine h_{i+1}.

The system of linear algebraic equations (13) is solved by the Tomas' algorithm

$$\overset{(\sigma+1)}{p_{s,i}^{j+1}} = \xi_{i+1} \overset{(\sigma+1)}{p_{s,i+1}^{j+1}} + \zeta_{i+1}, \quad (15)$$

where $\xi_{i+1} = \dfrac{\overset{(\sigma)}{C_i}}{\overset{(\sigma)}{B_i} - \overset{(\sigma)}{A_i}\xi_i}$, $\zeta_{i+1} = \dfrac{\overset{(\sigma)}{F_i} + \overset{(\sigma)}{A_i}\zeta_i}{\overset{(\sigma)}{B_i} - \overset{(\sigma)}{A_i}\xi_i}$.

The starting values of the coefficients ξ_1 and ζ_1 are determined from the boundary condition (10), which have the form

$$\xi_1 = 1, \; \zeta_1 = \frac{c^0\left(p_{s,0}^j\right)\dfrac{\lambda_{p\ell}}{h_0\tau}\left(p_{s,0}^j - p_{s,1}^j\right) + v_0}{\dfrac{c^0\left(p_{s,0}^j\right)}{h_0}\left(1 + \dfrac{\lambda_{p\ell}}{\tau}\right)}. \quad (16)$$

IV. RESULTS

Numerical results with using (14), (15) were obtained with the following values initial parameters: $v_0 = 10^{-4}$ m/s, $p_A = 10^4$ Pa, $R_m = 10^{12}$ 1/m, $\mu = 10^3$ Pa·s, $k^0 = 0.8\cdot 10^{-13}$ m^2, $\varepsilon_s^0 = 0.20$, $\varepsilon_{s_0} = 0{,}0076$, $\beta = 0.13$, $\delta = 0.57$.

Some results are graphically shown below. The growth of the cake thickness for different values of the relaxation time $\lambda_{p\ell}$ is shown in Fig.2. As one can see the increasing of relaxation time $\lambda_{p\ell}$ leads to the faster growth of the cake thickness at all other constant conditions. Fig.3 shows the compression pressure profiles for different relaxation times for several fixed time values. On the graphs one can see the decrease in the values of the compression pressure with an increase in the values of the relaxation time. This decrease for large values of time becomes insignificant, which can be explained by the weakening of the influence of pressure relaxation. Thus, at $t = 450$ sec. (Fig. 3a), the difference in the compression pressure profiles is significant, and for $t = 1800$ sec. (Fig. 3c) the difference is already negligible.

With time the amount of the compression pressure at all points of the cake-layer increases. In particular, one can observe a significant increase in the point $x = 0$ for the case of $\lambda_{p\ell} = 0$ from $0.4\cdot 10^5$ Pa at $t = 0$ to $0.85\cdot 10^5$ at $t = 1800$ sec. With allowance for the relaxation of the pressure gradient, these values are lower than for the case $\lambda_{p\ell} = 0$. For large times this difference disappears, which is explained by the weakening of the influence of relaxation effects. With increasing time, i.e. with increasing thickness of the sediment, the distribution of the profiles also widens. Note that the graphs in Fig.3 have exact ends in the coordinate *x*, which coincides with the thickness of the cake layer.

Similar graphs for the liquid pressure are shown in Fig.4. The phenomena noted above with respect to the influence of the relaxation parameter are also preserved for p_ℓ. The liquid pressure rises from the point $x = 0$ along the thickness of the cake layer. In addition, it assumes that at the point $x = 0$ the pressure p_ℓ has a constant value in accordance with the second boundary condition in (4), i.e. $p_\ell\big|_{x=0} = \mu R_m v_0$. Thus, at the point $x = 0$ the sum of the pressure $p_\ell + p_s$ has an increasing dynamics due to growth of p_s, while at the same time p_ℓ has a constant value.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 1 , January 2018

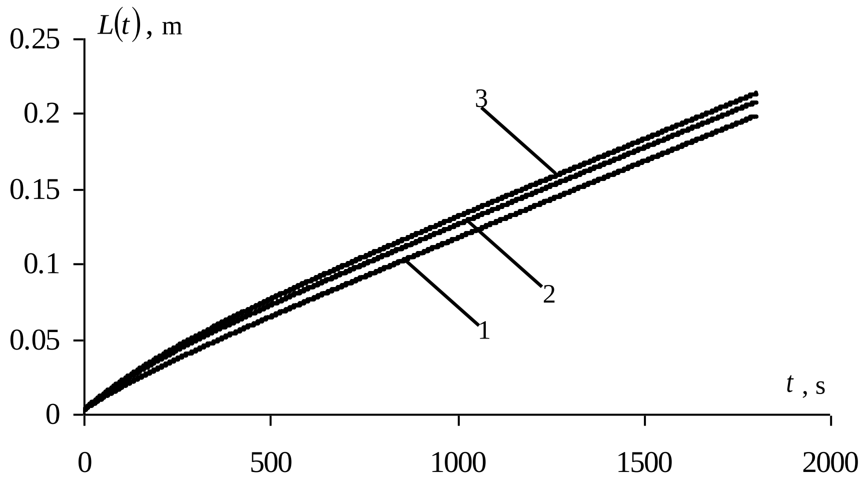

Fig.2. Dynamics of the cake thickness at $\lambda_{p\ell} = 0$ (1); 150 (2); 350 (3) c.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
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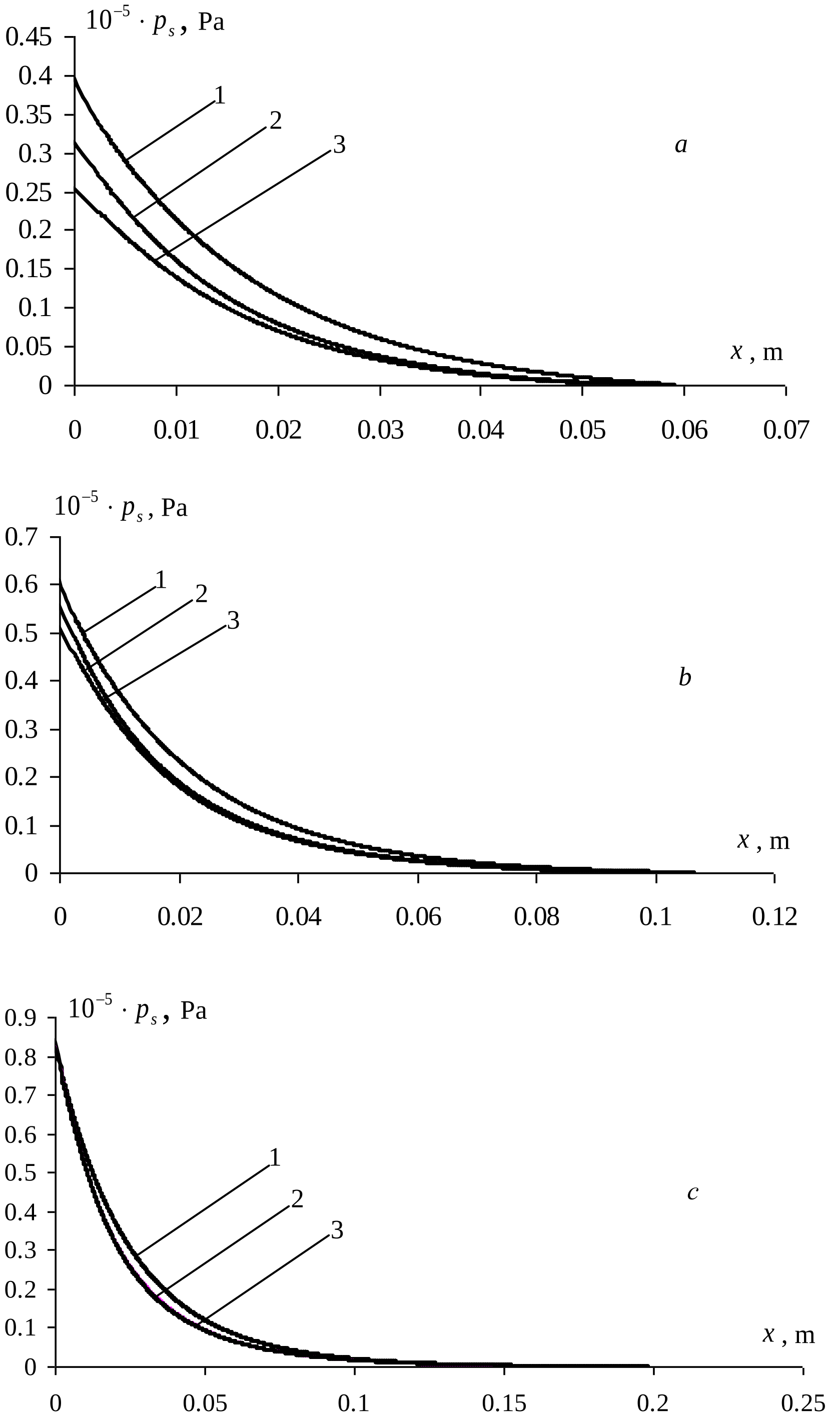

Fig 3. Profiles the compression pressure through thickness of the cake at $\lambda_{p\ell} = 0$ (1); 150 (2); 350 (3) s, $t = 450$ (a); 900 (b); 1800 (c) s.

ISSN: 2350-0328
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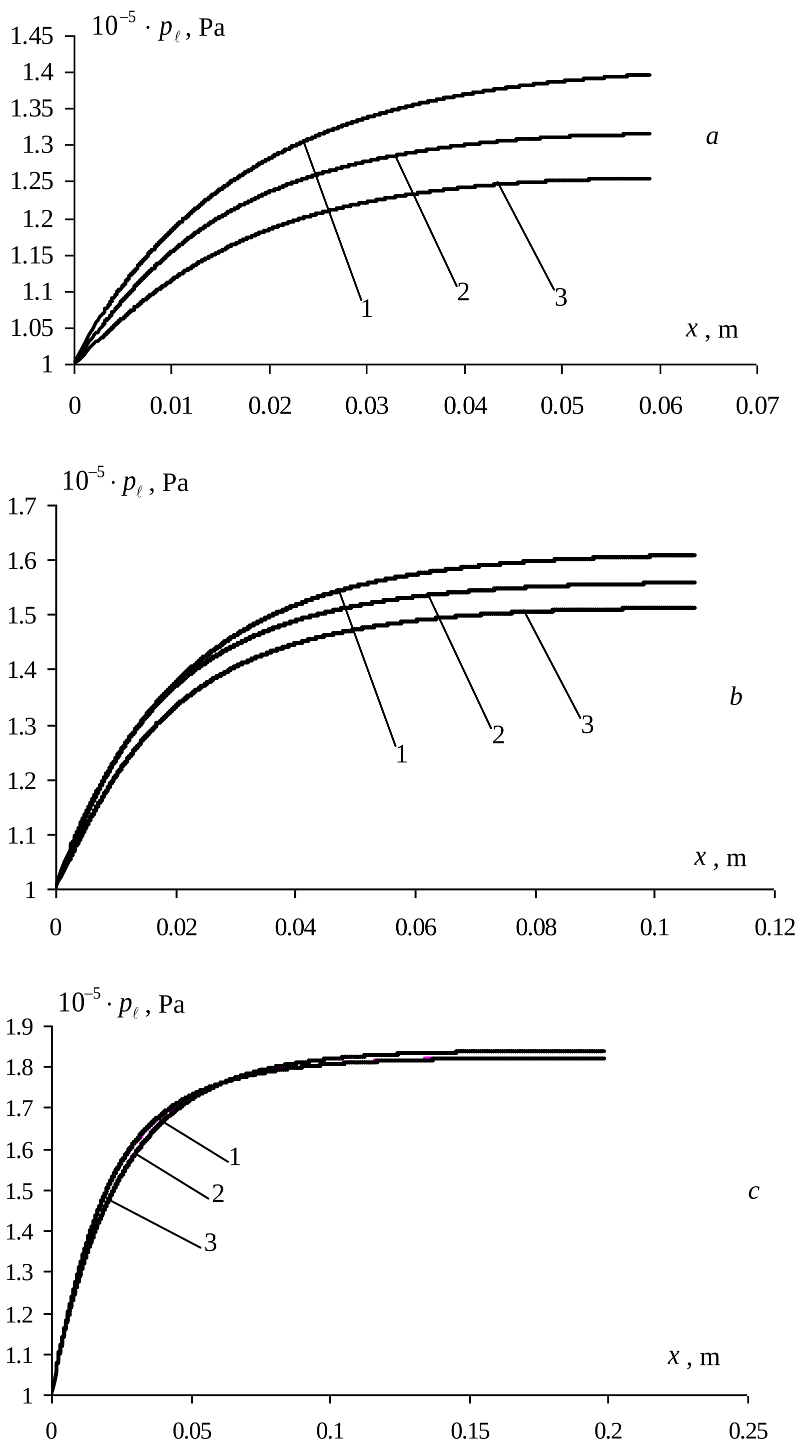

Fig. 4. Profiles of p_ℓ through thickness of the cake at $\lambda_{p\ell} = 0$ (1); 150 (2); 350 (3) s, $t = 450$ (*a*); 900 (*b*); 1800 (*c*) s.

V. CONCLUSION

On the basis of obtained results it can be concluded that the relaxation nature of the flow significantly alters both the growth of the cake thickness and its filtration characteristics. In particular, relaxation phenomena in filtration laws lead to the decreasing of compression pressure and liquid phase pressure distributions. At the developed time stage, when current times are considerably large then characteristic relaxation time the influence of relaxation phenomena run out.

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