Application of fixed-point iteration method in the design of Cassegrain antenna

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ABSTRACT: In this paper, fixed-point iteration method is used to design an optimal Cassegrain antenna structure. Initially, the antenna design relationship is introduced to minimize the effects of aperture obstruction, and its various parameters and variables are introduced. Then, the method of finding the root of this nonlinear equation is introduced using fixed-point iteration method. A design example is presented that accurately and quickly obtains the equation response using the investigated method, and the results are presented in the article.

KEY WORDS: Numerical solution of nonlinear equation, Fixed-point iteration method, Cassegrain antenna, Conical corrugated horn antenna

I.   INTRODUCTION

Reflector antennas have a wide range of applications in space and far way telecommunications. The Cassegrain antenna is a reflector antenna constructed according to the Cassegrain telescope structure [1]. This antenna has many advantages over the original reflector antennas, but since the secondary reflector is located in front of the main reflector, a portion of waves is blocked by the subreflector and therefore, the antenna efficiency decreases. In the design of this antenna, the secondary reflection dimensions should be calculated in such a way to minimize obstruction.

Usually in this system, conical corrugated horn antenna is used as feed antenna [2]. In [3], a relationship is presented to calculate the dimensions of the Cassegrain antenna and the dimensions of the antenna feed for which the efficiency of the aperture is optimal. This relationship is complex and non-linear, and has too many unknown parameters. If some of these parameters are optionally considered, this relation can be considered in terms of a single variable that relates to the dimensions of the sub-reflector.

As far as the author knows, so far this relationship has not been resolved using the fixed-point iteration method. As shown in this paper, this method can accurately obtain the response of this nonlinear equation.

In the second part of the paper, first, an explanation is given about the antenna design equation, and its various parameters are introduced. In the third part, the numerical method of fixed-point iteration is introduced and through the presentation of a design example in the fourth part of paper, the antenna design method is explained and, by solving the design equation, the optimal dimensions of the Cassegrain antenna and the antenna feed, which is in the form of a conical corrugated horn, are obtained.

II.   INTRODUCING CASSEGRAIN ANTENNA DESIGN EQUATION

Cassegrain antenna gain is proportional to aperture efficiency and the aperture efficiency depends on several factors which major factors are the field distribution on aperture plane and the hyperbolic subreflector dimension which blocks the incoming waves from the parabolic main reflector. It is shown that to improve the aperture efficiency it is required that the reflector edge relative field intensity with respect to reflector apex, be -11 dB or 0.2818 [5]. Also subreflector dimensions against main reflector must be as small as possible. If we use conical corrugated horn antenna as feed antenna , we can obtain a relation between Cassegrain antenna dimensions and feed antenna dimensions and electrical specifications that satisfies the optimization condition of aperture efficiency as following [3]

$$f(A) = 4F \tan\left(\frac{\tan^{-1}\left(\frac{a_f}{(2Ae - L_p)}\right)}{2}\right) - 2\frac{A(e^2 - 1)\sin(\psi_0)}{(1 + e\,\cos(\psi_0))}$$

(1)

Parameters of this equation are defined as

A: one of the hyperbolic surface geometric parameters.

F: Focal distance of parabolic reflector.

e: eccentricity of hyperbolic reflector which equals the eccentricity of the Cassegrain antenna system.

ψ_0 parabolic reflector subtended angle.

a_f feed antenna aperture radius considering the depth of corrugations.

L_P phase center displacement of the feed antenna relative to the feed aperture plane.

When designing an antenna, the main reflector diameter, D, is calculated according to the link budget, and usually a certain amount is specified. The ratio of focal length, F, and D is usually considered to be 0.5. Considering the governing equations on the parabolic reflection surfaces in the polar coordinate as

$$\psi_0 = 2\tan^{-1}\left(\frac{D}{4F}\right)$$

(2)

So ψ_0 is specified. We obtain a_f , considering that $a_f = a + d$, where a is the horn output radius neglecting the slot depth and d is the depth of the first slot adjacent to the feed antenna aperture which calculates from

$$d = \frac{\lambda}{4}\exp\left(\frac{1}{2.5ka}\right)$$

(3)

λ is the free space wavelength. 'a' is calculated from the required relative electric field intensity at reflectors edge to make highest radiation efficiency. According to studies done so far, the relative electric field strength of the conical corrugated horn antenna depends on a parameter called the horn antenna slant factor, S, and another parameter called K1, which are defined by (4) and (5) .

$$S = \frac{a^2}{2\lambda R}$$

(4)

$$K1 = 2\pi a \sin(\theta_0)/\lambda$$

(5)

In these equations 'a' is the same as defined in (3) and R is the horn slant radius and θ_0 is the subtended angle of subreflector. We use fig. (1) to extract 'a'. In this figure, the relative field intensity of the feed antenna for various S and $K1$ values in the range $0 \leq S \leq 0.7$ و $0 \leq K1 \leq 12$ are presented [6].
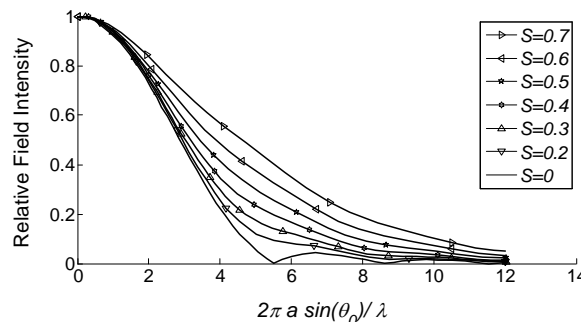

Figure 1. Relative field intensity of feed antenna versus different S and $K1$ values.

To obtain L_p in (1) , the phase center displacement of the feed antenna versus the aperture plane, table 1 is used which gives L_p / R versus different slant ratio values and R is the horn slant radius [7].

S	L_p/R	S	L_p/R
0.00	0.0	0.36	0.386
0.04	0.005	0.40	0.464
0.08	0.020	0.44	0.542
0.12	0.045	0.48	0.614
0.16	0.080	0.52	0.673
0.20	0.124	0.56	0.718
0.24	0.178	0.60	0.753
0.28	0.240	0.64	0.783
0.32	0.310	0.68	0.811

Table 1. L_p / R ratio versus different S values.

III. INTRODUCING THE FIXED-POINT ITERATION METHOD

In this method the equation $f(x)=0$ must be reformed to

$$x = g(x) \tag{6}$$
$$x_{i+1} = g(x_i) \tag{7}$$

Selecting a proper initial value , using iteration equation (7) we reach to the function root. Relation between (i+1)th step true error, $E_{t,i+1}$, and (i)th step true error $E_{t,i}$, in this method is as follows [4].

$$E_{t,i+1} = g'(\xi)E_{t,i} \tag{8}$$

For the convergence of this method, it is necessary that

$$|g'(\xi)| < 1 \tag{9}$$

When converting relation $f(x)=0$ to (6), it must be noted that the condition (9) holds. In Figure 2, the convergence process with condition $|g'(\xi)| < 1$ around the root is graphically. As you can see, starting from a suitable point, x_0 ,the next point x_1, is equal to the value of the function $g\ (x)$ at the point x_0. This process continues until we approach the intersection of the curves $y\ (x) = g\ (x),\ y\ (x) = x$. When the condition $|x_i - x_{i-1}| < \varepsilon$, is satisfied, the root finding process stops. The choice of ε depends on the accuracy required to solve a particular physical problem.
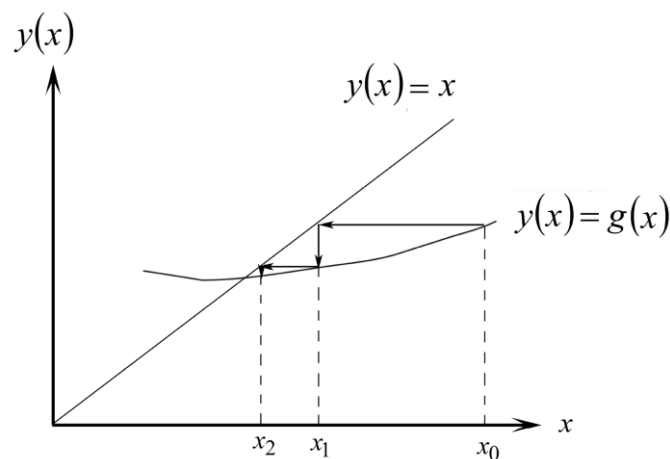

Figure 2 - Graphical description of the convergence of the fixed-point iteration method for the case where $|g'(\xi)| < 1$.

In the following, by presenting a specific design example, we have used this numerical approach to calculate the position and dimensions of the Cassegrain antenna.

IV. DESIGN AND PRESENTING THE APPLICATION OF THE FIXED-POINT ITERATION METHOD IN THE SOLUTION OF ANTENNA EQUATION

Suppose we are planning to design a Cassegrain antenna to receive the satellite signals at 1GHz center frequency, and its antenna feed is of conical corrugated type. Assume that the ratio ,$F / D = 0.5$. and the magnitude of the antenna, $M = 3$, and the Horn slant factor $S = 2$, and the parabolic reflector diameter calculated according to the link budget is 10 (m). Then according to the explanations given in the second part, we can calculate the factors in relation (1) and obtain a relation only in terms of the variable A. then by solving this relationship using the numerical method introduced in Section 3, the response of this equation will be calculated.

At first $F = 10 \times 0.5 = 5\ (m)$, using the following relation we get 'e' as

$$e = \frac{M+1}{M-1} = \frac{4}{2} = 2 \tag{10}$$

Then from (2),

$$\psi_0 = 2\tan^{-1}\left(\frac{D}{4F}\right) = 2\tan^{-1}\left(\frac{1}{2}\right) = 53.13° \tag{11}$$

In continue to get 'a', it is necessary to calculate θ_0, we can obtain θ_0 from effective focal length F_e which equals M times F. from (2) we have

$$\theta_0 = 2\tan^{-1}\left(\frac{D}{4F_e}\right) = 2\tan^{-1}\left(\frac{D}{4MF}\right) = 2\tan^{-1}\left(\frac{1}{4\times 0.5 \times 3}\right) = 18.92° \tag{12}$$

From Fig. 1 for relative field intensity equal to 0.2818 and for $S=0.2$, $K1$ approximately equals to 3.85, so from (5) and (12) we have,

$$a = \frac{\lambda K1}{2\pi\sin(\theta_0)} = \frac{3\times 10^8 \times 3.85}{2\pi \sin(18.92°)\times 1 \times 10^9} = 56.7\ (cm) \tag{13}$$

Then from (3)

$$d = \frac{\lambda}{4}\exp\left(\frac{1}{2.5ka}\right) = \frac{3\times 10^8}{4\times 10^9}e^{\left(\frac{3\times 10^8}{2.5\times 2\pi\times 0.567\times 10^9}\right)} = 7.8\ (cm) \tag{14}$$

Thus $a_f = a + d = 64.5\ (cm)$,

To obtain L_p from Table 1 , for S=0.2, we get $L_p / R = 0.124$. We take R from (4) as,

$$R = \frac{a^2}{2\lambda S} = \frac{(0.567)^2 \times 10^9}{2\times 3\times 10^8 \times 0.2} = 2.679\ (m) \tag{15}$$

Thus

$$L_p / R = 0.124 \rightarrow L_p = 0.124 \times 2.679 = 0.332\ (m) \tag{16}$$

Here all the parameters in (1) are specified and the only unknown variable in this relation is A, which can be obtained by using the fixed-point iteration method. For this purpose (1) can be considered as follows

$$A = 2F\tan\left(\frac{\tan^{-1}\left(\frac{a_f}{(2Ae - L_p)}\right)}{2}\right)\frac{(1 + e\cos(\psi_0))}{(e^2 - 1)\sin(\psi_0)} = g(A) \tag{17}$$

In Fig. 3, the functions $f(A) = A$ and $f(A) = g(A)$ are simultaneously drawn. As it is seen, the root occurs at around $A = 1$, and the function $f(A) = g(A)$ at the root position has a gradient close to one. Also, in the vicinity of $A = 0$, the function (17) is discontinuous. The algorithm mentioned in the previous section is executed using a computer program

written in MATLAB [8] Language. We start with the initial value of $A = 1$ and the intended accuracy is 0.005. This method does not answer for this function, and after 105 iterations, the inappropriate response becomes equal to $A = -0.8181$, which is physically not acceptable. So, in the following, we calculate 'A' from relation (1) in another way, and use this relation as the iteration formula.
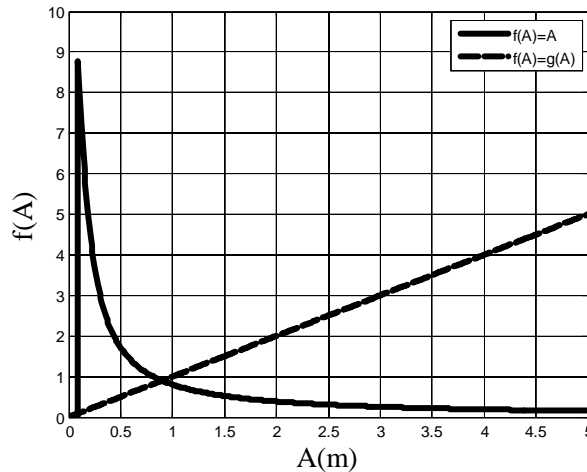

Figure 3. Drawings of functions $f(A) = A$, $f(A) = g(A)$, where $g(A)$ is defined in (17).

$$A = \frac{L_P}{2e} + \frac{a_f}{2e \tan\left(2\tan^{-1}\left(\frac{A(e^2 - 1)\sin(\psi_0)}{2F(1 + e\cos(\psi_0))} \right) \right)} = g(A) \qquad (18)$$

The function $f(A) = g(A)$ is plotted in Fig. 4 together with the function $f(A) = A$ and, as we see, the discontinuity problem is solved in the range about $A = 0$.
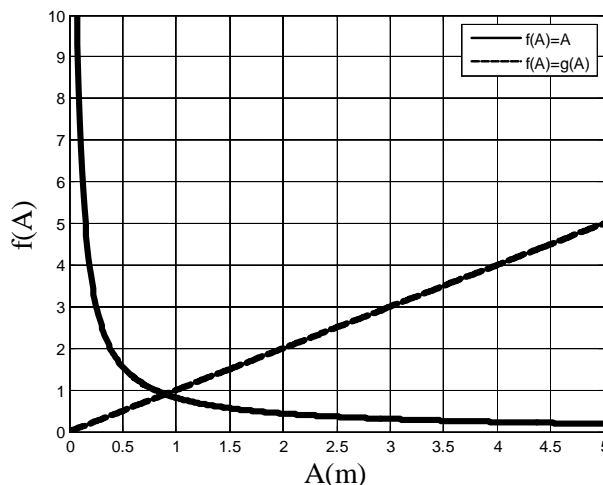

Figure 4. Drawings of functions $f(A) = A$, $f(A) = g(A)$, where $g(A)$ is defined in (18).

After using this function in the root finding algorithm and using the initial point $A = 1$ and the accuracy $\varepsilon = 0.005$, after 49 repetitions we reach to the correct root $A = 0.9004$, and the process of finding the root of the equation stops. Of course, in this function, the convergence is relatively slow since the function slope around the root is close to -1. After calculating A we can obtain the parameter C of the hyperbolic surface from the following equation

$$C / A = e \rightarrow C = Ae = 0.9004 \times 2 = 1.8 \ (m)) \qquad (19)$$

The process of finding the dimensions of the feed antenna, parabolic and hyperbolic reflecting surfaces for this example, ends in here.

V.   CONCLUSION

In this paper, the application of fixed-point iteration method is presented in designing and finding optimal dimensions of axially symmetric Cassegrain antenna. Initially, the Cassegrain antenna design equation was presented to optimize the aperture efficiency, and its various parameters were introduced. Then the fixed-point iteration method is discussed as the root finding method. Then, the antenna design equation is arranged in correspondence with the fixed-point iteration method. This method successfully calculates the root of this equation with proper accuracy and high speed. In order to further clarify the proposed method, a design example is presented to calculate the optimum dimensions of the satellite receiver antenna.

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