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Propagation of Horizontally Shear Waves in a Micro-Morphic Waveguide

Dr. A. Chandulal

Asst. Professor, Department of Mathematics,R.S. Vidyapeetha,Tirupati,Andra Pradesh , India

ABSTRACT: In this paper little attempt has been made to discuss the problem in layered media in the micro-morphic theory in classical theory of elasticity. In this paper the SH-waves propagation in a classical waveguide and also in micro polar wave guide is studied and also in micro polar waveguide. This problem is extended to micro morphic waveguide it is significant to note that is we have one additional wave in micro morphic medium and also there is any no counterpart to it in the classical and the micro polar theory.

KEYWORDS: SH- Waves, Elastic Plate, Cylindrical Shells, Waveguide, Longitudinal wave.

I.INTRODUCTION

Compared to the analysis of plane time harmonic waves in a half space, some complications enter if a body has a finite cross sectional dimension. The system of incident and reflected waves form a standing wave across the thickness of the layer, so that the propagation is essentially in the direction of the layer. This motivates the term wave guide for the layer or for any extended body with a cross-section of finite thickness. Thus the common feature of wave guides is, two or more parallel boundaries, which introduce one or more characteristic length into the problem and leads to wave dispersion, characterized in harmonic waves by a dependence of frequency on wave length. Consider an infinite elastic plate. This can be taken to be a half space, which is bounded in its depth by the introduction of a second parallel boundary. The P, SV and SH-waves reflect from one boundary to neighbouring boundary. The neighbouring parallel boundaries are in effect then guide the waves along the plate, rods, cylindrical shells and a layered elastic solids are other examples of wave guide. In this chapter we study the wave propagation in micromorphic elastic waveguide.

In the classical theory Tolstoy and Usdin [01] studied the propagation of longitudinal wave in waveguide by using the principle of constructive interference and obtained the frequency equation without postulating the displacements. Mindlin [02] studied the vibrations and wave propagation in isotropic plates in detail and obtained the frequency spectrum for the first time. Onoe, Niven and Mindlin [03] studied the axially symmetric waves in the infinite circular cylindrical rod. Wong, Miklowitz and Scott [04] studied waves in an infinite elastic rod of elliptic cross section. Mrithyumnaya Rao [05] studied the longitudinal wave propagation in micro polar waveguide. The SH-wave propagation in a classical waveguide is studied by See and Meetzler [06]. This problem is studied by Mrithyumjaya Rao [10] in micro polar waveguide.

II.BASIC EQUATION

The constitutive equations for this material are

$$t_{(km)} = A_1 \, e_{pp} \, \delta_{km} + 2A_2 \, e_{km} \qquad (1)$$

$$t_{(km)} = \sigma_{[km]} = 2A_3 \in_{pkm} \left(r_p + \theta_p\right) \qquad (2)$$

$$\sigma_{[km]} = -A_4 \, \theta_{pp} \, \delta_{km} - 2A_5 \, \theta_{(km)} \qquad (3)$$

$$t_{k[mn]} = B_1 \, \phi_{pp.k} \, \delta_{mn} + 2B_2 \, \phi_{(mn)k} \qquad (4)$$

$$m_{k1} = -2\left(B_5 \, \phi_{1,k} + B_4 \, \phi_{k,1} + B_5 \, \phi_{p,p} \, \delta_{k1}\right) \qquad (5)$$

Where ( ) denotes symmetric part and [ ] denotes anti-symmetric part, and

$$A_1 = \lambda + \sigma_1, \qquad B_1 = \tau_3$$

$$A_2 = \mu + \sigma_2, \qquad 2B_2 = \tau_7 + \tau_{10}$$

$$A_3 = \sigma_5, \qquad B_3 = 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10}$$

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$$A_4 = -\sigma_1, \qquad B_4 = -2\tau_4$$
$$A_5 = -\sigma_2, \qquad B_5 = -2\tau_9 \tag{6}$$

The equations of motion for this material are

$$(A_1 + A_2 - A_3)u_{p,pm} + (A_2 + A_3)u_{m,pp}$$
$$+2A_3 \ {}_{pkm}\,\phi_{p,k} + \rho f_m = \rho \frac{\partial^2 u_m}{\partial t^2} \tag{7}$$
$$B_1\,\phi_{pp,kk}\,\delta_{ij} + 2B_2\phi_{(i,j),kk} - A_4\phi_{pp}\delta_{ij}$$
$$-2A_5\ \phi_{(i,j)} + \rho f_{(i,j)} = \frac{1}{2}\ \rho_j \frac{\partial^2 \phi_{(i,j)}}{\partial t^2} \tag{8}$$
$$2B_3\,\phi_{p,mm} + 2(B_4 + B_5)\phi_{m,mp} - 4A_3(r_p + \phi_p)$$
$$-\rho 1_p = \rho_j \frac{\partial^2 \phi_p}{\partial t^2} \tag{9}$$

Where $\phi_p = \frac{1}{2} \in_{pkm} \phi_{km}.$

And $r_p = \frac{1}{2} \in_{pkm} u_{m,k}.$

and the couple stress tensor m_{kp} and the body couples 1_p are respectively

$$m_{kp} = \in_{pnm} t_{kmn}. \quad 1_{kp} = \in_{pnm} f_{mn}.$$

Where $\in_{pnm}$ is permutation symbol.

Decomposing the displacement $\vec{u}$ and micro-rotation $\vec{\phi}$ with scalar and vector potentials, we get

$$\vec{u} = \nabla s + \nabla \times \vec{U}, \qquad \nabla.\ \vec{U} = 0$$
$$\vec{\phi} = \nabla \phi + \nabla \times \psi, \qquad \nabla.\ \vec{\psi} = 0 \tag{10}$$

On substituting (10) in (7) and (8) the equations are identically satisfied if

$$\left(C_1^2 + C_3^2\right)\ \nabla^2 \phi = \ddot{S}, \tag{11}$$
$$\left(C_4^2 + C_5^2\right)\ \nabla^2 \phi - 2\omega_0^2\ \phi = \ddot{\phi}, \tag{12}$$
$$\left(C_2^2 + C_3^2\right)\ \nabla^2 \vec{U} + C_3^2\ \nabla \times \vec{\psi} = \ddot{\vec{U}} \tag{13}$$
$$C_4^2\ \nabla^2 \vec{\psi} - 2\omega_0^2\,\vec{\psi} + \omega_0^2 \nabla \times \vec{U} = \ddot{\vec{\psi}} \tag{14}$$

Where

$$C_1^2 = \frac{A_1 + 2A_2 - 2A_3}{\rho}, \qquad C_2^2 = \frac{A_2 - A_3}{\rho}$$
$$C_3^2 = \frac{2A_3}{\rho}, \qquad C_4^2 = \frac{2B_3}{\rho j},$$
$$C_5^2 = \frac{2(B_4 + B_5)}{\rho j}, \qquad \omega_0^2 = \frac{2A_3}{\rho j}, \tag{15}$$

III.SOLUTION OF THE PROBLEM

In this paper we study the propagation of SH-wave in a micro-morphic waveguide thickness of 2h. Since we are considering SH-wave the displacement vector u and micro-rotation ϕ are respectively $(0, v, 0)$, $(\phi_1, 0, \phi_3)$ and the micro-strains are ϕ_{22}, $\phi_{(12)}$, $\phi_{(13)}$. The x_1-axis is taken along the central plane of the layer and x_3-axis along the thickness of the layer.

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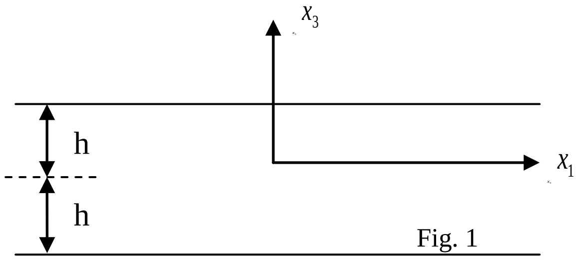

Fig. 1

Here $v, \phi_1, \phi_3, \phi_{22}, \phi_{(12)}$ and $\phi_{(13)}$ are functions of x_1, x_3 coordinates only. The displacement, micro-rotation are given in terms of potential functions are

$$v = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \qquad \phi_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi_2}{\partial x_3} \qquad \phi_3 = \frac{\partial \phi}{\partial x_3} - \frac{\partial \psi_2}{\partial x_1} \tag{16}$$

and the micro - strains are

$$\phi_{22} = \phi_{22}(x_1, x_3, t)$$

$$\phi_{(12)} = \phi_{(12)}(x_1, x_3, t) \tag{17}$$

$$\phi_{(13)} = \phi_{(13)}(x_1, x_3, t)$$

From (16), the equations (11) to (14) reduce to we get

$$\left(C_4^2 + C_5^2\right)\nabla^2\phi - 2\omega_0^2\phi = \phi \tag{18}$$

$$C_2^2\nabla^2\psi_2 - 2\omega_0^2\psi_2 + \omega_0^2 v = \ddot{\psi}_2 \tag{19}$$

$$\left(C_2^2 + C_3^2\right)\nabla^2 v - C_3\nabla^2\psi_2 = \ddot{v} \tag{20}$$

and the equation (8) reduces to we get

$$2B_2\nabla^2\phi_{22} - 2A_5\phi_{22} = \frac{1}{2}\rho j\frac{\partial^2\phi_{22}}{\partial t^2} \tag{21}$$

$$2B_2\nabla^2\phi_{(12)} - 2A_5\phi_{(12)} = \frac{1}{2}\rho j\frac{\partial^2\phi_{(12)}}{\partial t^2} \tag{22}$$

$$2B_2\nabla^2\phi_{(13)} - 2A_5\phi_{(13)} = \frac{1}{2}\rho j\frac{\partial^2\phi_{(13)}}{\partial t^2} \tag{23}$$

Where, $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$

The equation (18) is uncoupled whereas the equations (19) and (20) are coupled in v and ψ_2, $C_2^2, C_3^2, C_4^2, C_5^2$ and ω_0^2 are given by (15).

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A solution of (18) is given by

$$\phi = \left[A_1' \cos m_1 x_3 + A_2' \sin m_1 x_3 \right] e^{i(kx_1 - \omega t)} \tag{24}$$

where

$$m_1^2 = \frac{\omega^2}{C_2^2(\theta+\delta)} - \frac{2\in}{j(\theta+\delta)} - k^2, \qquad \theta = \frac{C_4^2}{C_2^2} \tag{25}$$

And $\delta = \dfrac{C_5^2}{C_2^2}$

Here k represents wave number, $\omega = kc$ and c the velocity of the wave.

For the equations (19) and (20) we seek solutions of the form

$$v = L_1 e^{px_3} + i(kx_1 - \omega t) \tag{26}$$

$$\psi_2 = L_2 e^{qx_3} + i(kx_1 - \omega t)$$

Substituting (26) in (19) and (20) and eliminating the arbitrary constants L_1 and L_2, we obtain

$$\left[\theta(q^2 - k^2) - \frac{2\in}{j\theta} + \frac{\omega^2}{C_2^2}\right]\left[(1+\in)(p^2 - k^2) + \frac{\omega^2}{C_2^2}\right] + \frac{\in^2}{j}(q^2 - k^2) = 0 \tag{27}$$

where $\in = \dfrac{\omega_0^2}{C_2^2}$

Neglecting $\in^2$ terms in the above equation, we obtain

$$-p^2 = \frac{\omega^2}{(1+\in)C_2^2} - k^2 \tag{28}$$

and

$$-q^2 = \frac{\omega^2}{\theta C_2^2} - \frac{2\in}{j\theta} - k^2 \tag{29}$$

Now the solutions of (26) become

$$V = \left[A_3' \cos m_2 x_3 + A_4' \sin m_2 x_3 \right] e^{i(kx_1 - wt)} \tag{30}$$

$$\psi_2 = \left[A_5' \cos m_2 x_3 + A_6' \sin m_2 x_3 \right] e^{i(kx_1 - wt)} \tag{31}$$

Where

$m_2 = iP,\ m_3 = iq$ *and* A_3' *to* A_6' are arbitrary constants.

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The modes of SH-wave propagation in the micro-morphic elastic layer will be split into two systems of symmetric and anti-symmetric modes respect to the x_1x_3- plane if

$$V = A_3' \cos(m_2x_3)e^{i(kx_1-wt)}$$

$$\varnothing = A_2' \sin(m_1x_3)e^{i(kx_1-wt)}, \quad (32)$$

$$\psi_2 = A_5' \cos(m_1x_3)e^{i(kx_1-wt)},$$

and

$$V = A_4' \sin(m_2x_3)e^{i(kx_1-wt)},$$

$$\varnothing = A_1' \cos(m_1x_3)e^{i(kx_1-wt)}, \quad (33)$$

$$\psi_2 = A_6' \sin(m_3x_3)e^{i(kx_1-wt)}$$ respectively.

The boundary conditions to be satisfied when the boundaries of the layer are stress free, are

$$t_{32} = m_{33} = m_{31} = 0 \; at \; x_3 = \pm h \quad (34)$$

Substituting (32) in (34) we obtain the three equations in A_3', A_2', A_5', viz.

$$(A_2 + A_3)\sin(m_2h)A_3' + [2A_3ik \sin(m_1h)]A_2' + \sin(m_3h)A_5' = 0,$$

$$[(B_3 + B_4 + B_5)m_1 + B_5k^2]\sin(m_1h)A_2' + (B_3 + B_4)ik \sin(m_3h)m_3 A_5' = 0$$

$$[(B_3 + B_4)ikm_1 \cos m_1h]A_2' + (B_3m_3^2 - B_4k^2)\cos(m_3h)A_5' = 0 \quad (35)$$

For the existence of the non-trivial solution of the system (35), the determinant of coefficients is zero i.e., (36).

$$\begin{vmatrix} 2A_3ik \sin(m_1h) & (A_2 + A_3)\sin(m_2h) & \sin(m_3h) \\ (B_3 + B_4 + B_5)m_1^2 + B_5k^2 & 0 & (B_3 + B_4)m_3ik \sin(m_3h) \\ (B + B)m \, ik \cos(m_1h) & 0 & (B_3m_3^2 - B_4k^2)\cos(m_3h) \end{vmatrix} = 0 \quad (36)$$

Simplifying the above determinant, we get

$$\sin(m_2h) = 0 \quad (37)$$

and

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$$\frac{\tan m_1h}{\tan m_3h} = \frac{K^2 m_1 m_3\left[-(\theta+\delta)+\frac{(B_5/\rho j)}{c_2^2}\right]\left[\theta+\frac{(B_4/\rho j)}{c_2^2}\right]}{\left[\theta m_3-\frac{(B_4/\rho j)}{c_2^2}k^2\right]\left[(\theta+\delta)m_1^2+\frac{k^2(B_5/\rho j)}{c_2^2}\right]} \tag{38}$$

Similarly for the anti-symmetric modes substituting (33) in (34) we obtain

$$cos(m_2h)=0 \tag{39}$$

and $$\frac{\tan m_1h}{\tan m_3h} = \frac{\left[\theta m_3^2-\frac{(B_4/\rho j)}{c_2^2}k^2\right]\left[(\theta+\delta)m_1^2+\frac{k^2(B_5/\rho j)}{c_2^2}\right]}{K^2 m_1 m_3\left[-(\theta+\delta)+\frac{(B_5/\rho j)}{c_2^2}\right]\left[\theta+\frac{(B_4/\rho j)}{c_2^2}\right]} \tag{40}$$

In equations (37) and (39), if the micro-morphic constants are equated to zero the result of corresponding classical problem is obtained as a particular case. The equations (38) and (40) correspond to the modes of micro polar wave. This wave is also obtained on a particular case of it, combining the equation (37) and (39), we have

$$m_2h = n\frac{\pi}{2}$$

Where n = 0, 2, 4, ...correspond to symmetric modes, n = 1, 3, 5, …… correspond to anti-symmetric modes.

The equations (21) to (23) admit solutions given by

$$\theta_{(22)} = \left[E_{22}\left(\sin\bar{1}\ x_3 + E_{22}^{'}\right)\cos\bar{1}\ x_3\right]e^{i(kx_1-wt)},$$

$$\theta_{(12)} = \left[E_{12}\sin\bar{1}\ x_3 + E_{12}^{'}\cos\bar{1}\ x_3\right]e^{i(kx_1-wt)}, \tag{41}$$

$$\theta_{(13)} = \left[E_{13}\sin\bar{1}\ x_3 + E_{13}'\cos\bar{1}\ x_3\right]e^{i(kx_1-wt)}$$

where $$\bar{1} = i\ \bar{n},\ \bar{n}^{2} = -\left[\frac{1}{2}\rho j\ w^2 - 2A_5 - w^2\right]$$

and $E_{22}, E_{22}', E_{12}, E_{12}', E_{13}, E_{13}'$ are arbitrary constants.

The modes of SH-wave propagation in the micro-morphic elastic layer will split into two systems of symmetric and anti-symmetric modes with respect to the x_1x_3 – plane if

$$\theta_{22} = E_{22}'\ \cos\left(\bar{1}\ x_3\right)e^{i(kx_1-wt)},$$

$$\theta_{(12)} = E_{12}'\ \cos\left(\bar{1}\ x_3\right)e^{i(kx_1-wt)}, \tag{42}$$

$$\theta_{(13)} = E_{13}'\ \cos\left(\bar{1}\ x_3\right)e^{i(kx_1-wt)},$$

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and $\theta_{22} = E_{22} \sin\left(\bar{1}\ x_3\right) e^{i(kx_1 - wt)}$,

$$\theta_{(12)} = E_{12} \sin\left(\bar{1}\ x_3\right) e^{i(kx_1 - wt)},$$

$$\theta_{(13)} = \mathrm{E}_{13} \sin\left(\bar{1}\ \mathrm{x}_3\right) \mathrm{e}^{\mathrm{i}(\mathrm{kx}_1 - \mathrm{wt})}. \tag{43}$$

When the boundary is tress free, the conditions to be satisfied are

$$t_{3(12)} = 0,\ t_{3(11)} = 0,$$

$$at\ x_3 = \pm h$$

$$t_{3(13)} = 0,\ t_{3(22)} = 0, \tag{44}$$

Substituting (42) in (44), we obtain

$$\sin \bar{1}\ h = 0 \tag{45}$$

Similarly substituting (43) in (44)

We obtain $$\cos \bar{1}\ h = 0 \tag{46}$$

The frequencies of (45) and (46) will take the form

$$\bar{1}\ h = n\frac{\pi}{2} \tag{47}$$

The symmetric modes correspond to n = 1, 3, 5, …..
and the anti-symmetric modes correspond to n = 0, 2, 4, …..
It is significant to note that, in the micro-morphic medium, we have one additional wave which is not appeared either in the classical or micro-polar theory.

IV.CONCLUSIONS

In this paper the micromorphic nature of elastic medium is taken into consideration, some interesting new results can be seen, the plane were propagation and surface wave propagation in half space, layered media and waveguides are studied. In all these problems some additional waves are noticed. In all these problems some additional waves are noticed, besides the waves which are the counterparts of classical and micro polar theories. All these additional waves are found to be dispersive and they depend only on the micromorphic elastic constants. It is noticed that the wave speeds are increased when the micro-effects are taken into consideration. It would be profitable if these problems are studied in transversely isotropic and another anisotropic media. It is also possible to study these problems by taking into account the thermal effects.

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