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Algorithms of Stable Evaluation of Initial Conditions for the Observing Device of Perturbations of the Linear System

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ABSTRACT: Algorithms for a stable estimation of the initial conditions for the observing device of perturbations of a linear system are given. An observation scheme is considered in which auxiliary variables are used as perturbations in the form of combinations of state variables. It is shown that, as a rule, the system of equations for determining the initial conditions of an observer is underdeveloped. Various algorithms for the stable computation of a matrix that is pseudo inverse to the matrix of outputs, using various matrix expansions, are analyzed. The above expressions allow us to synthesize simplified computational procedures for a stable estimation of the initial conditions for the observing device of perturbations.

I. INTRODUCTION

One of the modern trends in the theory of control, which has recently received significant development, has been the theory of constructing observers for the state of linear and nonlinear dynamical systems [1-9]. By the observer of a state, as is known, is meant a special dynamical system whose inputs are fed by the outputs of the observed dynamical system, and whose state vector asymptotically approaches the state vector of the observable system with time.

The approach [6,7] based on the expansion of the dynamics of the system based on data on the values of the input and output through the construction of an observer, a special dynamical system, whose state rapidly approaches the state of the initial system quickly enough, and the use of dynamic feedbacks in accordance with output, having the form of functions of the state of the observer and the input-output variables of the original system. In this case, the state of the observer at an arbitrary time is considered as an estimate of the state of the system at a given time.

We note the advantages of using asymptotic observers [4,5,7]. First, the ability to restore variables that are not available for measurement makes it possible to exclude the installation of additional sensors, which can improve the operational and cost characteristics of control systems. Secondly, the installation of physical sensors requires the additional dynamics of the sensors themselves to be taken into account in the control object model, which can cause problems in the synthesis of feedback due to excessive growth of the order of the control object model. Third, the structure of the asymptotic state observer for linear systems coincides with the Kalman filter structure, which allows the output variables to be filtered in the presence of noise in the measurement channels. Considering that, as a rule, the signals of physical sensors contain noises, the filtration issues without the use of asymptotic observers still have to be considered. Thus, the surveillance devices perform a dual function in control systems - on the one hand, they get information about the state vector and parameters of the control object, on the other hand they are filtering elements, which is important in the presence of noise in the measurement and control channels.

Thus, the idea of constructing asymptotic observers consists in introducing a dynamic subsystem into the feedback loop, whose structure coincides with the model of the control object. The observation task consists in "fitting" the observer's state vector to the current state value of the control object using the control object available for measurement of the output variables.

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II. FORMULATION OF THE PROBLEM

Consider the control object described by equations

$$x_{k+1} = Ax_k + Bu_k + Fw_k,$$
$$y_k = Cx_k, \tag{1}$$

where $x_k \in R^n$ – is the vector of state variables, $u \in R^m$ – is the vector of control actions, $y \in R^p$ – is the vector of measured output variables, $w \in R^r$ – is the vector function of perturbations of the external environment; $A \in R^{n\times n}$ – matrix of the dynamics of the control object, $B \in R^{n\times m}$ – matrix of control inputs, $C \in R^{p\times n}$ – matrix of outputs, $F \in R^{n\times r}$ – matrix of disturbance inputs.

To describe the external environment, its next model is usually taken [1,3]:

$$\xi_{k+1} = G\xi_k; \quad w_k = H\xi_k, \tag{2}$$

where $\xi \in R^v$ – is the vector of environmental state variables, $G \in R^{v\times v}$ – is the dynamics matrix, and $H \in R^{r\times v}$ – is the output matrix for the perturbations.

The matrix G in (2) is assigned based on the available information about the properties of the external environment, or is determined by the identification of the extended control object (1), (2).

It is necessary for the given input u_k, $\forall k \ge k_0$, according to observations y_k, to provide an estimation of both the current state of the control object x_k and the perturbations w_k acting on it. As is known, without loss of generality in (1), we can further assume that $u_k = 0$.

Following [3.10], let us consider the observing device for estimating perturbations acting on the control object (1), (2), which in the presence of any available measurements of x_k and w_k would have the following form:

$$\hat{\xi}_{k+1} = G\hat{\xi}_k + L(w_k - \hat{w}_k),$$
$$\hat{w}_k = H\hat{\xi}_k,$$

where $\hat{\xi}_k$ and $\hat{w}_k$ are the corresponding current estimates, and $L \in R^{v\times r}$ is the gain matrix. If $G - LH$ is a Hurwitz matrix, then $\Delta w_k \to 0$ at $k \to \infty$, where $\Delta w_k = H\Delta\xi_k$, $\Delta\xi_k = \xi_k - \hat{\xi}_k$.

Consider an observation scheme in which auxiliary variables are used as perturbations in the form of combinations of state variables x_k. For the control object (1) they have the form: $f = Px$, where the matrix $P \in R^{r\times n}$ is subject to the condition $PF = I_r$, and $I_r \in R^{r\times r}$ is the unit matrix.

The observer's equation will then have the form [10]:

$$z_{k+1} = (G - LH)z_k + [(G - LH)LP - LPA]x_k, \tag{3}$$

where $z_k = \hat{\xi}_k - Lf_k$. The solution of equation (3) for some $z_0 = z(k_0)$ delivers $\hat{\xi}_k = z_k + LPx_k$ and, consequently, the required estimate:

$$\hat{w}_k = H\hat{\xi}_k = H[z_k + LPx_k]. \tag{4}$$

The estimate (3) was obtained under the assumption of direct measurement of x_k [1]. Taking into account (2) for the control object (1), it is possible to construct an observational device in which some estimates are used for perturbations of the external environment, namely:

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$$\hat{x}_{k+1} = A\hat{x}_k + F\hat{w}_k + K(y_k - C\hat{x}_k),$$

where $K \in R^{n\times p}$ is the gain matrix.

Then, on the basis of the above expressions and following the methods of constructing observing devices [4, 10], we can write the following equations:

$$\hat{x}_{k+1} = (A - KC)\hat{x}_k + F\hat{w}_k + Ky_k, \quad (5)$$

$$\hat{z}_{k+1} = (G - LH)\hat{z}_k + [(G - LH)LP - LP(A - KC)]\hat{x}_k - LPKy_k, \quad (6)$$

$$\hat{w}_k = H[\hat{z}_k + LP\hat{x}_k] \text{ for any } k \ge k_0. \quad (7)$$

The initial conditions for (5) and (6) will be given in the form:

$$\hat{x}(t_0) = \hat{x}_0 = C^+ y(k_0) = C^T (CC^T)^{-1} y(k_0), \quad (8)$$

$$\hat{z}(k_0) = \hat{z}_0 = -LP\hat{x}_0$$

III. SOLUTION OF THE TASK

This is due to the fact that very often the system of equations (8) is, as a rule, underdeveloped. The accuracy of the synthesized observer essentially depends on the accuracy of the initial data $\hat{x}(t_0)$ and $\hat{z}(k_0)$. For this reason, the determination of the initial data for equations (5) - (7) is very important.

Let us consider the algorithms of stable computation C^+, using certain expansions of the matrix C [11, 12]. Taking into account that if $rankC = p$ ($C \in R^{p\times n}$ c $p \le n$), then the pseudo inverse of the matrix C is the matrix C^+ determined by the second Gaussian transformation:

$$C^+ = C^T (CC^T)^{-1}. \quad (9)$$

The validity of expression (9) is due to the fact that any matrix $C \in R^{p\times n}$ can be represented as a «skeletal» decomposition [13]:

$$C = B \cdot A$$

with matrices $B \in R^{p\times r}$ and $A \in R^{r\times p}$, where $r = rankC \le \min(p, n)$.

We now put

$$C^+ = A^+ \cdot B^+,$$

where according to (9) and $C^+ = (C^T C)^{-1} C^T$ at $p > n$ one can arrive at the relations

$$A^+ = A^T (AA^T)^{-1},$$

$$B^+ = (B^T B)^{-1} B^T.$$

Then

$$CC^+C = BAA^T (AA^T)^{-1} (B^T B)^{-1} B^T BA = BA = C.$$

If we take $U = A^T (AA^T)^{-1} (B^T B)^{-1} A$, it can be shown that $UC^T = A^+$.

Equation $C^+ = C^T V$ with $V = B(B^T B)^{-1} (AA^T)^{-1} (B^T B)^{-1} B$ is also valid. When the matrix C is accessed, a method based on the calculation of $Q = CC^T$ in expression (9) can also be used. Taking into account that Q is a symmetric non-negative definite matrix of order $p \times p$ of rank $r < p$, then

$$Q^+ = T^T (TT^T)^{-2} T, \quad (10)$$

where the matrix $T_{(p\times r)}$ of rank r is determined from the expansion

$$Q = T^T T. \quad (11)$$

The decomposition (11) is not unique in general [13,14]. However, the pseudo-inverse matrix $Q^+ = T^T(TT^T)^{-2}T$ is uniquely determined independently of the decomposition method $Q = T^T T$. Thus, expression (9) with regard to (10) can be written as:

$$C^+ = C^T Q^+ = C^T T^T (TT^T)^{-2} T .$$

In the case where the matrix Q is poorly conditioned, then in order to increase the stability of the pseudo-inversion procedure, it is expedient to use regular procedures [15-18] in (10):

$$C^+ = C^T T^T (TT^T + \alpha I)^{-2} T ,$$

where $\alpha > 0$ – is the regularization parameter, I – is the identity matrix.

The parameter of regularization α here is expedient to be determined on the basis of the method of model examples [18]. If the matrix $Q = CC^T$ is nondegenerate, then $Q^+ = Q^{-1}$ and expression (9) holds.

Very effective in solving equation (8) are also algorithmic procedures associated with the calculation of $Z = (CC^T)^{-1}$, $rankC = p$, which is often called a non-scaled covariance matrix.

We consider the following decompositions of the matrix C [11]:

$$DC = \left[R^T \; \vdots \; 0 \right]^T , \quad (12)$$

$$\tilde{D}CS = \left[\tilde{R}^T \; \vdots \; 0 \right]^T . \quad (13)$$

In the expressions (12), (13) $R_{p \times p}$ and $\tilde{R}_{p \times p}$ – the upper triangular matrices, D , $\tilde{D}$ – are the orthogonal matrices of the corresponding dimensions, and S – is the permutation matrix.

It can be shown [11, 14] that for (12) and (13) with $rankC = p$, respectively, the following relations hold:

$$Z = (CC^T)^{-1} = (R^{-1})^T R^{-1} , \quad (14)$$

$$Z = (CC^T)^{-1} = P\tilde{R}^{-1} (\tilde{R}^{-1})^T P^T .$$

Then, when the triangular matrix R is reversed, expressions can be used in (14).

$$t_{ii} = r_{ii}^{-1}, \quad i = 1, \ldots, p,$$

$$t_{ij} = -t_{jj} \sum_{l=i}^{j-1} t_{il} r_{lj}, \quad j = i+1, \ldots, p, \quad i = 1, \ldots, p-1.$$

For the case (13), it is also necessary to take into account the operations of left and right multiplication by the permutation matrices S and S^T, respectively.

IV. CONCLUSION

The above expressions allow us to synthesize simplified computational procedures for a stable estimation of the initial conditions for the observing device of perturbations.

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