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A Hydrodynamic Model of Physicochemical Water Flooding in Oil Reservoirs with Fractured Rocks

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ABSTRACT: In the paper a hydrodynamic model, describing physicochemical water flooding in fractured-porous oil reservoirs, is numerically analyzed. An element of fractured porous media is modeled as a single fracture and adjacent porous block. Concentration of solute in the water, adsorbed mass field, as well as relative mass flux of solute through common boundary of the fracture and porous matrix are determined and analyzed. Three types of adsorption isotherms are considered: linear isotherm (Henry law), Langmuir and Freundlich isotherms. Influences of isotherm type on solute transport characteristics are analyzed.

KEYWORDS: adsorption; fractured-porous medium; isotherm; mass transfer; solute; water flooding;

I. INTRODUCTION

Many of oil and gas reservoirs in the world have the fractured structure. The successful design and development of these reservoirs can be performed only on the basis of good understanding and characterization and modeling of fractures, fracture networks, and fractured porous media. It should be noted that when one speaks of a *fracture network*, fluid is only contained in the fractures, and the medium located in between the fractures is impervious. However, when one speaks of a *fractured porous medium*, the solid matrix located in between the fractures is porous and permeable; fluid flows inside the fractures and in the porous matrix and is transferred from one to the other according to different laws [1]. In [1] the macroscopic properties of fractures, fracture networks and fractured porous media from easily measurable quantities are estimated.

Fluid flow and contaminant transport in fractured porous media (FPM) are quite different from that in simple porous media [2]. The most commonly used model of FPM represents the media as a network of fractures surrounded by porous blocks [3,4,5]. Both fractures and porous blocks are assumed as two coexisting continua. In other words, this double-porosity model, is still useful instrument to study fluid flow in FPM, although no longer it is viewed as the only practicable model, there are different models and approaches simulating flow and transport in large-scale fractured porous media [2].

Fractured aquifers are an important source of water for public, industrial and agricultural use [6]. Many of aquifers have also fractured structure. Modeling physical, mechanical and chemical processes in such fractured aquifers has become increasingly important in water resources development and groundwater contamination. In [6] concepts and methods for characterizing flow and transport in FPM from an experimental and modeling perspective on the basis of aquifer-analogue concept are presented.

Fractures of FPM play a preferential flow and transport paths then porous blocks. These features create distinctiveness in the transport characteristics of solute migration and can be represented by models based on a dual porous medium approach [7]. Diffusion of solutes in non-homogeneous porous media consisting of high and low-permeability zones has been studied extensively. For instance, diffusion of solutes from fractures into micro-fissures and porous matrix is

known to significantly retard radionuclide transport within bedrock surrounding a repository [8,9]. So, diffusion of the solute from liquid flowing along the fractures into the porous blocks is the most important retardation mechanism. This two zone model approach or “mobile – immobile” approach was developed and studied by many authors [10, 11, 12, 13, 14, 15, 16]. In FPM two pore domains can be considered: 1) fracture domain or inter-aggregate pore space with faster flow and domain and 2) rock porous matrix or intra-aggregate pore space domain with slower or no flow [17]. The difference in flow and solute transport in the two domains may be accounted for by using a physical non equilibrium concept [18]. In general, it is better to consider such non-homogeneous media as dual-velocity or dual-permeability media. In such a media faster fluid flow and solute transport in inter-aggregate pore space or fractures and slower flow and solute transport in the intra-aggregate pore space can occur. The transport mechanism for solutes will be advection and dispersion in the two pore domains and diffusive transfer between the domains [17]. In “mobile-immobile” models in the mobile zone the transport governed by convection and diffusion mechanisms whereas in the immobile zone the transport mechanism is only diffusive. So, the dual-porosity approach and mobile-immobile approach are similar.

Models for solute transport in FPM can be conditionally divided into two groups: 1) without determination of the structure of the medium 2) with concretizing the geometry of the medium. In [14] an analytical solution for describing nonreactive solute transport in fractured media where the solute diffuses from the fracture into the adjacent porous matrix is presented. The governing conditions include one-dimensional advective transport in the fracture and one-dimensional diffusive transport (normal to the fracture) into the porous matrix. The medium consists of single fracture and adjacent porous matrix and therefore the model is of the second group. The use of the solution and the effect of matrix diffusion is illustrated by simulation of the effluent breakthrough curve data from a laboratory study of chloride transport through a large, relatively undisturbed column of fractured till. Long-range transport through a single discrete fracture is illustrated with a hypothetical example in the context of deep geologic disposal of radioactive or toxic wastes [14]. We use the schematization of the FPM [14] to study solute transport with different kinds of adsorption.

Currently, mathematical modelling is frequently applied to the mechanics of continuous media and in particular to subsurface fluid dynamics. The latter deals with such important theoretical and practical issues as water flow through dams, soil salinization, the spread of pollution by groundwater flows, oil production, groundwater flow into artesian wells and many others. The similarity of the physical processes involved in these phenomena means that their models also have many similarities, although the model equations all have their own special characteristics. It is in fact these special characteristics that make it very difficult to validate the models and solve the equations [19].

In this paper a mathematical model of solute transport in a medium consisting of a fracture and an adjacent infinity porous block is developed. In the fracture, that treated as a one dimensional media, solute transport governed by convection, diffusion, adsorption and mass-transfer to the porous block effects. End in the porous medium no convection occurs, solute transport mechanisms are only diffusion and adsorption. Different types of adsorption isotherms are considered: linear (Henry law), Langmuir and Freundlich isotherms. A solute transport problem is posed and numerically solved. Solute concentration distribution and adsorbed mass distribution, as well as local, cumulative and total solute mass transfer from the fracture to the porous block are determined and illustrated.

II. FORMULATION OF THE PROBLEM

Consider the solute transport in an element of fractured-porous media, consisting of a single fracture and an adjacent porous block (Figure 1).

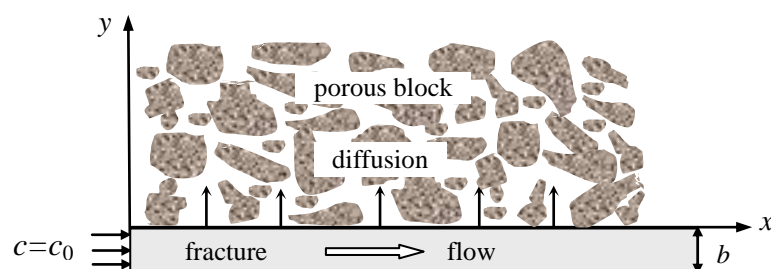


Fig.1. Schematic representation of the solute transport and the flow of liquid in fractured-porous media.

Let the convection-diffusion solute transport takes place along the fracture, and only the diffusion transport—in the porous block. Fluid flow in the fracture is created with a velocity V averaged over the cross section of the fracture. Then the solute transport in the fracture and the porous block, taking into account the adsorption effects, is described by a system of equations like to [20,21]

$$b \left(\frac{\partial c_f}{\partial t} + \rho \frac{\partial s_f}{\partial t} + V \frac{\partial c_f}{\partial x} \right) = b D_f^* \frac{\partial^2 c_f}{\partial x^2} + \theta_m D_m^* \frac{\partial c_m}{\partial y} \Big|_{y=0}, \quad 0 < x < \infty, \quad (1)$$

$$\frac{\partial c_m}{\partial t} + \frac{\rho}{\theta_m} \frac{\partial s_m}{\partial t} = D_m^* \frac{\partial^2 c_m}{\partial y^2}, \quad 0 < y < \infty, \quad (2)$$

where $c_m = c_m(t, x, y)$ – solute concentration in porous block (matrix), non-dimensional, $c_f = c_f(t, x)$ – averaged solute concentration over the cross section of the fracture, non-dimensional, θ_m – porosity of matrix, D_m^* – effective diffusion coefficient in the matrix, m^2/s , D_f^* – convective diffusion coefficient in the fracture m^2/s , V – averaged velocity of fluid motion in the fracture, m/s , ρ – bulk density of the total saturated medium – fractured-porous media, kg/m^3 , s_m, s_f – the amount of adsorbed solute in the matrix and in the fracture, respectively, m^3/kg , b – width of fracture, m , t – time, s .

Let in the left end of the fracture together with water the solute with concentration c_0 is injected. At the initial moment, both the fracture and the porous block were filled with pure water. After injecting from the point $(0,0)$, the solute will spread along the fracture and through the fracture into the porous block. At the common boundary of the fracture and the porous block, the concentrations of the solute must be equal both from the side of the fracture and from the porous block, i.e., the continuity of the concentration is preserved. At an infinite distance along the fracture, i.e. $x = \infty$, the flow of solute is absent, which corresponds to the case when the concentration field is not spread to infinity. In fact, this assumption corresponds to those time intervals of the study, when the concentration front occupies the final region along the axis x , i.e. along the fracture. A similar condition is also adopted in the porous block at infinity along the axis y , i.e. when $y \rightarrow \infty$ there is no flow of solute.

Under the constraints formulated above, the initial and boundary conditions have the form

$$c_f(0, x) = c_m(0, x, y) = 0, \quad (3)$$

$$c_f(t, 0) = c_m(t, 0, 0) = c_0, \quad c_f(t, x) = c_m(t, x, 0), \quad (4)$$

$$\frac{\partial c_m(t, x, \infty)}{\partial y} = 0, \quad \frac{\partial c_f(t, \infty)}{\partial x} = 0. \quad (5)$$

With the use of such schematization several solute transport problem were considered in [11,21], but without taking into account adsorption and diffusion effects.

The process of solute adsorption from an aqueous solution to the surface of a medium is assumed to be equilibrium. We use the following adsorption isotherms [22,23]

$$\begin{aligned} s &= kc, & k &= \text{const}, & \text{- Henry isotherm;} \\ s &= kc^N, & k, N &= \text{const}, & \text{- Freundlich isotherm;} \\ s &= \frac{a_1 c}{1 + a_2 c}, & a_1, a_2 &= \text{const}, & \text{- Langmuir isotherm.} \end{aligned}$$

In addition to the fields of concentration of solute c and concentration of adsorbed solute s , the relative solute mass transfer was determined through the common boundary of the fracture and the porous block

$$Q = -\theta_m D_m^* \left. \frac{\partial c_m}{\partial y} \right|_{y=0}$$

and relative summary Q_{sum} and total solute transfer Q_{tot}

$$Q_{sum} = \int_0^{\infty} Q dx, \quad Q_{tot} = \int_0^t \int_0^{\infty} Q dx dt.$$

III. NUMERICAL SOLUTION OF THE PROBLEM

To solve the problem (1) - (5), we apply the finite difference method [24].

In the domain $D_1 = \{x, x \in [0, \infty)\}$ and $D_2 = \{(x, y), 0 \leq x < \infty, 0 \leq y < \infty\}$ we introduce the following net,

$$\omega_{h_1} = \{x_i, x_i = ih_1, i = 0, 1, 2, \dots, k = \overline{0, K}, j = 0, 1, \dots, \tau = T/K\},$$

$$\omega_{h_1 h_2} = \{(x_i, y_j) \mid x_i = ih_1, i = 0, 1, \dots, y_j = jh_2, j = 0, 1, \dots\},$$

where h_1, h_2 steps of net by directions x and y , respectively. With respect to time, we introduce a net

$$\omega_{\tau} = \{t_k, t_k = \tau k, k = 0, 1, \dots\}, \quad \tau - \text{step of net by time.}$$

Thus, equation (1) is approximated on the net $\omega_{\tau} \cup \omega_{h_1}$, and (2) – on the net $\omega_{\tau} \cup \omega_{h_1 h_2}$. To approximate equation (1), we use an implicit "anti-flow" scheme, and for (2) – a purely implicit scheme or an advanced scheme. For the Henry isotherm, then the approximations of the equations have the form

$$R_f \frac{cf_i^{k+1} - cf_i^k}{\tau} + V \frac{cf_i^{k+1} - cf_{i-1}^{k+1}}{h_1} = D_f^* \frac{cf_{i+1}^{k+1} - 2cf_i^{k+1} + cf_{i-1}^{k+1}}{h_1^2} + \frac{\theta_m}{b} D_m^* \frac{cm_{i,1}^k - cm_{i,0}^k}{h_2}, \quad (6)$$

$$R_m \frac{cm_{i,j}^{k+1} - cm_{i,j}^k}{\tau} = D_m^* \frac{cm_{i,j-1}^{k+1} - 2cm_{i,j}^{k+1} + cm_{i,j+1}^{k+1}}{h_1^2}, \quad (7)$$

$$i = 1, 2, \dots, j = i = 1, 2, \dots, k = 0, 1, \dots,$$

where $R_f = 1 + \rho k_f$, $R_m = 1 + \frac{\rho}{\theta_m} k_m$, cf_i^k , $cm_{i,j}^k$ – net values of concentrations cf and cm in points of net

(t_k, x_i) and (t_k, x_i, y_j) , respectively.

Equations (6) and (7) are reduced to three-point net equations

$$A_1 cm_{i,j-1}^{k+1} - B_1 cm_{i,j}^{k+1} + E_1 cm_{i,j+1}^{k+1} = -F_{i,j}^{(1)}, \quad i = \overline{0, I}, j = \overline{1, J-1}, k = \overline{0, K-1}, \quad (8)$$

$$A_2 cf_{i-1,j}^{k+1} - B_2 cf_{i,j}^{k+1} + E_2 cf_{i+1,j}^{k+1} = -F_{i,j}^{(2)}, \quad i = \overline{1, I-1}, j = \overline{0, J}, k = \overline{0, K-1},$$

(9)

where I, J – enough large integers, taken in such a way that the concentration and adsorption fronts do not reach the points $x_I, (x_I, y_J)$.

$$A_1 = \frac{D_m^*}{h_2^2}, \quad B_1 = 2 \frac{D_m^*}{h_2^2} + \frac{R_m}{\tau}, \quad \dot{A}_1 = \frac{D_m^*}{h_2^2}, \quad F_{i,j}^{(1)} = \frac{R_m}{\tau} cm_{i,j}^k,$$

$$A_2 = \frac{D_f^*}{h_1^2} + \frac{V}{h_1}, \quad B_2 = \frac{R_f}{\tau} + \frac{V}{h_1} + 2\frac{D_f^*}{h_1^2}, \quad E_2 = \frac{D_f^*}{h_1^2}, \quad F_{ij}^{(2)} = \frac{R_f}{\tau} cf_i^k + \frac{\theta_m D_m^*}{bh_2} (cm_{i,1}^k - cm_{i,0}^k).$$

The initial and boundary conditions have the form

$$\begin{aligned} cf_i^0 &= cm_{i,j}^0 = 0, \\ cf_0^{k+1} &= cm_{0,0}^{k+1} = c_0, \\ cf_i^{k+1} &= cm_{i,0}^{k+1}, \\ (10) \\ cf_I^{k+1} &= cf_{I-1}^{k+1}, \\ cm_{i,J}^{k+1} &= cm_{i,J-1}^{k+1}. \end{aligned}$$

For solving equations (8), (9) we use Thomas algorithm (tridiagonal matrix algorithm). For this solution were present in the form

$$cm_{i,j}^{k+1} = \alpha_{j+1}^{(1)} + \beta_{j+1}^{(1)} cm_{i,j+1}^{k+1}, \quad i = \overline{0, I-1}, \quad j = \overline{0, J-1}, \quad k = \overline{0, K-1}, \quad (11)$$

$$cf_i^{k+1} = \alpha_{i+1}^{(2)} + \beta_{i+1}^{(2)} cf_{i+1}^{k+1}, \quad i = \overline{1, I-1}, \quad k = \overline{0, K-1}, \quad (12)$$

where $\alpha_{j+1}^{(1)}, \beta_{j+1}^{(1)}, \alpha_{i+1}^{(2)}, \beta_{i+1}^{(2)}$ – coefficients of Thomas algorithm.

To determine the starting values of the coefficients, we use the boundary conditions (10). From (12) we have $cf_0^{k+1} = \alpha_1^{(2)} + \beta_1^{(2)} cf_1^{k+1} = c_0$ from which $\alpha_1^{(2)} = c_0, \beta_1^{(2)} = 0$. From (11) for $j = 0$ we get $cm_{i,0}^{k+1} = \alpha_1^{(1)} + \beta_1^{(1)} cm_{i,1}^{k+1} = cf_i^{k+1}$, from which $\alpha_1^{(1)} = cf_i^{k+1}, \beta_1^{(1)} = 0$.

Calculations are carried out in the following sequence. From (8) we determine cf_i^{k+1} by using Thomas algorithm, then from (9) also by the Thomas algorithm we determine $cm_{i,j}^{k+1}$.

After determining the concentration fields from the isotherms, it is possible to determine the adsorption fields

$$sf_i^{k+1} = k_f cf_i^{k+1}, \quad sm_{i,j}^{k+1} = k_m cm_{i,j}^{k+1}.$$

For other adsorption laws, the adsorption field is determined from the corresponding dependences. In this case, the expressions R_f and R_m will also be different:

$$R_f = 1 + \rho k_f N c_f^{N-1}, \quad R_m = 1 + \frac{\rho}{\theta_m} k_m N c_m^{N-1} \text{ - for Freundlich isotherm and}$$

$$R_f = 1 + \frac{a_1}{(1 + a_2 c_f)^2}, \quad R_m = 1 + \frac{\rho a_1}{\theta_m (1 + a_2 c_m)^2} \text{ - for Langmuir isotherm.}$$

Knowing the concentration fields it is possible to determine the mass transfer through the common boundary of fracture and the porous block

$$Q = -\theta_m D_m^* \frac{\partial c_m}{\partial y} \Big|_{y=0},$$

and its approximation

$$Q_i^{k+1} = -\theta_m D_m^* \frac{cm_{i,1}^{k+1} - cm_{i,0}^{k+1}}{h_2}.$$

Relative summary mass transfer through the common boundary is defined as

$$Q_{sum} = \int_0^\infty Q dx, \tag{13}$$

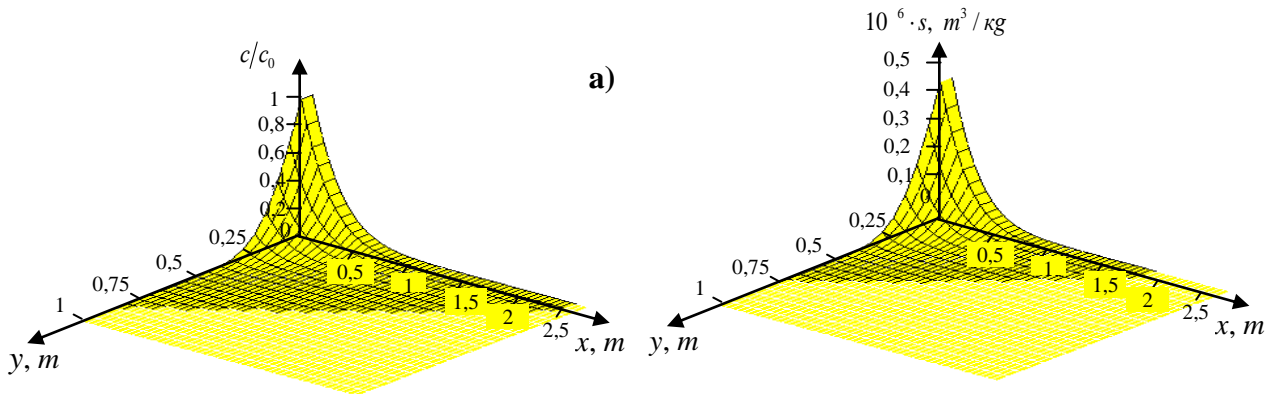
and total one as:

$$Q_{tot} = \int_0^\infty \int_0^\infty Q dxdt \tag{14}$$

The integrals (13), (14) were calculated numerically using the net values of the relative mass transfer Q_i^{k+1} .

IV. RESULTS

On the basis of the above numerical solution, calculations were carried out with the following initial data [11,21,22]:
 $c_0 = 0,1$, $\theta_m = 0,3$, $D_m^* = 10^{-6} m^2/s$, $D_f^* = 7 \cdot 10^{-5} m^2/s$, $V = 10^{-3} m/s$, $b = 4 \cdot 10^{-4} m$,
 $\rho = 2,5 \cdot 10^3 kg/m^3$.



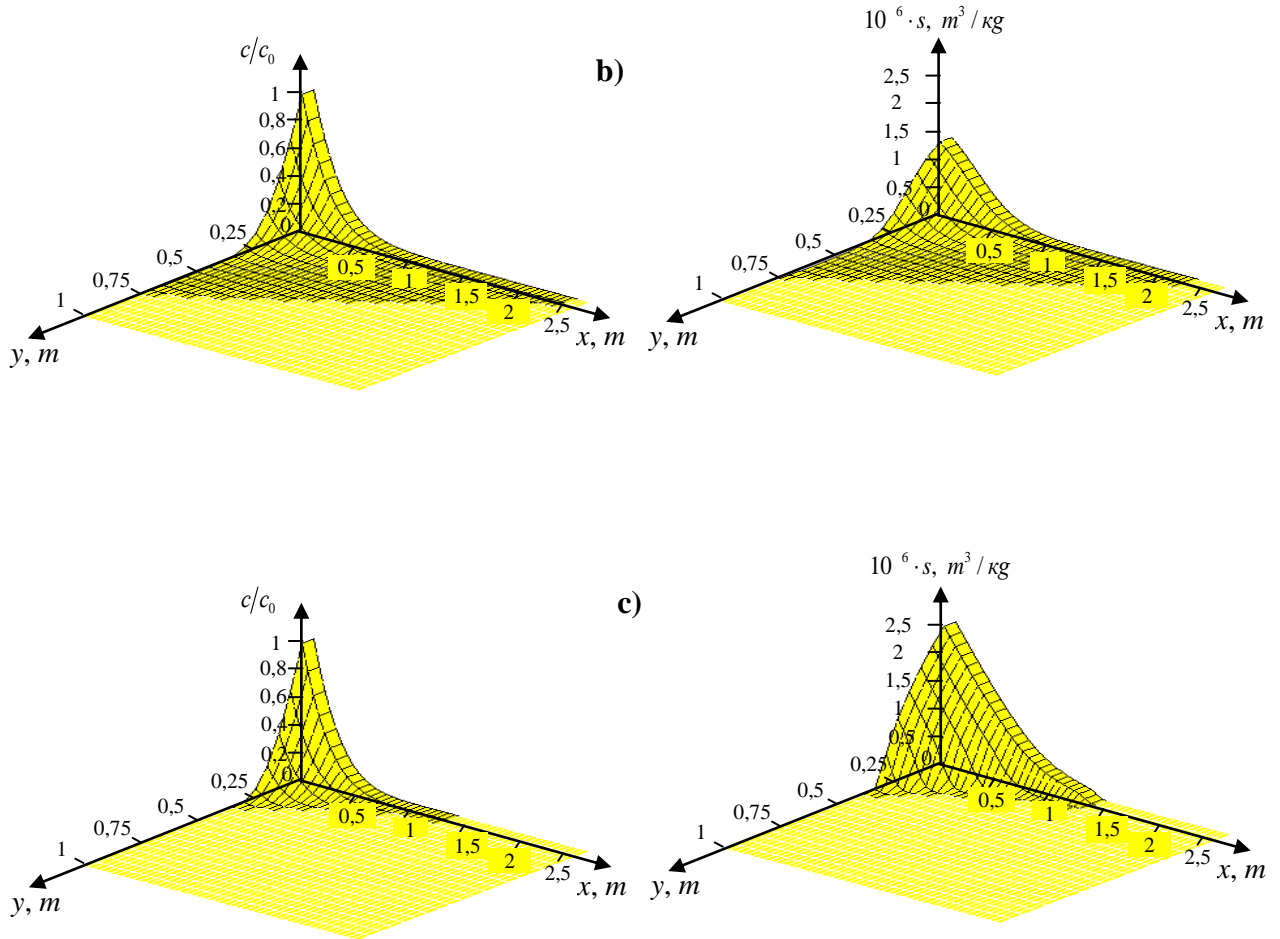
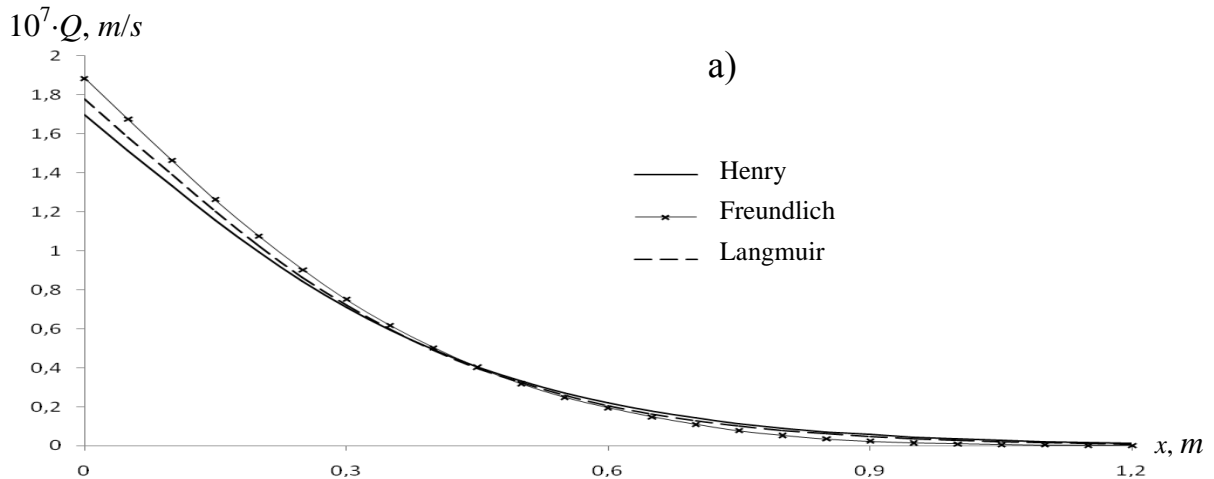


Fig.2. Concentrations and adsorption surfaces for isotherms Henry (a), Langmuir (b) and Freundlich (c) at $D_m^* = 10^{-6} m^2/s$, $D_f^* = 5 \cdot 10^{-3} m^2/s$, $V = 10^{-3} m/s$, $k = 5 \cdot 10^{-3} m^3/kg$, $N = 0,3$, $a_1 = 4,1 \cdot 10^{-2} m^3/kg$, $a_2 = 20$, $t = 10000s$.



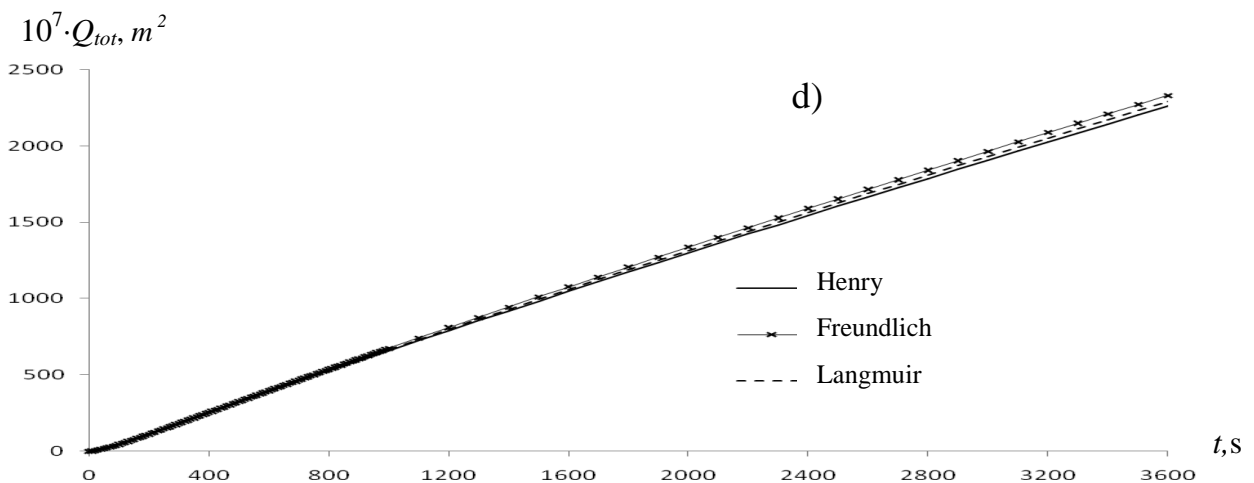
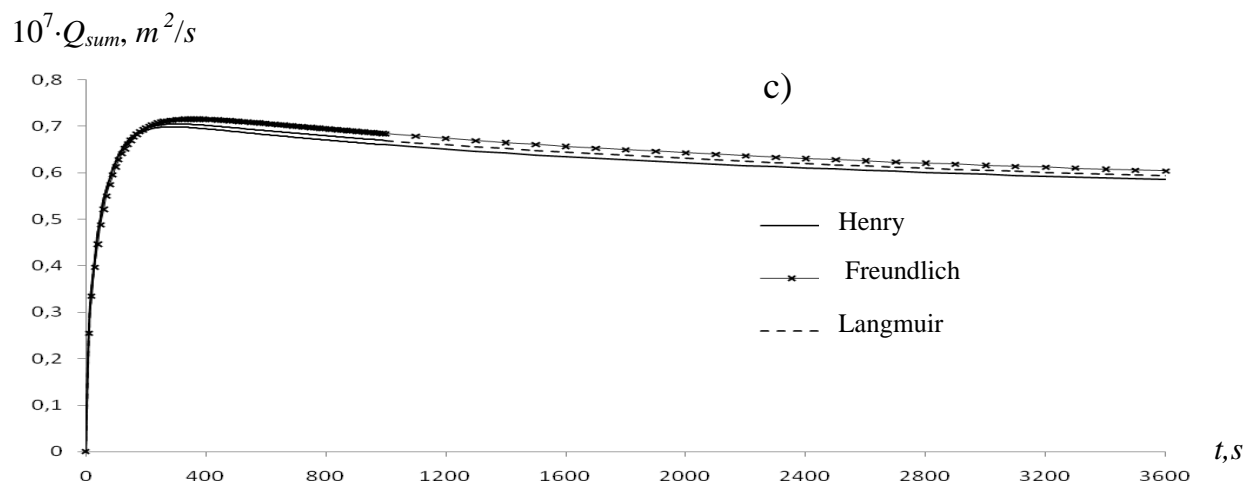
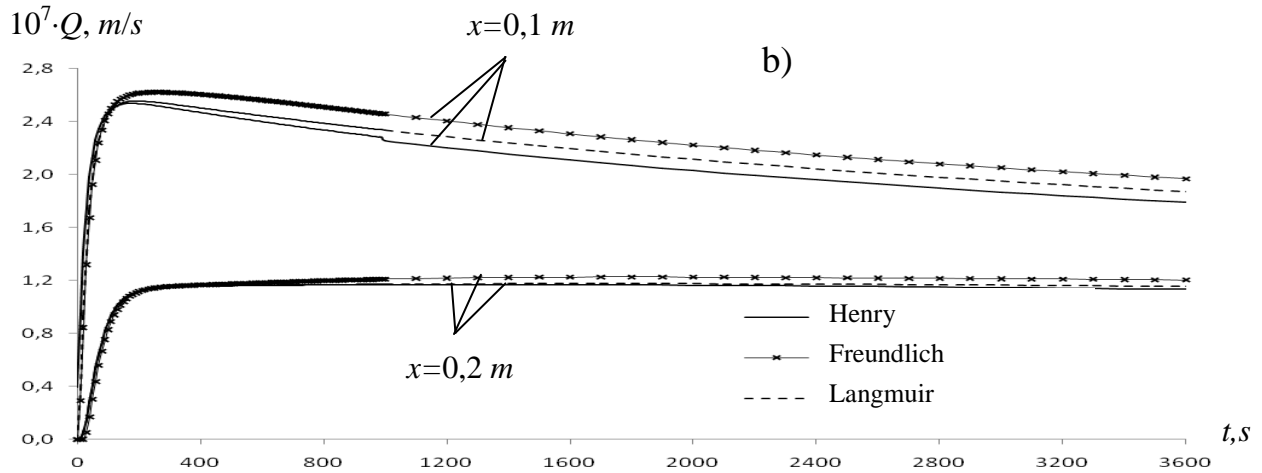


Fig.3. Dependences of the current (a), at a fixed point (b), cumulative (c), total (d) mass transfer for different adsorption isotherms at $D_m^* = 10^{-6} \text{ m}^2/\text{s}$, $D_f^* = 7 \cdot 10^{-3} \text{ m}^2/\text{s}$, $V = 10^{-3} \text{ m}/\text{s}$, $k = 5 \cdot 10^{-3} \text{ m}^3/\text{kg}$, $N = 0,3$, $a_1 = 4,1 \cdot 10^{-2} \text{ m}^3/\text{kg}$, $a_2 = 20$.

For some model parameters, different multivariant computations of the various solute transport parameters were carried out.

Some results of calculations are shown in Fig. 2 and 3. As can be seen from the presented data, the concentration fields with increasing adsorption intensity have a limited distribution; the range of concentration changes is narrowed (Figure 2). For the some other initial Both the values and the adsorption propagation region for the Langmuir isotherm are greater than for the Henry isotherm. In the case of Freundlich isotherm, this circumstance intensifies even more. Thus, if the maximum value of adsorption for the Henry isotherm at the point $(0,0)$ was equal $0,5 \cdot 10^{-6} \text{ m}^3/\text{kg}$, then for the Langmuir and Freundlich isotherms these values are, respectively, $1,4 \cdot 10^{-6}$ and $2,5 \cdot 10^{-6} \text{ m}^3/\text{kg}$.

The adsorption fields also progress in the x and y directions at the transition from the Henry isotherm to the Langmuir and Freundlich isotherms. An analysis is made for the case in which the diffusion coefficient in the fracture takes larger values. In this case, one can notice a progression of the concentration and adsorption fronts in comparison with the previous cases.

The increase in the diffusion characteristics in the fracture thus affects not only the concentration field of the substance, but also the adsorption field. It should be noted that the quantitative characteristics of adsorption in this case practically do not change.

In Fig. 3a are presented profiles of the relative current solute mass transfer through the common fracture boundary and the porous block for various adsorption isotherms. As can be seen from the figure, up to certain x distances, this is less for a linear isotherm then for the Langmuir and Freundlich isotherms.

At fixed points, the growth of the relative mass transport is shown in Fig. 3b. Here, too, a previous pattern is observed for different adsorption isotherms. It can also be noted that, at points farther from the point of feed, the flow varies with the lag, without changing the regularity noted above. In Fig.3c and Fig.3d. the summary and total relative mass transfer across the common boundary of the fracture and porous block are presented. The summary mass transfer is characterized by the presence of a maximum, which is explained by a change in the concentration gradient at this boundary. This phenomenon is not observed for the total mass transfer. Apparently, the total relative mass transfer represents a time-integral characteristic, which smooths the non-monotonicity of the dependencies.

V. CONCLUSION

In the paper a mathematical model, describing solute transport in a fractured porous medium is presented. As an element of the fractured porous medium a medium consisting of single fracture and adjacent porous block is considered. Solute transport in the fracture is governed by convection, diffusion and solute mass transfer into the porous block, while in the porous block – only by diffusion. In both media equilibrium adsorption with different isotherms of solute can occur. The mathematical model is numerically solved. It was shown, that for all adsorption isotherms increasing of adsorption effects leads to the increasing of adsorption concentration characteristics. In turn, the propagation of the solute concentration field in the direction of motion decreases.

As can be seen from the data obtained, the adsorption of the solute leads to a significant lag in the propagation of the solute concentration field.

From the practical point of view adsorption of the chemicals at water flooding in oil reservoirs plays negative role decreasing the effectiveness of oil displacement process.

The elementary schematization of the fractured porous media in the form of a single fracture and an adjacent porous block, used in this work, can provide qualitatively useful information that allows deeper understanding of the mechanisms of physicochemical flooding of oil reservoirs with fractured-porous rocks.



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