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Symmetrical Vibrations of a Three-Layer, Longitudinally Covered Plate

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ABSTRACT:. A theory of nonstationary vibrations of a three-layer viscoelastic plate is developed on the basis of a flat formulation of the problem on the basis of exact solutions of the equations of the linear theory of viscoelasticity in transformations. Equations of vibration of symmetric vibrations of an infinite three-layer plate in terms of two auxiliary functions that are the main parts of the displacements of some intermediate surface of the middle layer. An algorithm is proposed that allows one to uniquely determine the VAT of an arbitrary layer of a plate.

KEYWORDS: plate, geometrical nonlinearity, deformation, rib displacement, pulse loading, deflection, force.

I. INTRODUCTION

The theory of elastic and viscoelastic plastics is one of the sections of the three-dimensional theory of elasticity. In this section, we consider that problems of their calculation, under which the boundary conditions on the side surfaces of the plate are given in the stresses. In this case, the construction of the basic relations of the theory of plates consists in reducing the three-dimensional problem to a two-dimensional one. Different methods and approaches are used to achieve the goal. Absolutely, various simplifying hypotheses and assumptions. These hypotheses and assumptions lead to significant shortcomings and errors with the simplifications, [1].

In the last decades, multi-layered, in particular three-layered saucers are widely used in various fields of engineering and construction. In many cases, dynamic calculations of plates are carried out according to the classical theory of Kirchhoff. Therefore, very often, such calculations prove to be suitable only for low-frequency oscillatory processes. To such belong a considerable amount of research. An analysis of some of them is given in [2,3,4].

These shortcomings include both classical and refined shield vibration theories. Therefore, many researchers have attempted to refine the differential equations of vibration [5,6]. At the same time, we try to derive the equations of vibrations that take into account various physical, mechanical or geometric factors. Further development and refinement of the classical theory can be divided into two areas: development of asymptotic theories and theories of the type of Timoshenko and Reissner [7]. In addition, depending on the factors considered, the methods for deriving the differential equations of vibration based on the dynamic theory of elasticity are also divided into other directions.

One of them is the method of using general solutions of three-dimensional problems of the dynamic theory of elasticity, which is very popular [4]. A significant and successful application to the problems of dynamics, this method was obtained in the works of I.G.Filippov and his students [8,9]. It is based on the use of integral transformations in coordinate and time. It effectively uses the general solutions of the three-dimensional problems of the theory of elasticity (viscoelasticity) in transformations. Subsequently, these solutions are decomposed into power series to approximate the satisfaction of the dynamic conditions given on the boundary surfaces of the plate [10, 11].

The essence of the method is reduced to the study of the constructed solutions for various types of external influences. The clarification of the conditions under which the displacements or their “principal parts” satisfy simple partial differential equations forms the basis of the method [12]. It includes the creation of an algorithm that allows one to calculate the approximate values of dislodgment and stress fields from the field of these "main parts".

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II. EQUATIONS OF MOTION

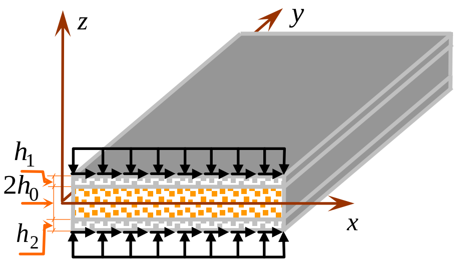

Fig. 1.

We consider an infinite isotropic rectangular three-layer shield, referred to a rectangular Cartesian coordinate system to a rectangular Cartesian coordinate system $Oxyz$, $-\infty < x, y < +\infty$. It is assumed that the space between the rigid marginal layers is filled with a lighter, and therefore less rigid, material (filler) that holds the layers at this distance and carries out their joint work. It is also assumed that the contacts between the load-bearing layers and the filler are rigid.

Considering the unlimited size of the shield, it is further assumed that it is in a plane deformation condition (Fig. 1). The axis Ox is directed along the cross section Oxz on its middle line, and the axis Oz – up. The layers of the plate are numbered as shown in Fig. the upper carrier layer is called the first layer, the lower carrier layer is the second one, and the filler is the zero layer. Let, h_1, $2h_0$ and h_2 be the thicknesses of the first, zero and second layers; λ_m, μ_m - the Lamé coefficients and ρ_m - bulk density of layers $(m = 0,1,2)$. Stress dependencies $\sigma_{ij}^{(m)}$ from deformations $\varepsilon_{ij}^{(m)}$ at the points of the layers, the surfaces are described by Hooke's law. In this case, the equations of motion of the points of the layers in the Cartesian coordinate system are greatly simplified by introducing functions $\varphi_m(x,z,t)$ and $\psi_m(x,z,t)$. Here φ_m and ψ_m are the potentials of the longitudinal and transverse waves, respectively. In the case of planar deformation, taking into account that the displacement vectors of the points of the layers $\vec{U}^m = \vec{U}^m(U_m, W_m)$ decompose only by unit vectors $\vec{i}$, $\vec{k}$ the equations of motion are reduced to the wave equations

$$\Delta\varphi_m = \ddot{\varphi}_m / a_m^2; \qquad \Delta\psi_m = \ddot{\psi}_m / b_m^2, \tag{1}$$

where a_m, b_m - the velocity of longitudinal and transverse waves in the layers; Δ - two-dimensional differential Laplace operator. In this case, the components of the dislodgment vectors, as well as stress tensors and deformations of the layers, are expressed in terms of the introduced functions φ_m and ψ_m.

It is assumed that at $t < 0$ the plate was at rest, and at the time $t = 0$, the dynamic surfaces are subjected to dynamic effects. Because of the linearity of the problem, it is possible to represent mixing fields, in the form of an overlap of a symmetric and antisymmetric parts

$$\vec{U}_m = \vec{U}_m^s + \vec{U}_m^a,$$

where $\vec{U}_m^s$ - symmetrical (longitudinal), $\vec{U}_m^a$ - antisymmetric (bending) parts of the displacement fields of the layers of the plate. In this case, the symmetric parts must satisfy the boundary conditions for $z = (-1)^{i-1} h_i^*$, $h_i^* = h_0 + h_i$,

$$\sigma_{xz}^{(i)} = f_x^{(i)}; \ \sigma_{zz}^{(i)} = f_z^{(i)}; \quad (i = 1,2). \tag{2}$$

In addition, dynamic and kinematic contact conditions occur on the aggregate surfaces:

$$\text{при } z = h_0 \qquad \sigma_{xz}^{(0)} = \sigma_{xz}^{(1)}, \quad \sigma_{zz}^{(0)} = \sigma_{zz}^{(1)}, \quad U_0 = U_1, \quad W_0 = W_1;$$

$$\text{при } z = -h_0 \qquad \sigma_{xz}^{(0)} = \sigma_{xz}^{(2)}, \quad \sigma_{zz}^{(0)} = \sigma_{zz}^{(2)}, \quad U_0 = U_2, \quad W_0 = W_2. \tag{3}$$

The initial conditions are zero.

III. SOLUTION METHOD

It is necessary to give expressions for the functions $f_x^{(1,2)}(x,t)$ and $f_z^{(1,2)}(x,t)$ from the boundary conditions in solving the problem,. Following [5], the functions of external influences can be represented in the form

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$$f_x^{(1,2)}(x,t)=\int_0^\infty \begin{Bmatrix} \cos kx \\ \sin kx \end{Bmatrix} dk \int_{(l)} \tilde{f}_x^{(1,2)}(k,p)e^{pt}dp,$$
$$f_z^{(1,2)}(x,t)=\int_0^\infty \begin{Bmatrix} \sin kx \\ -\cos kx \end{Bmatrix} dk \int_{(l)} \tilde{f}_z^{(1,2)}(k,p)e^{pt}dp, \qquad (4)$$

where $f_x^{(1,2)}(k,p)$, and $f_z^{(1,2)}(k,p)$ - are functions regular for $\operatorname{Re} p \geq 0$, that have a finite number of poles and take arbitrary values within some region $\Omega(k,p)$ containing an interval $(-i\omega_0, i\omega_0)$ of the imaginary axis, decreasing for $p \to \mp i\infty$ no slower than $|p|^{-n_0}$, where $n_0 >> 1$, and such that outside $\Omega(k,p)$ their values are negligible are small. In addition, the functions $\tilde{f}_x^{(1,2)}(k,p)$ and $\tilde{f}_z^{(1,2)}(k,p)$ are analytic, taking arbitrary values in intervals $(0,k_0)$, decreasing for $k \to \infty$, as k^{-n_0}, and negligible for $k > k_0$; (l) – a contour $\operatorname{Re} p = \nu > 0$ on the complex plane (p), leaving the region to the right $\Omega(k,p)$ of itself.

According to the accepted representations of the external action functions, the solution of the problem is also sought in the form (4). This allows us to obtain from (1) ordinary second-order differential equations. In case of symmetric influences, when there are longitudinal vibrations of the plate, the solution of the equations obtained will be

$$\tilde{\varphi}_m(z,k,p)=A_m^{(1)} ch\alpha_m z, \quad \tilde{\psi}_m(z,k,p)=B_m^{(1)} sh\beta_m z. \quad (m=0,1,2) \qquad (5)$$

were

$$\alpha_m^2 = k^2 + p^2/a_m^2 \ ; \qquad \beta_m^2 = k^2 + p^2/b_m^2 .$$

The displacements U_m and W_m can also be represented in the form (4) and substituting together with (5) in the displacement expressions, for the transformed $\tilde{U}_m$ and $\tilde{W}_m$ we have expressions in terms of hyperbolic functions and integration constants. Further, using standard expansions of hyperbolic functions in power series, we obtain

$$\tilde{U}_m = \sum_{n=0}^\infty \left[k\alpha_m^{2n}\cdot A_m^{(1)} - \beta_m^{2n+1}B_m^{(1)}\right]\frac{z^{2n}}{(2n)!}; \quad \tilde{W}_m = \sum_{n=0}^\infty \left[\alpha_m^{2n+2}\cdot A_m^{(1)} - k\beta_m^{2n+1}B_m^{(1)}\right]\frac{z^{2n+1}}{(2n+1)!}. \qquad (6)$$

As the unknown functions in the equations of vibration of a three-layered plate, we take the principal parts of the transformed displacements $\tilde{U}_0$ and $\tilde{W}_0$ of the surface of the zero layer, the distance from the surface $z=0$ of which is given by

$$\xi = \chi \cdot h_0 \qquad\qquad -1 \leq \chi < 0; \quad 0 \leq \chi < 1$$

were χ - a constant number satisfying the inequality $-1 \leq \chi \leq 1$. To this end, we take in (6) $z=\xi$, $m=0$ and $n=0$. Then, introducing the notation $\tilde{U}_0^{(0)}$ and $\tilde{W}_0^{(0)}$, we obtain

$$\tilde{U}_0^{(0)} = kA_0^{(1)} - \beta_0 B_0^{(1)}; \qquad \tilde{W}_0^{(0)} = \left[\alpha_0^2 A_0^{(1)} - k\beta_0 B_0^{(1)}\right]\xi . \qquad (7)$$

Having solved the system with respect $A_0^{(1)}$ and $\beta_0 B_0^{(1)}$, we express them through $\tilde{U}_0^{(0)}$ and $\tilde{W}_0^{(0)}$. From the contact conditions (3) there are expressions for the constants $A_m^{(1)}$ and $B_m^{(1)}$ with $m=1,2.$ Then they are substituted into the boundary conditions (2). This makes it possible to obtain the equations of symmetric vibrations of a three-layer plate in the following form

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$$A_1[\frac{\partial}{\partial x}W_0^{(0)}] + B_1[U_0^{(0)}] = S_1\ [f_x^{(1)}],$$

$$A_2[W_0^{(0)}] + B_2[\frac{\partial}{\partial x}U_0^{(0)}] = S_2\ [f_z^{(1)}], \quad (8)$$

were A_k, B_k, S_k - differential operators of the same structure, having the form

$$D_k = D_{k1}\frac{\partial^4}{\partial t^4} + D_{k2}\frac{\partial^4}{\partial x^2 \partial t^2} + D_{k3}\frac{\partial^4}{\partial x^4} + D_{k4}\frac{\partial^2}{\partial t^2} + D_{k5}\frac{\partial^2}{\partial x^2} + D_{k6};$$

D_{kj} are equal A_{kj}, B_{kj} or S_{kj}:

$$\ldots\ldots\ A_{13} = -(1 - q_1 + 2q_0)\frac{z_1 h_0^4}{12} - (4 + 6q_1 + 4q_0 - 9q_0 q_1)\frac{z_1^3 h_0^2}{36};\ \ldots\ldots\ A_{26} = 1 - q_2;$$

$$B_{11} = -\xi\left(\frac{3(1-q_1)(q_0 - 1)}{a_0^2} - \frac{1}{b_1^2}\right)\frac{1}{a_1^2}\frac{z_1 h_0^2}{6} - \xi\frac{q_1 - 1}{a_1^2 b_1^2}\frac{z_1^3}{6};\ \ldots\ldots\ B_{26} = -\xi(1 + q_2);$$

$$S_{i1} = \xi\,\mu_i^{-1}\frac{1}{a_i^2 b_i^2}\frac{h_0^4}{12};\ \ldots\ldots\ldots\ldots\ S_{i6} = \xi\,\mu_i^{-1},$$

where $(i = 1, 2)$; $z_1 = h_0 + h_1$; $z_2 = h_0 + h_2$; $q_m = 1 - \dfrac{\lambda_m}{\mu_m}$; a_m , b_m - respectively, the velocity of longitudinal and transverse waves in the plate material. Thus the displacements of the plate points are determined by the formulas

$$U_0(x,z,t) = \left[(1 - q_0)\frac{z^2}{2}\frac{\partial^2}{\partial t^2} - (1 - q_0)\frac{z^2}{2}\frac{\partial^2}{\partial x^2} + 1\right]U_0^{(0)}(x,z,t) - \frac{1}{\xi}q_0\frac{z^2}{2}\frac{\partial}{\partial x}W_0^{(0)}(x,z,t);$$

$$W_0(x,z,t) = \frac{1}{\xi}\left[\left(\frac{1}{b_0^2} + q_0\right)\frac{z^3}{6}\frac{\partial^2}{\partial t^2} - (1 + q_0)\frac{z^3}{6}\frac{\partial^2}{\partial x^2} + z\right]W_0^{(0)}(x,z,t) + \quad (9)$$

$$+ q_0\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right]\frac{z^3}{6}\frac{\partial}{\partial x}U_0^{(0)}(x,z,t).$$

IV. APPLIED STATEMENT PROBLEM ITS DECISION

Let's consider the problem of symmetric vibrations of a shield clamped in the longitudinal direction, at $x = 0$ and $x = l$, where l - length plate in the direction of the axis Ox. As vibration equations, we take the system (8). The boundary conditions of the problem have the form

$$U_0^{(0)} = 0;\ \frac{\partial^2 U_0^{(0)}}{\partial x^2} = 0;\ \frac{\partial W_0^{(0)}}{\partial x} = 0;\ \frac{\partial^3 W_0^{(0)}}{\partial x^3} = 0.$$

The initial conditions are assumed to be zero.

The solution of the system of equations (8), which includs the conditions for fixing the ends, and also the functions of external actions, is represented in the form.

$$U_0^{(0)} = \sum_{m=1}^{\infty} u(t)\sin\frac{m\pi x}{l};\ W_0^{(0)} = \sum_{m=1}^{\infty} w(t)\cos\frac{m\pi x}{l};\ f_x = \sum_{m=1}^{\infty} f_{xm}(t)\sin\frac{m\pi x}{l};\ f_z = \sum_{m=1}^{\infty} f_{zm}(t)\cos\frac{m\pi x}{l}; \quad (10)$$

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The substitution of (10) into (8) leads to a system of two fourth-order differential equations with respect to the functions $u(t)$ and $w(t)$. The problem was solved numerically at the following values of the physico-mechanical and geometric parameters of the three-layer plate: $\xi = 0.9h_0$; $l = 0.4\,m$; $h_0 = 0.04\,m$; $h_1 = 0.001\,m$; $h_2 = 0.001\,m$; $\rho_0 = 30\,kg/m^3$; $\rho_1 = 2700\,kg/m^3$; $\rho_2 = 2700\,kg/m^3$; $E_0 = 0.165\cdot 10^9\,Pa$; $E_1 = 69\cdot 10^9\,Pa$; $E_2 = 69\cdot 10^9\,Pa$; $\nu_0 = 0.03125$; $\nu_1 = 0.33$; $\nu_2 = 0.33$; $f_{xm}(t) = t^2$; $f_{zm}(t) = 3t^2$. The results are shown in Fig. 2-5 in the form of graphs of the longitudinal and transverse displacements of the points of the middle layer and normal stresses in its various sections.

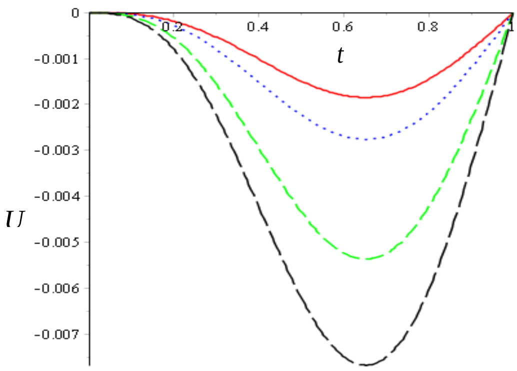

Fig. 2. Dependence of displacement U on time at $x = 0.2$(—); 0.3 (·····); 0.4(– –); 0.6(—·).

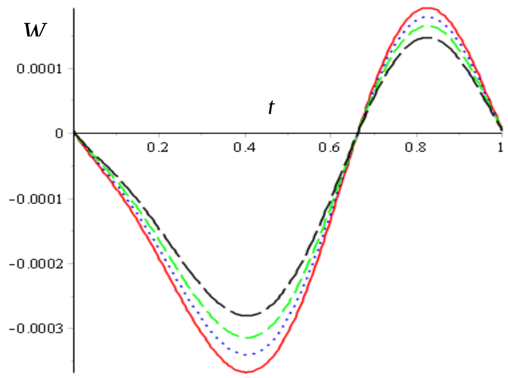

Fig. 3. Dependence of displacement W on time at $x = 0.2$(—); 0.3 (·····); 0.4 (– –); 0.6(—·).

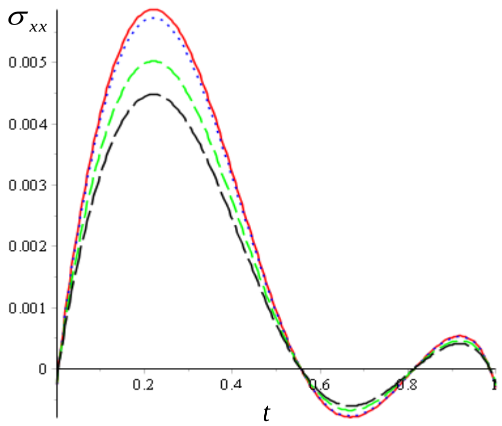

Fig. 4. Dependence of displacement σ_{xx} on time $x = 0.2$(—); 0.3 (·····); 0.4 (– –); 0.6(—·).

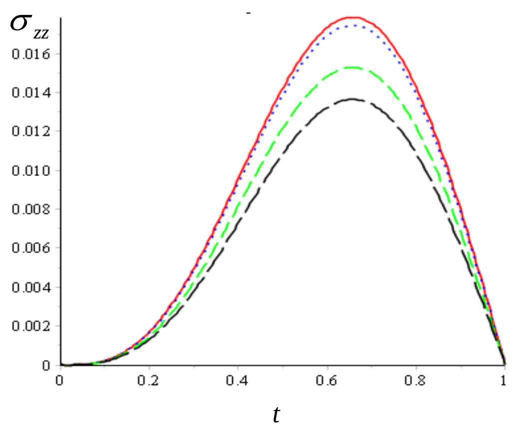

Fig. 5. Dependence of displacement σ_{zz} on time at $x = 0.2$(—); 0.3 (·····); 0.4 (– –); 0.6(—·).

V. CONCLUSION

From the presented graphs in Fig. 2-3 it follows that the longitudinal displacement of the points of different sections reaches their maximum at values between 0.6 and 0.8 of the dimensionless time. Negative values of longitudinal displacement indicate that the shield for weight the period of action of the external load undergoes compression. The transverse displacement of the point of the cross sections has a sinusoidal character as a function of time. At the same time, it reaches its maximum at a value of the dimensionless time close to 0.4. The maximum value of the longitudinal displacement corresponds to the zero value of the lateral displacement. In addition, at the beginning of the process and

further to the time value of 0.63-0.66, the transverse displacement is negative, and at 0.65 <t <0.7. Further, it remains positive with a relative maximum at 0.82.
Following graphs (Figs. 4-5) are in good agreement with the dependencies of displacements, having relative maxima at the points where the displacements are minimal. At the points of maximum displacement values, it should be noted that corresponding stresses are minimal.

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