



Condition for Double Line Graph – Using FUZZY GRAPH

A.R.Shoba, Hemanath S, R.Malathi

M.Phil Scholar, SCSVMV, Kanchipuram.
3rd year, CSE, Sastra University, Tanjore.
Asst Prof of Mathematics, SCSVMV, Kanchipuram.

ABSTRACT: In this paper, we will be learning some new concepts of notions of fuzzy graph, union, intersection of two fuzzy graphs and a few properties relating to finite union and intersection of fuzzy graphs are established here. We are also going to find the conditions for complete graph.

KEY WORDS: Finite intersection, Finite union, fuzzy graph, notions of fuzzy graph.

I. INTRODUCTION

There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets, etc. which can be considered as mathematical tools to deal with uncertainties. But all these theories have their own inherent difficulties. The theory of probabilities can deal only with possibilities. The most appropriate theory to deal with uncertainties is the theory of fuzzy sets, developed by Zadeh in 1965. But it has an inherent difficulty to set the membership function in each particular cases. Also the theory of intuitionistic fuzzy set is more generalized concept than the theory of fuzzy set, but also there have same difficulties. Thereafter many researchers have applied this concept on different branches of mathematics, like group theory decision making problems, relations topology etc[5].

In 1736, Euler first introduced the concept of graph theory. The theory of graph is extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operation research, optimization and computer science, etc.. In 1975, Rosenfeld introduced the concept of fuzzy graphs. Thereafter many researchers have generalized the different notions of graph theory using the notions of fuzzy sets

In this paper, our aim is to introduce the notion of fuzzy graph and then a few operations, like union, intersection.

II. PRELIMINARIES

Definition: 1.1

Let V be a nonempty finite set and $\sigma: V \rightarrow [0; 1]$.
Again, let $\mu: V \times V \rightarrow [0; 1]$ such that $\mu \leq (\sigma(x), \sigma(y)) \quad \forall (x, y) \in V \times V$
Then the pair $G := (\sigma; \mu)$ is called a fuzzy graph over the set V

Here σ and μ are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph $(\sigma; \mu)$.

A fuzzy graph $G := (\sigma; \mu)$ over the set V is called strong fuzzy graph if

$$\mu(x, y) = \sigma(x) \wedge \sigma(y) \quad \forall (x, y) \in V \times V.$$

Definition: 1.2

Let $H := (\rho; \nu)$ and $G := (\sigma; \mu)$ be two fuzzy graphs over the set V . Then H is called the fuzzy subgraph of the fuzzy graph of G if

$$\rho(x) \leq \sigma(x) \text{ and } v(x; y) \leq \mu(x; y) \quad \forall x; y \in V:$$

Definition: 1.3

Let $G_1 := (\sigma_1; \mu_1)$ and $G_2 := (\sigma_2; \mu_2)$ be two fuzzy graphs over the set V .

Then the union of G_1 and G_2 is another fuzzy graph $G_3 := (\sigma_3; \mu_3)$ over the set V ,

where $\sigma_3(x) = \sigma_1 \vee \sigma_2$ and $\mu_3 = \mu_1 \vee \mu_2$;

i.e: $\sigma_3(x) = \max\{\sigma_1(x); \sigma_2(x)\} \quad \forall x \in V$

and $\mu_3(x; y) = \max\{\mu_1(x; y); \mu_2(x; y)\} \quad \forall x; y \in V$

Definition: 1.4

Let $G_1 := (\sigma_1; \mu_1)$ and $G_2 := (\sigma_2; \mu_2)$ be two fuzzy graphs over the set V .

Then the intersection of G_1 and G_2 is another fuzzy graph

$G_3 := (\sigma_3; \mu_3)$ over the set V , where

$$\sigma_3 = \sigma_1 \wedge \sigma_2 \text{ and } \mu_3 = \mu_1 \wedge \mu_2;$$

i.e: $\sigma_3(x) = \min\{\sigma_1(x); \sigma_2(x)\} \quad \forall x \in V$

and $\mu_3(x; y) = \min\{\mu_1(x; y); \mu_2(x; y)\} \quad \forall x; y \in V$

Let U be an initial universal set and E be a set of parameters. Let IU denotes the collection of all fuzzy subsets of U and $A \subseteq E$.

Definition: 1.5

The complement of a fuzzy graph $G : (\sigma, \mu)$ is also a fuzzy graph and denoted as

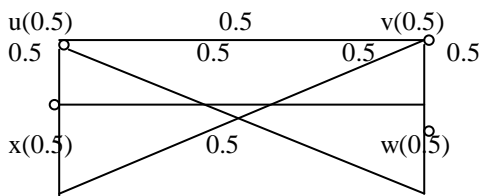
$G^c : (\sigma^c, \mu^c)$

where $\sigma^c = \sigma$ and $\mu^c(x, y) = \sigma(x) \wedge \sigma(y) - \mu(x, y) \quad \forall x, y \in V$ [4]

Definition: 1.6

A fuzzy graph $G : (\mu, \rho)$ is complete graph if $\rho(u, v) = \mu(u) \wedge \mu(v) \quad \forall u, v \in V$

Example of the complete graph:[3]



Definition: 1.7

Let V be a nonempty finite set and $\sigma: V \rightarrow [0; 1]$.

Again, let $\mu: V \times V \rightarrow [0; 1]$ such that

$$\mu(x; y) = \min\{ \sigma(x), \sigma(y) \} \quad \forall (x; y) \in V \times V$$

Then the triplet $G := (V, \sigma, \mu)$ is called a fuzzy graph over the set V .

Here V and μ are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph $(\sigma; \mu)$.



Definition 1.8

Let V be a nonempty finite set and $\sigma: V \rightarrow [0; 1]$. $C: [0,1] \rightarrow [0,1]$

Again, let $\mu: V \times V \rightarrow [0; 1]$ such that
 $\mu^c(x; y) = 1 - [\min\{\sigma(x), \sigma(y)\}] \quad \forall (x; y) \in V \times V$

Definition 1.9

Let V be a nonempty finite set and $\sigma: V \rightarrow [0; 1]$.

Again, let $v: V \times V \rightarrow [0; 1]$ such that
 $v(x; y) = \max\{\sigma(x), \sigma(y)\} \quad \forall (x; y) \in V \times V$

Then the triplet $G := (V, \sigma, v)$ is called a fuzzy graph over the set V . Here V and v are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph $(\sigma; v)$.

Definition 1.10

Let V be a nonempty finite set and $\sigma: V \rightarrow [0; 1]$ & $C: [0,1] \rightarrow [0,1]$

Again, let $v: V \times V \rightarrow [0; 1]$ such that $v^c(x; y) = 1 - [\max\{\sigma(x), \sigma(y)\}] \quad \forall (x; y) \in V \times V$

Example

Consider $V = \{x_1, x_2, x_3\}$ and
 $E = \{e_1, e_2, e_3\}$

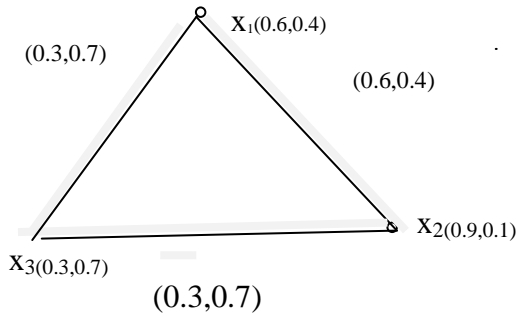
Here $G_{E,V}$ is described by the below table 1

$V(x_i, x_j) \in V \times V \setminus \{(x_1, x_2), (x_1, x_3), (x_2, x_3)\}$ and for all $e \in E$

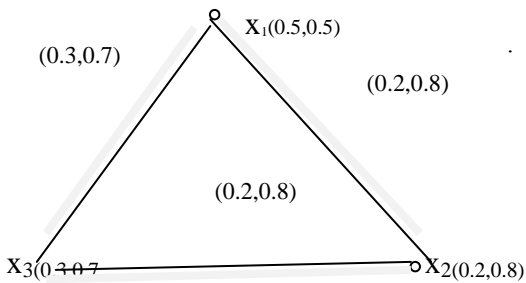
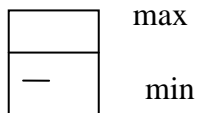
TABLE I

σ (μ, μ^c)	x_1	x_2	x_3
e_1	0.5,0.5	0.2,0.8	0.3,0.7
e_2	0.8,0.2	0.7,0.3	0.4,0.6
e_3	0.6,0.4	0.9,0.1	0.3,0.7

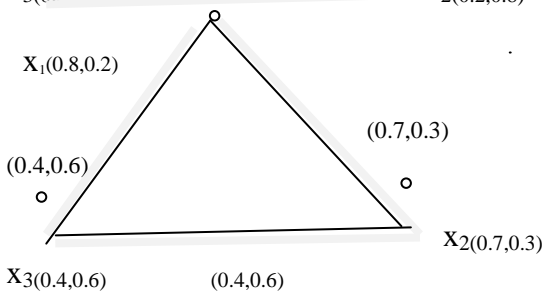
$\min \mu, \max \mu^c$	(x_1, x_2)	(x_1, x_3)	(x_2, x_3)
e_1	0.2,0.8	0.3,0.7	0.2,0.8
e_2	0.7,0.3	0.4,0.6	0.4,0.6
e_3	0.6,0.4	0.3,0.7	0.3,0.7



corresponding to e_3



corresponding to e_1



corresponding to e_2

III. CONCLUSION

In this paper, we have discussed about the notions of fuzzy graph, union and intersection of two fuzzy graphs and also conveyed few properties relating to finite union and intersection of fuzzy graphs. We have also found the conditions for double line complete graph.

IV. RESULT

If $0 < \sigma < 1$ then graph of $\{\min(\mu), \max(\mu^c)\}$ is a double line complete graph.



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