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# Minimization of Hiring Cost of the Machines in 2 Stage Flow Shop Scheduling With Breakdown Interval.

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**ABSTRACT:** The paper deals  $n \times 2$  flow shop scheduling problem, under specified rental policy with break down interval. The objective of the paper is to obtain optimal sequence of jobs in order to minimize the rental cost of machines including. The method is clarified with the help of numerical illustration.

**KEYWORDS:** Flow shop scheduling, Rental policy, Breakdown interval

## I. INTRODUCTION

Scheduling theory beginning from Johnson's work in 1954. Johnson had considered the effect of break-down of machines on the completion times of jobs in an optimal sequence. Later on many researchers such as Adiri[1], Akturk and Gorgulu[2], Schmid[3], Chandramouli[4], Singh T. P. [5], Belwal and Mittal [6] etc. have discussed the various concepts of break-down of machines. The functioning of machines for processing the jobs on them is assumed to be smooth with having no disturbance on the completion times of jobs. But there are feasible sequencing situations in flow shops where machines while processing the jobs get sudden break-down due to failure of a component of machines for a certain interval of time or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as stop of flow of electric current to the machines may be a government policy due to shortage of electricity production. In each case this may be well observed that working of machines is not continuous and is subject to break for a certain interval of time. Various Researchers have done a lot of work in this direction. Johnson [3], Ignall and Schrage[7], Szwarc[8], Chandra Shekhran[9], Maggu & Das [10], Bagga P. C.[11], Singh T. P., Gupta Deepak [12], Sharma Sameer etc.

Bagga and Narain[15] studied  $n \times 2$  general flow shop problem to minimize rental cost under a pre-defined rental policy. We have extended the study made by Singh T.P., Gupta Deepak [13] by introducing the concept of break-down interval. We have developed an algorithm minimizing the utilization time of second machine combined with Johnson's algorithm in order to minimize the rental cost of the machines.

## II. PRACTICAL SITUATION

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology.

Another event which is mostly considered in the models is the break-down of machines. There may also be delays due to material, changes in release and tail dates, tools unavailability, failure of electric current, the shift pattern of the

facility and fluctuations in processing times. All of these events complicate the scheduling problem in most cases. Hence the criterion of break-down interval becomes significant.

### III. NOTATIONS

$S$  : Sequence of jobs 1,2,3,...,n  
 $A_i$  : Processing time of  $i$ th job on machine A.  
 $B_i$  : Processing time of  $i$ th job on machine B. .  
 $L$  : Length of the break-down interval.  
 $S_i$  : Sequence obtained from Johnson's procedure to minimize rental cost.  
 $C_j$  : Rental cost per unit time of machine  $j$ .  
 $U_i$  : Utilization time of B (2 nd machine) for each sequence  $S_i$   
 $t_1(S_i)$  : Completion time of last job of sequence  $S_i$  on machine A.  
 $t_2(S_i)$  : Completion time of last job of sequence  $S_i$  on machine B.  
 $R(S_i)$  : Total rental cost for sequence  $S_i$  for all machines.  
 $CT(S_i)$  : Completion time of 1 st job of each sequence  $S_i$  on machine A.

### IV. ASSUMPTIONS

We assume rental policy that all the machines are taken on rent as and when they are required and are returned as when they are no longer required for processing. Under this policy second machine is taken on rent at time when first job completes its processing on first machine. Therefore idle time of second machine for first job is zero.

### V. ALGORITHM

On considering the effect of break-down interval (a,b) on different jobs, the algorithm which minimize the total rental cost of machines under specified rental policy with

**Step 1:** Using Johnson's (1954) technique and find an optimal schedule of given jobs.

**Step 2:** Prepare a flow time table for the sequence obtained in step 1 and read the effect of break down interval(a ,b) on different jobs on the lines of Singh T.P.

**Step 3:** Form a reduced problem with processing times "

$A$  and  $B$  .

If the break-down interval (a, b) has effect on job  $i$  then

$$A' = A + L \quad \& \quad B' = B + L \quad \text{where } L = b - a, \text{ the length of break-down interval.}$$

If the break-down interval (a, b) has no effect on job  $i$  then

$$A' = A, \quad B' = B$$

**Step 4:** Now repeat the procedure to get the sequence  $S_i$ , using Johnson's two machine algorithm as in step 1.

**Step 5:** Observe the processing time of 1 st job of  $S_1$  on the first machine A. Let it be  $\alpha$ .

**Step 6:** Obtain all the jobs having processing time on A greater than  $\alpha$ . Put these job one by one in the 1 st position of the sequence  $S_1$  in the same order. Let these sequences be  $S_2, S_2, S_3, \dots, S_r$ .

**Step 7:** Prepare in-out flow table only for those sequence  $S_i$  ( $i=1,2,\dots,r$ ) which have job block  $\beta(k, m)$  and evaluate total completion time of last job of each sequence, .i.e.  $t_1(S_i)$  &  $t_2(S_i)$  on machine A & B respectively.

**Step 8:** Evaluate completion time  $CT(S_i)$  of 1 st job of each of above selected sequence  $S_i$  on machine A.

**Step 9:** Calculate utilization time  $U_i$  of 2 nd machine for each of above selected sequence  $S_i$  as:

**International Journal of Advanced Research in Science,  
Engineering and Technology**

**Vol. 5, Issue 9 , September 2018**

$$U_i = t_1(S_i) - CT(S_i) \text{ for } i=1, 2, 3, \dots, r.$$

**Step 10:** Find  $\text{Min } \{U_i\}$ ,  $i=1, 2, \dots, r$ . let it be corresponding to  $i = m$ , then  $S_m$  is the optimal sequence for minimum rental cost.

$$\text{Min rental cost} = t_1(S_m) \times C_1 + U_m \times C_2,$$

where  $C_1$  &  $C_2$  are the rental cost per unit time of 1 st & 2 nd machines respectively.

**VI. NUMERICAL ILLUSTRATION**

Let us consider 5 jobs and 2 machines problem to minimize the rental cost.

Jobs	A	B
1	11	8
2	15	11
3	14	15
4	17	16
5	12	18

Table -1

Rental costs per unit time for machines A & B are 16 and 14 units respectively. Also given that the break-down interval is (20,25).

**Solution**

**Step 1:** Using *Johnson's* two machines algorithm, the optimal sequence is

$S = 5, 3, 4, 2, 1$ .

**Step 2:** The in-out flow table for the sequence  $S = 5- 3- 4- 2- 1$  is as follows:

Jobs	A In - Out	B In - Out
5	0 - 12	<b>12 - 30</b>
3	<b>12 - 26</b>	30 - 45
4	26 - 43	45 - 61
2	43 - 58	61 - 72
1	58 - 69	72 - 80

Table -2

**Step3:** On considering the effect of break down interval (20, 25), the revised processing times  $A'$  and  $B'$  of machines A and B are as follows:

Jobs	A	B
1	11	8
2	15	11
3	19	15
4	17	16
5	12	23

Table -3

**Step 4:** Using *Johnson's* two machines algorithm, the optimal sequence is  $S_1 = 5 - 4 - 3 - 2 - 1$  **Step 5:** The processing time of 1st job on  $S_1 = 12$  .i.e.  $\alpha = 12$

**Step 6:** The other optimal sequences for minimizing rental cost are

$$S_2 = 4 - 5 - 3 - 2 - 1$$

$$S_3 = 3 - 5 - 4 - 2 - 1$$

$$S_4 = 2 - 5 - 4 - 3 - 1$$

$$S_5 = 1 - 5 - 4 - 3 - 2$$

**Step 7:** The in-out flow tables for sequences  $S_1, S_2, S_3, S_4$  and  $S_5$  are as follows:

For  $S_1 = 5 - 4 - 3 - 2 - 1$

Jobs	A	B
1	11	8
2	15	11
3	19	15
4	17	16
5	12	23

Table 4

Total time elapsed on machine A =  $t_1(S_1) = 74$

Total time elapsed on machine B =  $t_2(S_1) = 85$

Utilization time of 2nd machine (B) =  $U_1 = 85 - 12 = 73$ .

$$S_2 = 4 - 5 - 3 - 2 - 1$$

<b>Jobs</b>	<b>A In- Out</b>	<b>B In -Out</b>
4	0 – 17	17 – 33
5	17 – 29	35 – 56
3	29 – 48	56 – 71
2	48 – 63	71 – 82
1	63 – 74	82 – 90

Table -5

Total time elapsed on machine A =  $t_1(S_2) = 74$   
 Total time elapsed on machine A =  $t_2(S_2) = 74$   
 Utilization time of 2nd machine (B) =  $U_2 = 90 - 17 = 73$ .

$S_3 = 3 - 5 - 4 - 2 - 1$

<b>Jobs</b>	<b>A In- Out</b>	<b>B In -Out</b>
3	0 – 19	19 – 34
5	19 – 31	34 – 57
4	31 – 48	57 – 73
2	48 – 63	73 – 84
1	63 – 74	84 – 92

Table -6

Total time elapsed on machine A =  $t_1(S_3) = 74$   
 Total time elapsed on machine B =  $t_2(S_3) = 90$   
 Utilization time of 2nd machine (B) =  $U_3 = 92 - 19 = 73$ .

$S_4 = 2 - 5 - 4 - 3 - 1$

$$S_4 = 2 - 5 - 4 - 3 - 1$$

<b>Jobs</b>	<b>A In- Out</b>	<b>B In -Out</b>
2	0 – 15	15 – 26
5	15 – 27	27 – 50
4	27 – 44	50 – 66
3	44 – 63	66 – 81
1	63 – 74	81 – 89

Table - 7

Total time elapsed on machine A =  $t_1(S_4) = 74$

Total time elapsed on machine B =  $t_2(S_4) = 89$

Utilization time of 2nd machine (B) =  $U_4 = 89 - 15 = 74$ .

$$S_5 = 1 - 5 - 4 - 3 - 2$$

<b>Jobs</b>	<b>A In- Out</b>	<b>B In -Out</b>
1	0 – 11	11 – 19
5	11 – 23	23 – 46
4	23 – 40	46 – 62
3	40 – 59	62 – 77
2	59 – 74	77 – 88

Table - 8



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Vol. 5, Issue 9 , September 2018

Total time elapsed on machine A=  $t_1(S_5) = 74$

Total time elapsed on machine B=  $t_2(S_5) = 88$

Utilization time of 2nd machine (B)=  $U_5 = 88 - 11 = 77$ .

The total utilization of machine A is fixed 74 units and minimum utilization of B is 90 units for the sequence  $S_2$ .

Therefore the optimal sequence is  $S_2 = 4 - 5 - 3 - 2 - 1$ .

Therefore minimum rental cost is =  $74 \times 16 + 90 \times 14 = 2444$  units.

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