Unavailability in n×2 Flow Shop Scheduling To Minimize Rental Cost with Job Block Criteria

Deepak Gupta, Poojadeep Sehgal

Prof. & Head, Dept. of Mathematics, Maharishi Markandeshwar (Deemed to be University) Mullana, Ambala, india
Research   Scholar, Maharishi Markandeshwar (Deemed to be University) Mullana, Ambala, India

ABSTRACT: The present paper studies job block concept in n×2 stage   flow shop scheduling problem to minimize the rental cost. The objective of the study is to get an optimal sequence of jobs in order to minimize the rental cost of machines including breakdown interval and equivalent job block criteria. The algorithm is made clear by numerical illustration.

KEYWORDS: Flow shop scheduling, Equivalent job block, Rental policy, Breakdown interval.

I. INTRODUCTION

In flow-shop scheduling, the object is to obtain a sequence of jobs which when processed in a fixed order of machines, will optimize some well defined criteria. Scheduling theory beginning from Johnson' s work in 1954. Johnson had considered the effect of break-down of machines on the completion times of jobs in an optimal sequence. Later on many researchers such as *Adiri* [1], *Akturk and Gorgulu* [2], *Schmidt*3, *Chandramouli* [4], *Singh T. P.*[5], *Belwal and Mittal* [6] etc. have discussed the various concepts of break-down of machines. The functioning of machines for processing the jobs on them is assumed to be smooth with having no disturbance on the completion times of jobs. But there are feasible sequencing situations in flow shops where machines while processing the jobs get sudden break-down due to failure of a component of machines for a certain interval of time or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as stop of flow of electric current to the machines may be a government policy due to shortage of electricity production. In each case this may be well observed that working of machines is not continuous and is subject to break for a certain interval of time. Various Researchers have done a lot of work in this direction. *Johnson* [*3*] **,** *Ignall* and *Scharge* [7] ,*Szwarch* [8] *Chandra Shekhran* [9] **,** *Maggu & Das* [10] ,*Bagga P. C.*[11], *Singh T. P., Gupta Deepak* [12] ,*Sharma Sameer* etc. *Maggu & Das* **[10]** introduced the concept of equivalent-job blocking in the theory of scheduling. The concept is useful and significant in the sense to create a balance between the cost of providing priority in service to the customer and cost of giving services with non priority customers. The decision maker may decide how much to charge extra from the priority customer

Bagga and Narain**[15]** studied *n* x 2 general flow shop problem to minimize rental cost under a pre-defined rental policy. Further Narain .L., GuptaD. Et.al.[16] made an attempt to minimize the rental cost of machines including job block through simple heuristic approach. We have extended the study made by *Singh T.P., Gupta Deepak* [13] by introducing the concept of break-down interval. We have developed an algorithm minimizing the utilization time of second machine combined with Johnson's algorithm in order to minimize the rental cost of the machines

II. PRACTICAL SITUATION

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the

recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology.

Sometimes the priority of one job over the other is preferred. It may be because of urgency or demand of its relative importance, the job block criteria becomes important. Another event which is mostly considered in the models is the break-down of machines. There may also be delays due to material, changes in release and tail dates, tools unavailability, failure of electric current, the shift pattern of the facility and fluctuations in processing times. All of these events complicate the scheduling problem in most cases. Hence the criterion of break-down interval becomes significant.

III. NOTATIONS

S : Sequence of jobs 1,2,3,….,n
M_j: Machine j, j= 1,2,…….
A_i : Processing time of ith job on machine A.
B_i : Processing time of ith job on machine B.
$β$: Equivalent job for job – block.
L : Length of the break-down interval.
S_i: Sequence obtain from johnson's procedure to minimize rental cost.
C_j : Rental cost per unit time of machine j.
U_i : Utilization time of B (2 nd machine) for each sequence S_i
$t_1(S_i)$: Completion time of last job of sequence o S_i n machine A.
$t_2(S_i)$: Completion time of last job of sequence o S_i n machine B.
$R(S_i)$: Total rental cost for sequence o S_i f all machines.
$CT(S_i)$:Completion time of 1 st job of each sequence o S_i n machine A.

IV. ASSUMPTIONS

1. We assume rental policy that all the machines are taken on rent as and when they are required and are returned as when they are no longer required for processing. Under this policy second machine is taken on rent at time when first job completes its processing on first machine. Therefore idle time of second machine for first job is zero.

2. Jobs are independent to each other.

3. Machine break-down interval is deterministic, .i.e. the break-down intervals are well known in advance. This simplifies the problem by ignoring the stochastic cases where the break-down interval is random.

4. Pre- emption is not allowed, .i.e. once a job started on a machine, the process on that machine can't be stopped unless the job is completed.

V. ALGORITHM

Based on the equivalent job block theorem by *Maggu & Das* and by considering the effect of break-down interval (a ,b) on different jobs, the algorithm which minimize the total rental cost of machines under specified rental policy with the minimum makespan can be depicted as below**:**

Step 1: Define expected processing time of job block β = (k ,m) on machine
A & B using equivalent job block given by *Maggu & Das* i.e. find $A_β$ and $B_β$ as follows:
$A_β = A_k + A_m – \min (B_k , A_m)$

$B_β = B_k + B_m – \min (B_k , A_m)$

Step 2: Using *Johnson's* two machine algorithm 3 obtain the sequence S,while minimize the total elapsed time.

Step 3: Prepare a flow time table for the sequence obtained in step 3 and read the effect of break-down interval (a ,b) on different jobs on the lines of *Singh T.P.*

Step 4: Form a reduced problem with processing times A_i and B_i
If the break-down interval (a, b) has effect on job i then

$A^{'} = A^{'} + L,\ B^{'} = B^{'} + L$ where L = b – a, the length of break-down interval.
If the break-down interval (a, b) has no effect on job i then
$A^{'} = A\ ,\ B^{'} = B$

Step 5: Find the processing times $A^{'}_{\beta}$ and $B^{'}_{\beta}$ of job-block $\beta(k,m)$ on machine A and B using equivalent job-block β as in step 1.

Step 6: Now repeat the procedure to get the sequence S_i, using Johnson'stwo machine algorithm as in step 2.

Step 7: Observe the processing time of 1 st job of S_1 on the first machine A.Let it be α
.
Step 8: Obtain all the jobs having processing time on A greater than α. Putthese job one by one in the 1 st position of the sequence S_1 in the same order.Let these sequences be S_2, S_3, S_4 ,……,S_5.

Step 10: Prepare in-out flow table only for those sequence S_i (i=1,2,…r)which have job block $\beta(k, m)$ and evaluate total completion time of last job of each sequence, .i.e. $t_1(S_i)$ & $t_2(S_i)$ on machine A & B respectively.

Step 11: Evaluate completion time CT (S_i) of 1 st job of each of above selected sequence S_i on machine A.

Step 12: Calculate utilization time U_i of 2 nd machine for each of above selected sequence S_i as:
$U_i = t_2 (Si) – CT (S_i)$ for i=1, 2 , 3,…r.

Step 13: Find Min {U_i}, i=1, 2, …r. let it be corresponding to i = m, then S_m is the optimal sequence for minimum rental cost.
Min rental cost = $t_1(S_m) \times C_1 + U_m \times C_2$,
where C_1 & C_2 are the rental cost per unit time of 1 st & 2 nd machines
respectively.

VI. NUMERICAL ILLUSTRATION

Let us consider 5 jobs and 2 machines problem to minimize the rental cost.

Jobs	A	B
1	6	5
2	6	2
` 3	7	4
4	10	3
5	8	2

Table-1

Rental costs per unit time for machines A & B are 11 and 13 units respectively and jobs (3, 5) are to be processed as an equivalent group job. Also given that the break-down interval is (30,35).

Step1: The processing times of equivalent job block β = (3,5) by using *Maggu* and *Das* criteria are given by:
$A_{\beta} = 7 + 8 – 4 = 11$ and
$B_{\beta} = 4 + 2 - 4 = 2$

Jobs	A	B
1	6	5
2	6	2
` β	11	2
4	10	3

Table-2

Step 2: Using *Johnson's* two machines algorithm, the optimal sequence is
S = 1, 4, β, 2, i.e. S = 1, 4, 3, 5, 2.

Step 3: The in-out flow table for the sequence S = 1- 4- 3- 5- 2 is as follows:

Jobs	A In - Out	B In - Out
1	0 - 6	6 – 11
4	6 – 16	16 - 19
3	16 – 23	23 – 27
5	**23 – 31**	**31 – 33**
2	**31 – 37**	37 - 39

Table-3

Step4: On considering   the effect of break down interval (30, 35), the revised processing times $A^{'}$ and $B^{'}$ of machines A and B are as follows:

Jobs	$A^{'}$	$B^{'}$
1	6	5
2	11	2
3	7	3
4	10	2
5	13	7

Table-4

Step5 : The new processing times of equivalent job block β = (3,5) by using *Maggu* and *Das* criteria are given by

$A_\beta = 7 + 13 – 3 = 17$ and

$B_\beta = 3 + 7 - 3 = 7$

Jobs	A'	B'
1	6	5
2	11	2
` β	17	7
4	10	2

Table-5

Step 6: Using *Johnson's* two machines algorithm, the optimal sequence is
S = β, 1, 2, 4 i.e. S = 3, 5,1, 2,4.

Step7: The processing time of 4th job on S_1 = 10 .i.e. α = 10

Step 8: The other optimal sequences for minimizing rental cost are
$S_2 = 5 − 3 − 1 − 2 − 4$

$S_3 = 1 − 3 − 5 − 2 − 4$

$S_4 = 2 − 3 − 5 − 1 − 4$

$S_5 = 4 − 3 − 5 − 1 − 2$

Step 9: The in-out flow tables for sequences S_1, S_2 ,S_3, S_4 and S_5 are as follows:
For $S_1 = = 3 − 5 − 1 − 2 − 4$

Jobs	A In Out	B In Out
3	0-7	7-10
5	7-20	20-27
1	20-26	27-32
2	26-37	37-39
4	37-47	47-49

Table-6

Total time elapsed on machine A = $t_1(S_1)$ = 47
Total time elapsed on machine B = $t_2(S_1)$ = 49
Utilization time of 2nd machine (B)= U_1 = 49 − 7 = 42.
$S_2 = 5 − 3 − 1 − 2 − 4$

Jobs	A In- Out	B In -Out
5	0 – 13	13 – 26
3	13 – 20	26 – 29
1	20 – 26	29 – 34
2	26 – 37	37 – 39
4	37 – 47	47 – 49

Table-7

Total time elapsed on machine A = $t_1(S_2)$ = 47
Total time elapsed on machine A = $t_2(S_2)$ = 49
Utilization time of 2nd machine (B)= U_2 = 49 – 13 = 36
S_3 =1– 3 – 5 – 2 – 4

Jobs	A In- Out	B In –Out
1	0 – 6	6 – 11
3	6 – 13	13 – 16
5	13 – 26	26 – 33
2	26 – 37	37 – 39
4	37 – 47	47 – 49

Table-8

Total time elapsed on machine A = $t_1(S_3)$ = 47
Total time elapsed on machine B = $t_2(S_3)$ = 49
Utilization time of 2nd machine (B)= U_3 = 49 – 6 = 43.

S_4 = 2 – 3 – 5 – 1 – 4

Jobs	A In- Out	B In -Out
2	0 – 11	11 – 13
3	11 – 18	18 – 21
5	18 – 31	31 – 38
1	31 – 37	38 – 43
4	37 – 47	47 – 49

Table-9

Total time elapsed on machine A = $t_1(S_4)$ = 47
Total time elapsed on machine B = $t_2(S_4)$ = 49
Utilization time of 2nd machine (B)= U_4 = 49 – 11 = 38.

S_5 = 4 – 3 – 5 – 1 – 2

Jobs	A In- Out	B In -Out
4	0 – 10	10 – 12
3	10 – 17	17 – 20
5	17 – 30	30 – 37
1	30 – 36	37 – 42
2	36 – 47	47 – 49

Table-10

Total time elapsed on machine A = $t_1(S_5)$ = 47

Total time elapsed on machine B = $t_2(S_5)$ = 49

Utilization time of 2nd machine (B)= U_5 = 49 – 10 = 39.

The total utilization of machine A is fixed 47 units and minimum utilization of B is 49 units for the sequence S_2. Therefore the optimal sequence is **S_2 = 5 – 3 – 1 – 2 – 4**.

Therefore minimum rental cost is = 47 x 11 + 49 x 13 =   1154 units.

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