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# Ro-Lindel of Space

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**ABSTRACT:**In this paper give a generalization of Lindelof spaces is called ro-Lindelof ( resp., ro-almost Lindelof) if every regular open cover $\Omega$ of $(X,\tau)$ has a countable subcover (resp. a countable subfamily $\{A_n : n \in \mathbb{N}\}$ satisfy $X = \bigcup_{n\in\mathbb{N}} Cl(A_n)$). Some characterizations and properties of ro-Lindelof spaces are given.

**KEY WORDS**: Topological Space, ro-Lindelof, ro-almost Lindelof,

## I.INTRODUCTION

In [1,2,3,4,6 and 7], some generalizations of Lindelof space were introduced and studied, a topological space $(X,\tau)$ is called almost Lindelof (resp.,nearly Lindelof) if every open cover $\Omega$ of $(X,\tau)$ has a countable subfamily $\{A_n : n \in \mathbb{N}\}$ satisfy $X = \bigcup_{n\in\mathbb{N}} Cl(A_n)$) (resp., $X = \bigcup_{n\in\mathbb{N}} int(Cl(A_n))$) ). Also $(X,\tau)$ is called $I$-Lindelofif every regular closed cover $\Omega$ of $(X,\tau)$ has a countable subfamily $\{A_n : n \in \mathbb{N}\}$ satisfy $X = \bigcup_{n\in\mathbb{N}} int(A_n)$. Throughout this paper denote the interior and closure of any subset $A$ of $X$ by $int(A)$ and $Cl(A)$ respectively. A subset $G$ of a space $(X,\tau)$ is called regular open (resp., regular closed) if $G = int\big(Cl(G)\big)$ (resp., $G = Cl\big(int(G)\big)$. In [1], a topological space $(X,\tau)$ is called extremally disconnected (e.d.) if $Cl(G)$ is open for each open set $G$ and this equivalently, $Cl(A) \cap Cl(B) = \emptyset$ for every regular open sets $A$ and $B$ with $A \cap B = \emptyset$. In this work, we introduce the concept ro-Lindelof space and some characterizations.

## II. MAIN RESULTS

Firstly, we introduce the following definition:

**Definition 1**. A topological space $(X,\tau)$ is called ro-Lindelof ( resp. ro-almost Lindelof) if every regular open cover $\Omega$ of $(X,\tau)$ has a countable subcover (resp. a countable subfamily $\{A_n : n \in \mathbb{N}\}$ satisfy $X = \bigcup_{n\in\mathbb{N}} Cl(A_n)$).

Clearly, every ro-Lindelof space is ro-almost Lindelof. In the following some results on this classes of topological spaces:

**Theorem 2.** The following hold forany topological space $(X,\tau)$:

(1) $(X,\tau)$ is ro-almost Lindelof iff every family $\Omega$ of regular closed subset of $X$ with $\bigcap_{U\in\Omega} U = \emptyset$ contains a countable subfamily $\Gamma$ such that $\bigcap_{U\in\Gamma} int(U) = \emptyset$

(2) $(X,\tau)$ is ro-Lindelof iff every family $\Omega$ of regular closed subset of $X$ with $\bigcap_{U\in\Omega} U = \emptyset$ contains a countable subfamily $\Gamma$ such that $\bigcap_{U\in\Gamma} U = \emptyset$.

**Proof.** (1) Let $\Omega = \{U_\alpha :\ \alpha \in I\}$ be a family of regular closed subsets of $(X,\tau)$ such that $\cap\{U_\alpha :\ \alpha \in I\} = \emptyset$. Then the family $\{X - U_\alpha :\ \alpha \in I\}$ forms a cover of the $(X,\tau)$ by regular open subsets, since $(X,\tau)$ is ro-almost Lindelof, so $I$ contains a countable subset $I'$ such that $X = \cup\{Cl(X - U_\alpha) :\ \alpha \in I'\}$, and this implies that $\emptyset = X - \cup\{Cl(X - U_\alpha) :\ \alpha \in I'\} = \cap\{X - (Cl(X - U_\alpha)) :\ \alpha \in I'\} = \cap\{int(U_\alpha) :\ \alpha \in I\}$.

Conversely, let $\psi = \{G_\alpha :\ \alpha \in I\}$ be a cover by regular open sets of $(X,\tau)$, then $\cap\{X - G_\alpha :\ \alpha \in I\} = \emptyset$. By assumption there exists $I' \subseteq I$ with $\cap\{int(X - G_\alpha) :\ \alpha \in I'\} = \emptyset$, so $X = X - \cap\{int(X - G_\alpha) :\ \alpha \in I'\} = \cup\{X - int(X - G_\alpha) :\ \alpha \in I'\} = \cup\{Cl(G_\alpha) :\ \alpha \in I'\}$.

(3) Similarly . ■

**Theorem 3.** Every nearly Lindelof space is ro-Lindelof.

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**Proof.** Let $\{U_\alpha : \alpha \in I\}$ be a cover of $(X,\tau)$ by regular open sets, thus there is countable subset $I'$ of $I$ such that $X = \cup \{int(Cl(U_\alpha)) : \alpha \in I'\}$ because the topological space is Lindelof. Since $int(Cl(U_\alpha)) = U_\alpha$, hence $X = \cup \{U_\alpha : \alpha \in I'\}$.■

**Theorem 4.** Every e.d. ro-almost Lindelof space is nearly Lindelof.
**Proof.** Let $\{U_\alpha : \alpha \in I\}$ be an open cover of $(X,\tau)$. Since $(X,\tau)$ is e.d. , then $\{Cl(U_\alpha) : \alpha \in I\}$ forms regular open cover. Thus there exists countable subset $I'$ of $I$ such that $X = \cup \{Cl(U_\alpha) : \alpha \in I\} = \cup \{int(Cl(U_\alpha)) : \alpha \in I\}$. ■

By Theorem 3, Theorem 4 and [1, Theorem 1.19], we have the following diagram:

I-Lindelof ⟺ e.d. nearly Lindelof ⟺ e.d. ro-Lindelof ⟺ e.d. ro-almost Lindelof

**Theorem 5.** If a space $(X,\tau)$ is a countable union of closed ro-almost Lindelof subspaces then it is ro-almost Lindelof.
**Proof.** Assume that $X = \cup \{F_n : n \in \mathbb{N}\}$, where $(F_n, \tau_{F_n})$ is closed ro-almost Lindelof subspaces for all $n \in \mathbb{N}$. Let $\Gamma$ be a regular open cover of $(X,\tau)$, then the family$\{A \cap F_n : A \in \Gamma\}$ is a regular open cover of $(F_n, \tau_{F_n})$. By hypothesis, $F_n = \cup \{Cl_{F_n}(A \cap F_n) : A \in \Gamma\}$. So, we have that $X = \cup_{n\in\mathbb{N}} \left\{ \cup \left\{ Cl_{F_n}(A \cap F_n) : A \in \Gamma \right\} \right\} = \cup_{n\in\mathbb{N}} \{ \cup \{ Cl_X(A \cap F_n) : A \in \Gamma = n \in \mathbb{N} \cup ClXA : A \in \Gamma$.Therefore *(X,τ)* is ro-almost Lindelof.

**Definition 6 [4].** Let $(X,\tau)$ and $(Y,\sigma)$ are topological spaces and $f: X \longrightarrow Y$ function, we call that:

(1) $\delta$-continuous if for every $x \in X$ and each regular open set $V$ in $Y$ containing $f(x)$ there is regular open set $U$ in $X$ containing $x$ such that $f(U) \subseteq V$.

(2) Almost $\delta$-continuous if for every $x \in X$ and each regular open set $V$ in $Y$ containing $f(x)$ there is regular open set $U$ in $X$ containing $x$ such that $f(U) \subseteq Cl(V)$.

**Theorem 7.** Let $f: (X,\tau) \longrightarrow (Y,\sigma)$ be $\delta$-continuous surjection map, if $X$ is ro-Lindelof then $Y$ is ro-Lindelof.
**Proof.** Let $\{V_\alpha : \alpha \in \Delta]$ be a regular open cover of $Y$. Let $x \in X$ and $V_{\alpha_x}$ be an regular open set in $Y$ such that $f(x) \in V_{\alpha_x}$. Since $f$ is $\delta$-continuous then there is regular open set $U_{\alpha_x}$ of $X$ containing $x$ such that $f(U_{\alpha_x}) \subseteq V_{\alpha_x}$. Now, $\{U_{\alpha_x} : x \in X\}$ is a regular open cover of $X$, so by hypothesis there exists countable subcover $\{U_{\alpha_{x_n}} : n \in \mathbb{N}\}$ and hence $Y = f(X) = f\left(\cup_{n\in\mathbb{N}} U_{\alpha_{x_n}}\right) = \cup_{n\in\mathbb{N}} f\left(\cup_{n\in\mathbb{N}} U_{\alpha_{x_n}}\right) \subseteq \cup_{n\in\mathbb{N}} V_{\alpha_{x_n}}$ . ■

**Theorem 8.** Let $f: (X,\tau) \longrightarrow (Y,\sigma)$ be almost $\delta$-continuous surjection map, if $X$ is ro-Lindelof then $Y$ is ro-almost Lindelof.
**Proof.** Let $\{V_\alpha : \alpha \in \Delta]$ be a regular open cover of $Y$. Let $x \in X$ and $V_{\alpha_x}$ be an regular open set in $Y$ such that $f(x) \in V_{\alpha_x}$. Since $f$ is almost $\delta$-continuous then there is regular open set $U_{\alpha_x}$ of $X$ containing $x$ such that $f(U_{\alpha_x}) \subseteq Cl(V_{\alpha_x})$. Now, $\{U_{\alpha_x} : x \in X\}$ is a regular open cover of $X$, so there exists countable subfamily $\{U_{\alpha_{x_n}} : n \in \mathbb{N}\}$ such that $X = \cup_{n\in\mathbb{N}} U_{\alpha_{x_n}}$. Therefore$Y = f(X) = f\left(\cup_{n\in\mathbb{N}} U_{\alpha_{x_n}}\right) = \cup_{n\in\mathbb{N}} f\left(\cup_{n\in\mathbb{N}} U_{\alpha_{x_n}}\right) \subseteq \cup_{n\in\mathbb{N}} Cl(V_{\alpha_{x_n}})$ . ■

**Theorem 9.** The following statements are equivalent for any topological space $(X,\tau)$:

(1) $X$ is ro-Lindelof.

(2) Every cover of $X$ by form $\{U \subseteq X : \forall x \in U\ \exists\ V_x$ regular open set containing $x$ such that $V_x - U$ is a countable set$\}$ has a countable subcover.

**Proof.** (1) ⇒(2) Let $\{G_\alpha : \alpha \in \Delta\}$ be a hypothesis cover in (2). For every $x \in X$, there exists $\alpha_x \in \Delta$ such that $x \in G_{\alpha_x}$. Thus there exists regular open set $V_{\alpha_x}$ containing $x$ and $V_{\alpha_x} - G_{\alpha_x}$ is countable. It is clear that $\{V_{\alpha_x} : x \in X\}$ is a regular open cover of $X$, so there exists countable subcover $\{V_{\alpha_{x(n)}} : n \in \mathbb{N}\}$. Therefore,

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$$X = \bigcup_{n\in\mathbb{N}} V_{\alpha_{x(n)}}$$
$$= \bigcup_{n\in\mathbb{N}} ((V_{\alpha_{x(n)}} - G_{\alpha_{x(n)}}) \cup G_{\alpha_{x(n)}})$$
$$= (\bigcup_{n\in\mathbb{N}} (V_{\alpha_{x(n)}} - G_{\alpha_{x(n)}})) \cup (\bigcup_{n\in\mathbb{N}} G_{\alpha_{x(n)}})$$

For each $n \in \mathbb{N}, V_{\alpha_{x(n)}} - G_{\alpha_{x(n)}}$ is a countable set , so there exists a countable subset $\Delta(n)$ of $\Delta$ such that $V_{\alpha_{x(n)}} - G_{\alpha_{x(n)}} \subseteq \cup\{G_\alpha: \ \alpha \in \Delta(\mathrm{n})\}$ and this leads to $X \subseteq (\bigcup_{n\in\mathbb{N}}(\cup\{G_\alpha: \ \alpha \in \Delta(\mathrm{n})\})) \cup (\bigcup_{n\in\mathbb{N}} G_{\alpha_{x(n)}})$ and this implies that (2) is hold.

(2)⇒(1) Since every regular open set $U$, we have $U - U = \emptyset$ countable, thus every regular open cover of $X$ admits a countable subcover. ■

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