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An EOQ Inventory Model for time-dependent deteriorating items under price-dependent ramp type demand with shortages

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ABSTRACT: In this paper the Inventory models for deteriorating items are considered in which inventory is depleted not only by demand but also by decay, such as, direct spoilage as in fruits, vegetables and food products, physical depletion as in highly volatile liquids, drugs, pharmaceuticals, and chemicals etc., or deterioration as in electronic components. We have described different models in which depletion over time is the result of the combined effect of demand usage and decay. Economic order quantities under conditions of price dependence of demand are considered for study.

In Model-1, the rate of deterioration is taken to be time-proportional and a power law form of the price dependence of demand is considered. In Model-2, the inventory model has been formulated in stochastic environment. Here we have assumed that the time where the inventory depletes to zero (and subsequent starting of shortage) is a random point of time. In this situation the problem is no longer deterministic but stochastic in nature. If the probability distribution of the point is known, then we can calculate the expectation of the cost and minimizing the cost function we can find the optimal price.

Hence, we derive the EOQ model for inventory of item that deteriorates at a time-proportional decay rate, assuming the demand rate a ramp type function of time. Finally a numerical example has been studied along with its sensitivity.

KEYWORDS: Inventory**,** EOQ model, Price Dependence, Ramp type demand, Deterioration, Shortages, Stochastic Environment .

I. INTRODUCTION

The classical inventory model of Harris-Wilson [22] considers the ideal care in which depletion of inventory is caused by a constant demand rate alone. But subsequently, it was noticed that depletion of inventory may take place due to deterioration also. Almost all items deteriorate with time excepting items as steel, hardware, glassware etc. For these items, the rate of deterioration is so small that there is hardly any need to consider the effect of deterioration. On the other hand, all food items, chemicals etc. deteriorate quite rapidly with the passing of time and become useless for consumption. This loss must have to be taken into account while analyzing the inventory system. In this connection, studies of many researchers like Chakrabarti [6], Covert and Philip [13] developed a two parameter Weibull distribution deterioration for an inventory model. This investigation was followed by Datta and Pal [14], Jalan et al. [25], Dixit and Shah [16], Giri et al. [20], Shah et al. [38] etc.Various types of inventory models with a constant deterioration rate and a linearly time varying demand rate were discussed by Chakrabarti *et al* [5], Chakrabarti[7], Giri *et al* [6], etc.. The assumption of constant demand rate is not always appropriate for many inventory items. Chakrabarti *et al* [8] discussed an order inventory model with variable rate of deteriorating and alternating replenishing rates considering shortage. Inventory models for deteriorating items with quadratic time varying demand and shortages in all cycles was studied by Chakrabarti *et al* [9]. Giri *et al* [19] developed a note on a lot sizing heuristic for deteriorating items with time-varying demands and shortages.The works done by Donaldson [17], Silver [39], Mandal[31], Ritchie [35], Pal and Mandal [34] are to be mentioned regarding time dependent demand rates. In the present paper, efforts have been made to analyze an EOQ model for inventory of items that deteriorate at a time-

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dependent rate assuming the demand rate a ramp type function of time. Such ramp type demand pattern is generally seen in the case of any new brand of goods (as Boroline, Dettol, Nirma washing powder, Maruti-800 etc.) coming to the market. The demand rate for such items increases with time (in the present model we have assumed a linear trend) up to certain time and then ultimately stabilizes and becomes constant. Shortages are also allowed. It is believed that such type of demand rate is quite realistic [cf. 23]. At the end, a numerical example with a sensitivity study has been presented.

In Model-1, we develop an inventory model taking a time-proportional deterioration rate and a power law form of the price-dependent demand rate. Here we have also considered shortage. It is seen that deterioration of a product increases with time. Hence we have taken the deterioration rate to be time-proportional. The price-dependence of the demand function is taken to be nonlinear. The price-dependent demand is that when price of a commodity increases, demand decreases and when price of a commodity decreases, demand increases. The condition for the convexity of the cost function is established. A numerical example is discussed to illustrate the procedure of solving the model.

In Model-2, we extend the inventory model of model-1 in stochastic environment. Here we have assumed that the point, where the inventory depletes to zero and shortage starts, is a random point. In this situation the problem is no longer deterministic but stochastic in nature. If the probability distribution of the point is known, then we can calculate the expectation of the cost and minimizing the cost function we can find the optimal price.

II. LITERATURE SURVEY

It is observed that, the demand rate of an item is influenced by the selling price of an item, as, whenever the selling price of an item increases, the demand for decreases and vice-versa. Generally, this type of demand is seen for finished goods. Several author have investigated this type of inventory model. According to the market research, it is observed that time to time advertisement of an item also changed its demand. The demand rates of these items may be dependent on displayed stock level. Such types of demand in different forms were considered by Maiti, Chung, and others. All these models considered either linear or non-linear form of demand and derived results. In this area, a lot of research papers have been published by several researchers viz. Agarwal[1], Balkhi[2], Bhowmick[3], Bhunia[4], Chang[10], Chang[11], Chung[12], Dave[15], Hariga[21], Karmakar[26], Leea[28], Mondal[32], Sahoo[36], Sajadieh[37], Teng[41],Wu[43], etc. Garg[18] developed an EPQ model with price discounting for non-instantaneous deteriorating item with ramp-type production and demand rates. Jain[24] described an EOQ inventory model for items with ramp-type demand, three parameter Weibull distribution deterioration and starting with shortage. Karmakar[26] discussed inventory Models with ramp-type demand for deteriorating items with partial backlogging and time-varying holding cost. Kundu[27] developed a production lot size model with fuzzy-ram type demand and fuzzy deterioration rate under permissible delay in payments. Lin[29] discussed improved solution process for inventory model with ramp type demand under stock dependent consumption rate. Mahata[30] described a fuzzy replenishment policy for deteriorating items with ramp type demand rate under inflation. Skouri[40] developed inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. Uthaykumar[42] described fuzzy economic production quantity model for weibull deteriorating items with ramp type of demand.

III. ASSUMPTIONS AND NOTATIONS

The fundamental assumptions and notations used in this paper are given as follows:
(i) Replenishment size is constant .
(ii) Lead time is zero.
(iii) T is the fixed length of each production cycle.
(iv) C_1 is the inventory holding cost per unit per unit time.
(v) C_2 is the shortage cost per unit per unit time.
(vi) C_3 is the cost of each deteriorated unit.
(vii) Deterioration rate $\theta(t) = \theta_1 + \theta_2 t$, $0 < \theta_1 , \theta_2 < 1$.
(viii) Shortages are allowed and completely backlogged.

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(ix) The demand rate $R(t)$ is $R(t) = d(p)\alpha\beta[t-(t-\mu)H(t-\mu)]^{\beta-1}$; $0 < \alpha < 1$, $\beta > 0$.
where the well known Heavisides' function $H(t-\mu)$ is defined as

$$H(t-\mu) = \begin{cases} 1\,, & when\ t \geq \mu \\ 0\,, & when\ t < \mu \end{cases}$$

IV. MODEL DEVELOPMENT

In Model-1, we develop an inventory model taking a time-proportional deterioration rate and a power law form of the price-dependent demand rate. Here we have also considered shortage. It is seen that deterioration of a product increases with time. Hence we have taken the deterioration rate to be time-proportional. The price-dependence of the demand function is taken to be nonlinear. The price-dependent demand is that when price of a commodity increases, demand decreases and when price of a commodity decreases, demand increases. The condition for the convexity of the cost function is established. A numerical example is discussed to illustrate the procedure of solving the model.
In Model-2, we extend the inventory model of model-1 in stochastic environment. Here we have assumed that the point, where the inventory depletes to zero and shortage starts, is a random point. In this situation the problem is no longer deterministic but stochastic in nature. If the probability distribution of the point is known, then we can calculate the expectation of the cost and minimizing the cost function we can find the optimal price.

A. Model – 1

Let Q be the total amount of inventory produced or purchased at the beginning ofeach period and after fulfilling backorders let us assume we get an amount $S(>0)$ asinitial inventory. Due to reasons of market demand and deterioration of the items, theinventory level gradually diminishes during the period $(0, t_1)$and ultimately falls tozero at$t = t_1$. Shortages occur during the period (t_1, T) which are fully backlogged.
The instantaneous state of the inventory level $I(t)$ at any time t is described by the differential equations :

$$\frac{dI(t)}{dt} + (\theta_1 + \theta_2 t)\, I(t) = -\, d(p)\alpha\beta t^{\beta-1} \ ;\ 0 \leq t < \mu \ , \qquad \ldots\ldots\ldots (1)$$

$$\frac{dI(t)}{dt} + (\theta_1 + \theta_2 t)\, I(t) = -\, d(p)\alpha\beta \mu^{\beta-1} \ ;\ \mu \leq t < t_1 \ , \qquad \ldots\ldots\ldots (2)$$

$$\frac{dI(t)}{dt} = -\, d(p)\alpha\beta t^{\beta-1} \ ; \qquad t_1 \leq t < T \ , \qquad \ldots\ldots\ldots (3)$$

The boundary conditions are $I(t_1) = 0$ and $I(0) = S$.
The solutions of the above equations are

$$I(t) = d(p)\alpha\left(-t^{\beta} + \frac{\theta_1 t^{\beta+1}}{\beta+1} + \frac{\theta_2 t^{\beta+2}}{\beta+2}\right) + S\left(1 - \theta_1 t - \frac{\theta_2 t^2}{2}\right) ;$$

$$0 \leq t < \mu \ , \qquad \ldots\ldots\ldots (4)$$

$$I(t) = \left\{S + d(p)\alpha(\beta-1)\mu^{\beta}\right\}\left(1 - \theta_1 t - \frac{\theta_2 t^2}{2}\right) + \frac{d(p)\alpha\beta(\beta-1)\mu^{\beta+1}}{2}\left\{\frac{\theta_1}{\beta+1} + \frac{\theta_2\mu}{3(\beta+2)}\right\} + d(p)\alpha\beta\mu^{\beta-1}t\left\{-1 + \frac{\theta_1 t}{2} + \frac{\theta_2 t^2}{3}\right\} ;$$

$$\mu \leq t < t_1 \ , \qquad \ldots\ldots\ldots (5)$$

$$I(t) = -d(p)\alpha\beta\mu^{\beta-1}(t - t_1) \ ; \qquad t_1 \leq t < T \ , \qquad \ldots\ldots\ldots (6)$$

Since $I(t_1) = 0$, we get from equation (5) the following

$$S = d(p)\alpha\beta\mu^{\beta-1}t_1\left(1 + \frac{\theta_1 t_1}{2} + \frac{\theta_2 {t_1}^2}{6}\right) - d(p)\alpha(\beta-1)\mu^{\beta}\left\{1 + \frac{\beta\theta_1\mu}{2(\beta+1)} + \frac{\beta\theta_2\mu^2}{6(\beta+2)}\right\}, \qquad \ldots\ldots\ldots (7)$$

The total amount of deteriorated units is$D = S - \int_0^{t_1} R(t)dt$

$$D = \frac{d(p)\alpha\beta\ \ ^{\beta-1}t_1^2}{6}(3\theta_1 + \theta_2 t_1) - \frac{d(p)\alpha\beta(\beta-1)\mu^{\beta+1}}{6}\left(\frac{3\theta_1}{\beta+1} + \frac{\theta_2\mu}{\beta+2}\right), \qquad \ldots\ldots\ldots (8)$$

The average total cost per unit time is given by

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$$C(T,S,t_1,p) = \frac{C_3 D}{T} + \frac{C_1}{T}\int_0^{t_1} I(t)dt - \frac{C_2}{T}\int_{t_1}^{T} I(t)dt$$

$C(T,t_1,p) = \frac{d(p)C_3\alpha\beta\mu^{\beta-1}}{6T}\left[3\theta_1 t_1^2 + \theta_2 t_1^3 - (\beta-1)\mu^2\left(\frac{3\theta_1}{\beta+1} + \frac{\theta_2\mu}{\beta+2}\right)\right] + \frac{d(p)C_1\alpha\mu^{\beta-1}}{T}\left[\mu t_1\left(-1+\theta_1 t_1 + \frac{\theta_2 t_1^2}{6}\right) + \beta\left\{\frac{(1-\beta)\mu^2}{2(\beta+1)} + t_1^2 + \frac{(1-\beta)\theta_1\mu^3}{6(\beta+2)} + \frac{(2-\beta^2-\beta)\theta_2\mu^4}{12(\beta+2)(\beta+3)} + \frac{(\beta^2-1)t_1\mu^2}{6}\left(\frac{3\theta_1}{\beta+1} + \frac{\mu\theta_2}{\beta+2}\right) + \mu t_1\beta\left(1 - \frac{\theta_1 t_1}{2} - \frac{\theta_2 t_1^2}{2}\right) + \frac{t_1^2}{12}(2\theta_1 t_1 + \theta_2)\right\}\right] + \frac{d(p)C_2\alpha\beta^{\ \beta-1}}{T}(T-t_1)^2$, ... (9)

Our problem is now to determine the values of T, t_1 and p which minimize $C(T,t_1,p)$. However, the functional form of $d(p)$ must be prescribed to proceed further.

A.1 Solution of the Problem

$d(p) = ap^{-b}$, where $a(>0)$ is a scale parameter and $b(>0)$ is a shape parameter.
$t_1 = \gamma T$, $0 < \gamma < 1$.
We have

$$\begin{aligned} C(T,p) = \frac{ap^{-b}\alpha\mu^{\beta-1}}{T}\Bigg[& C_3\beta\left\{\gamma^2T^2(3\theta_1 + \theta_2\gamma T) - (\beta-1)\mu^2\left(\frac{3\theta_1}{\beta+1} + \frac{\theta_2\mu}{\beta+2}\right)\right\} \\ & + C_1\Bigg\{\mu\gamma T\left(-1 + \theta_1\gamma T + \frac{\theta_2\gamma^2T^2}{6}\right) \\ & + \beta\Bigg(\frac{(1-\beta)\mu^2}{2(\beta+1)} + \gamma^2T^2 + \frac{(1-\beta)\theta_1\mu^3}{6(\beta+2)} + \frac{(2-\beta^2-\beta)\theta_2\mu^4}{12(\beta+2)(\beta+3)} + \frac{(\beta^2-1)\gamma\mu^2T}{6}\left(\frac{3\theta_1}{\beta+1} + \frac{\mu\theta_2}{\beta+2}\right) \\ & + \mu\gamma\beta T\left(1 - \frac{\theta_1\gamma T}{2} - \frac{\theta_2\gamma^2T^2}{2}\right) + \frac{\gamma^2T^2}{12}(2\theta_1\gamma T + \theta_2)\Bigg)\Bigg\} + C_2\beta T^2(1-\gamma)^2\Bigg] \end{aligned}$$

The necessary conditions for minimization of $C(T,p)$ are
$\frac{\partial C(T,p)}{\partial T} = 0$ and $\frac{\partial C(T,p)}{\partial p} = 0$ . . . . (10)
The sufficient condition for minimization of $C(T,p)$ requires that it must be a convex function for $T > 0, p > 0$.
Now the function will $C(T,p)$ be convex if

$$\begin{vmatrix} \frac{\partial^2 C(T,p)}{\partial T^2} & \frac{\partial^2 C(T,p)}{\partial T\partial p} \\ \frac{\partial^2 C(T,p)}{\partial p\partial T} & \frac{\partial^2 C(T,p)}{\partial p^2} \end{vmatrix} > 0. \qquad \text{. . . . . (11)}$$

Equations(10) can be solved simultaneously by some computer oriented numerical technique to obtain optimal pricep^* and optimal cycle time T^*. As an illustration, we take up a numerical example.

A..2 Numerical Example

To illustrate the model the following example is considered.
Let $a = 16 \times 10^7$, $b = 3.21$, $C_1 = 5$,$C_2 = 20$, $C_3 = 10$, $\theta_1 = 0.1$,$\theta_2 = 0.02$,$\alpha = 0.02$, $\beta = 1.5$, $\mu = 0.14$, $\gamma = 0.6$ in appropriateunits.
Equations (10) are now solved simultaneously for the above parameter values using a gradient based non-linear optimization technique (LINGO), which yields the local optimal solution:
Optimal cycle time $(T^*) = 0.578$ unit, optimal price $(p^*) = 72.561$ unit,
optimal cost $[C^*(T,p)] = 7672.253$ unit, optimal quantity ordered $(Q^*) = 98.769$unit.
It is numerically verified that this solution satisfies the convexity condition for $C\ (T,p)$.

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A.3 Sensitivity Analysis

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by changes or errors in its input parameter values. In this model, we study the sensitivity of the optimal cycle length T^*, price p^*, cost C^* with respect to the changes in the values of the parameters a, C_1, α, β . The results are shown in Table-1. The following points are noted from Table-1:
(i) The model is moderately sensitive to change in parameters a.
(ii) The model is slightly sensitive to changes in the parameters α, β .
(iii) The model is almost insensitive to changes in the parameter C_1.

Table 1 : Sensitivity analysis

Parameter	% change in parameter values	T^*	p^*	C^*
a	-50	0.478	72.012	11500.12
	-20	0.528	72.464	9681.432
	+20	0.696	72.824	5762.713
	+50	0.825	73.467	3727.347
C_1	-50	0.572	72.152	7642.652
	-20	0.574	72.172	7636.791
	+20	0.581	72.091	7668.257
	+50	0.588	72.014	7704.172
α	-50	0.568	72.632	7546.212
	-20	0.564	72.471	7612.478
	+20	0.567	72.235	7714.614
	+50	0.558	72.074	7782.924
β	-50	0.573	71.715	7913.314
	-20	0.577	72.274	7842.702
	+20	0.584	73.134	7453.513
	+50	0.589	73.692	7368.672

B. Model-2

In this section, the inventory model has been formulated in stochastic environment. We now minimize the average cost per unit time $C(T, t_1, p)$.
t_1 is a random point of time.

B.1 The Problem and Solution

In model-1, the average cost function $C(T, t_1, p) = \frac{d(p)\alpha\mu^{\beta-1}}{T}\Big[\frac{C_3\beta}{6}\Big\{3\theta_1 t_1^2 + \theta_2 t_1^3 - (\beta-1)\mu^2\left(\frac{3\theta_1}{\beta+1} + \frac{\mu\theta_2}{\beta+2}\right)\Big\} + C_1\Big\{\mu t_1\left(-1 + \theta_1 t_1 + \frac{\theta_2 t_1^2}{6}\right) + \beta\Big(\frac{(1-\beta)\mu^2}{2(\beta+1)} + t_1^2 + \frac{(1-\beta)\theta_1\mu^3}{6(\beta+2)} + \frac{(2-\beta^2-\beta)\theta_2\mu^4}{12(\beta+2)(\beta+3)} + \frac{(\beta^2-1)t_1\mu^2}{6}\left(\frac{3\theta_1}{\beta+1} + \frac{\mu\theta_2}{\beta+2}\right) + \mu t_1\beta\left(1 - \frac{\theta_1 t_1}{2} - \frac{\theta_2 t_1^2}{2}\right) + \frac{t_1^2}{12}(2\theta_1 t_1 + \theta_2)\Big)\Big\} + C_2\beta(T - t_1)^2\Big]$

Let us assume that $T = \gamma t_1 (\gamma > 1), (p) = ap^{-b}$, $a > 0 , b > 0$.

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a is a scale parameter and b is a shape parameter.
Therefore, the average cost function (AVC) becomes

$$AVC = C(t_1,p) = \frac{ap^{-b}\alpha\mu^{\beta-1}}{\gamma}\left[\frac{C_3\beta}{6}\left\{3\theta_1 t_1 + \theta_2 t_1^2 - \frac{(\beta-1)\mu^2}{t_1}\left(\frac{3\theta_1}{\beta+1}+\frac{\mu\theta_2}{\beta+2}\right)\right\} + C_1\left\{\mu\left(-1+\theta_1 t_1 + \frac{\theta_2 t_1^2}{6}\right) + \beta\left(t_1 + \frac{1}{t_1}\left(\frac{(1-\beta)\mu^2}{2(\beta+1)} + \frac{(1-\beta)\theta_1\mu^3}{6(\beta+2)} + \frac{(2-\beta^2-\beta)\theta_2\mu^4}{12(\beta+2)(\beta+3)} + \frac{(\beta^2-1)\mu^2}{6}\left(\frac{3\theta_1}{\beta+1}+\frac{\mu\theta_2}{\beta+2}\right) + \mu\beta\left(1-\frac{\theta_1 t_1}{2}-\frac{\theta_2 t_1^2}{2}\right) + \frac{1}{12}(2\theta_1 t_1^2 + \theta_2 t_1)\right)\right)\right\} + C_2\beta t_1(\gamma-1)^2\right]. \quad \text{........ (12)}$$

In this case, the average cost function $C(t_1,p)$ is a random variable with respect to t_1 . So the expected average cost per unit time is $E(AVC) = \xi(p)$

$$= \frac{ap^{-b}\alpha\mu^{\beta-1}}{\gamma}\left[\frac{C_3\beta}{6}\left\{3\theta_1 E(t_1) + \theta_2 E(t_1^2) - (\beta-1)\mu^2 E\left(\frac{1}{t_1}\right)\left(\frac{3\theta_1}{\beta+1}+\frac{\mu\theta_2}{\beta+2}\right)\right\} + C_1\left\{\mu\left(-1+\theta_1 E(t_1) + \frac{\theta_2 E(t_1^2)}{6}\right) + \beta\left(E(t_1) + E\left(\frac{1}{t_1}\right)\left(\frac{(1-\beta)\mu^2}{2(\beta+1)} + \frac{(1-\beta)\theta_1\mu^3}{6(\beta+2)} + \frac{(2-\beta^2-\beta)\theta_2\mu^4}{12(\beta+2)(\beta+3)} + \frac{(\beta^2-1)\mu^2}{6}\left(\frac{3\theta_1}{\beta+1}+\frac{\mu\theta_2}{\beta+2}\right) + \mu\beta\left(1-\frac{\theta_1 E(t_1)}{2}-\frac{\theta_2 E(t_1^2)}{2}\right) + \frac{1}{12}(2\theta_1 E(t_1^2) + \theta_2 E(t_1))\right)\right)\right\} + C_2\beta E(t_1)(\gamma-1)^2\right]. \quad \text{........ (13)}$$

Now, assume that the distribution function of $t_1$to be Erlang distribution. Then its probability density function f(x) is given by

$$f(x) = \begin{cases} \frac{\lambda^r x^{r-1}e^{-\lambda x}}{\Gamma(r)} & ;\ 0 < x < \infty \\ 0 & ;\ otherwise \end{cases}$$

where λ and r are positive constants (given parameters) and $\Gamma(r)$ is a gamma function,defined by $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}\,dx$.
Since gamma function is tabulated, the value of $\Gamma(r)$ being a constant can be easily obtained for the given value of r.
If r has an integer value, then $\Gamma(r) = (r-1)!$.
$E(t_1) = \frac{r}{\lambda}$, $E\left(\frac{1}{t_1}\right) = \frac{\lambda}{r-1}$, $E(t_1^2) = \frac{(r+1)r}{\lambda^2}$.
Therefore the expected average cost per unit time is as follows:

$$E(AVC) = \xi(p)$$

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$$= \frac{ap^{-b}\alpha\mu^{\beta-1}}{\gamma}\left[\frac{C_3\beta}{6}\left\{3\theta_1\frac{r}{\lambda}+\theta_2\frac{(r+1)r}{\lambda^2}-(\beta-1)\mu^2\frac{\lambda}{r-1}\left(\frac{3\theta_1}{\beta+1}+\frac{\mu\theta_2}{\beta+2}\right)\right\}+C_1\left\{\mu\left(-1+\theta_1\frac{r}{\lambda}+\frac{\theta_2}{6}\frac{(r+1)r}{\lambda^2}\right)+\beta\left(\frac{r}{\lambda}+\frac{\lambda}{r-1}\left(\frac{(1-\beta)\mu^2}{2(\beta+1)}+\frac{(1-\beta)\theta_1\mu^3}{6(\beta+2)}+\frac{(2-\beta^2-\beta)\theta_2\mu^4}{12(\beta+2)(\beta+3)}\right)+\frac{(\beta^2-1)\mu^2}{6}\left(\frac{3\theta_1}{\beta+1}+\frac{\mu\theta_2}{\beta+2}\right)+\mu\beta\left(1-\frac{\theta_1}{2}\frac{r}{\lambda}-\frac{\theta_2}{2}\frac{(r+1)r}{\lambda^2}\right)+\frac{1}{12}\left(2\theta_1\frac{(r+1)r}{\lambda^2}+\theta_2\frac{r}{\lambda}\right)\right)\right\}+C_2\beta\frac{r}{\lambda}(\gamma-1)^2\right]. \qquad \ldots\ldots\ldots(14)$$

The necessary condition for $\xi(p)$ to be minimum is that $\frac{\partial\xi(p)}{\partial p}=0$. ....(15)

As an illustration, we take up a numerical example.

The sufficient condition for $\xi(p)$ to be minimum is that $\frac{\partial^2\xi(p)}{\partial p^2}>0$.

B.2 Numerical Example

To illustrate the model the following example is considered.
Let $a=16\times10^7$, $b=3.21$, $C_1=5$,$C_2=20$, $C_3=10$, $\theta_1=0.1$, $\theta_2=0.02$, $\alpha=0.02$, $\beta=1.5$, $\mu=0.14$, $\gamma=2$, $\lambda=10$, $r=5$ in appropriate units.
An equation(15) is now solved for the above parameter values using a gradient based non-linear optimization technique (LINGO), which yields the Global optimal solution:
Optimal cost = $\xi(p)=6814.651$, Optimal price = $p^*=42.468$.
It is numerically verified that this solution satisfies the convexity condition for $\xi(p)$.

V. CONCLUSION

In the present model, we consider that the inventory is depleted not only by ramp type demand, but also by time-dependent deterioration. In the inventory literature, models with price-dependent dependent as well as deterioration are not very common. Many researchers discussed an inventory policy by taking a linear price dependence demand and constant rate of deterioration. A linear price-dependence demand is rarely encountered in the real market. In model-1, we have taken the deterioration rate to be time-proportional and the price-dependence of demand to be non-linear. In model-2, the inventory model has been formulated in stochastic environment. In all these models shortages are also considered. Moreover, we find that the optimal reorder time of the proposed EOQ model is unique.

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