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Parallel Computational Algorithm for Solving Gas Filtration Problems in Porous Media

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ABSTRACT: A mathematical model and a parallel computational algorithm for solving the problem of gas filtration in porous media for arbitrary regions are presented. The proposed algorithm is based on the replacement of differential operators by a finite-difference conservative scheme, and the resulting systems of linear algebraic equations are solved by the Gauss method. This parallel algorithm provided a 20-fold acceleration of calculations on a multiprocessor cluster when solving the problem of gas filtration in porous media.

KEY WORDS: Mathematical model, numerical method, parallel computational algorithm, filtration, gas, porous medium, FastMPJ, mpiJava.

I.INTRODUCTION

During numerical integration of problems of mass transfer in porous medium, which are described by multidimensional systems of partial differential equations with lumped and distributed parameters there is a necessity to multiply solve of tri- or five-diagonal systems of linear algebraic equations (SLAE) with a constant coefficient matrix different right-hand sides.

II. LITERATURE SURVEY

Many scientists from different countries are involved with the solution to these problems, and to date they have produced significant results of fundamental and applied nature.

Particularly, the study [1] is devoted to the latest advances in the development of a continuous approach to the modeling of the dusty gas flow in isotropic porous structures. Over the past fifty years this approach has attracted considerable attention due to the need to develop models of the dusty gas stream that can describe different phenomena, including in relation to the subsurface transport of dissolved or suspended solids, structures of liquid-dust separators, analysis and design of filtration systems. To date developed a number of models for the gas particles flow through porous media, taking into account, as the behavior of a macroscopic flow and microscopic interactions that arise from the porous microstructure. A detailed study of porous microstructures leads to better understanding the interaction between the involved phases and the strength of the porous matrix, acting on the fluid phase. Models dusty gas flow and particle transport in porous structures are characterized by a mathematical idealization porous microstructures.

In [2], the results of numerical modeling of gas flow in a porous medium initially saturated with gas and water, accompanied by the formation of gas hydrate. It is shown that depending on the parameters of the environment at the outer edge (permeable or impermeable to gas flow) of hydrate formation may occur only once on the front edge and in the extended region.

K.Yu. Bogachev [3] proposed a mathematical model of fracturing that occurs around wells during hydraulic fracturing or when injection pressure is exceeded. A computational algorithm for solving the problem is developed for the hybrid MPI platform. The results demonstrate an acceleration of more than 50 times, compared with a sequential algorithm. The results obtained by the author can be used in the numerical solution of hydrocarbon feedstock development

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problems, as well as in solving other systems of partial differential equations on high performance computing (HPC) systems.

D.N. Morozov et al. [4] discussed the problems of using hybrid computing systems of ultrahigh performance for solving problems of mathematical physics. The authors describe a software package for modeling multiphase filtering processes that allows revealing full potential of high-performance systems based on graphic accelerators. On the example of test calculations of leakage problems, it was shown that the logical simplicity of the proposed computational algorithms and the method of software implementation provide high acceleration of calculations.

An explicit algorithm constructed by the analogy with the kinetically-consistent difference schemes was proposed in the paper [5] which discusses solving of three-phase filtration problems. The filtration model includes the energy equation and allows taking into account possible sources of heat emission. Parallel implementation is directed to HPC systems based on graphics accelerators. The computational domain decomposition was optimized to additionally speed up the calculations.

A.V. Tsepaev [6] solved the problem of multiphase fluid filtration in the presence of wells. The author presented algorithms based on the methods of tasks decomposition for implementation on HPC system that combines the power of the CPU and GPU using MPI.

The paper [7] is devoted to the methods of solving three-phase fluid flow problems in the reservoirs. It was assumed that in domain with high velocities Darcy's law is violated and a nonlinear filtration law was used. The methods for solving three-phase flow problems in porous media with a nonlinear filtration law based on the decomposition methods were proposed. The proposed methods were implemented on the heterogeneous HPC system.

Depending on substance filtering area and integration step of spatial integration procedure the order of matrices can range from hundreds to millions. Thus, in case of large-scale systems the solving is required considerable computing resources. Therefore, the problem of development of efficient parallel computing algorithms does not lose its relevance.

III. STATEMENT OF THE PROBLEM

To solve above mentioned problems, we consider the continuity equation for the area G, not including the wells:

$$\frac{\partial}{\partial x}(v_x\rho)+\frac{\partial}{\partial y}(v_y\rho)=-\frac{\partial}{\partial t}(\rho m) \tag{1}$$

and in particular nodes, where the well

$$\frac{\partial}{\partial x}(v_x\rho)+\frac{\partial}{\partial y}(v_y\rho)=-\frac{\partial}{\partial t}(m\rho)\pm Q\frac{\rho\, p_{at}}{p\cdot b\Delta x\Delta y} \tag{2}$$

Where Q - volumetric flow rate (at atmospheric pressure) in the wells of tasks, $Q\rho$ - mass flow, P_{at} - atmospheric pressure, ρ - the density, b – power of the stratum,

$$v_x=-\frac{k}{\mu}\frac{\partial p}{\partial x};\ v_y=-\frac{k}{\mu}\frac{\partial p}{\partial y} \tag{3}$$

Substituting equation (3) in (2) and taking into account the variability of the capacity of the reservoir will get:

$$\frac{\partial}{\partial x}(\frac{\kappa}{\mu}b\rho\frac{\partial p}{\partial x})+\frac{\partial}{\partial y}(\frac{\kappa}{\mu}b\rho\frac{\partial p}{\partial y})=\frac{\partial}{\partial t}(m\rho)\tilde{b}\pm Q\frac{\rho\, p_{at}}{p\Delta x\Delta y}\delta(x,y), \tag{4}$$

where $\tilde{b}$ - the average power value in the "square", k,μ - respectively the filtration coefficient and viscosity of the gas,

$$\delta(x,y)=\begin{cases}1 & when\ \ (x,y)\in\gamma_v\\ 0 & when\ \ (x,y)\notin\gamma_v\end{cases}$$

We assume that the gas is ideal and we obtain [12-15]:

$$\frac{\partial}{\partial x}\left(\frac{\kappa}{\mu}b\rho\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\kappa}{\mu}b\rho\frac{\partial p}{\partial y}\right)=\tilde{b}\frac{\partial}{\partial t}(mp)\pm\delta(x,y)\frac{Q\, p_{at}}{\Delta x\Delta y} \tag{5}$$

Equation (5) is valid for any law of filtration and any dependence on the density of the pressure.

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International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 12 , December 2019

If in the equation (5) all coefficients are constant, i.e., $k = const$, $\mu = const$, $b = const$, $m = const$, we obtain the well-known equation Leybenson

$$\frac{\partial^2 p^2}{\partial x^2} + \frac{\partial^2 p^2}{\partial y^2} = \frac{2m\mu}{k}\frac{\partial p}{\partial t} \pm \frac{Qp_{at}}{\Delta x \Delta y} \cdot \frac{2\mu}{b \cdot k}\delta(x, y) \tag{6}$$

To carry out numerical experiments on a computer, using the equation:

$$x = x^* L; \quad y = y^* L; \quad p = p^* p_H;\ Q^* = \frac{2\mu\, p_{at}}{bk\, p_H^{\,2}} \cdot Q;\ \Delta t^* = \frac{k\, p_H}{2m\mu L^2}\Delta t,$$

we give the dimensionless form to the equation (6):

$$\frac{\partial^2 p^2}{\partial x^2} + \frac{\partial^2 p^2}{\partial y^2} = \frac{1}{2p}\frac{\partial p^2}{\partial t^2} \pm \delta(x, y) \cdot Q \frac{1}{\Delta x \Delta y} \tag{7}$$

To solve the formulated problem to the equation (7) we add initials, boundary and internal conditions:

$$p(x, y, t)\big|_{t=0} = p_{\square}, \quad (x, y) \in G \tag{8}$$

$$\left.\frac{\partial p}{\partial n}\right|_{\Gamma} = 0; \ \int \frac{k}{\mu} b \cdot \frac{\partial p}{\partial n} ds = cQ_{\nu}. \tag{9}$$

As can be seen it is quite difficult to get the analytical solution of this problem.

IV. SOLUTION METHOD

One of the effective methods for solving this problem is to replace the differential operators in equation to finite-difference conservative scheme.Using the finite-difference approximation, we will get a SLAE in the end. After solving this SLAE the gas-dynamic parameters of an object can be determined.To do this we will introduce a uniform grid in x and y:

$$L = n \cdot \Delta x;\ \Delta x^* = \frac{\Delta x}{L} = \frac{1}{n};\ \Delta y^* = \frac{\Delta y}{L} = \frac{a}{m}, \text{ if } \Delta x \neq \Delta y$$

then using the scheme of longitudinal-transverse directions by ox_i, oy_i and linearizing nonlinear terms of finite-difference equation we will obtain [12-15]:

$$\left(2p_{i+1,j}^{(s)} p_{i+1,j}^{(k+0,5)} - p_{i+1,j}^{2(s)}\right) - \left(2 + \frac{\gamma_1}{2\tilde{p}_{i,j}^{(k+0,5)}}\right)\left(2p_{i,j}^{(s)} p_{i,j}^{(k+0,5)} - p_{i,j}^{2(s)}\right) +$$
$$+\left(2p_{i-1,j}^{(s)} p_{i-1,j}^{(k+0,5)} - p_{i-1,j}^{2(s)}\right) = \left(L_1 \overline{p}_{i,j}^{2(k)} + \gamma_1 \frac{\overline{p}_{i,j}^{2(k)}}{2\tilde{p}_{i,j}^{(k+0,5)}} \pm \delta_{i,j} Q_{\nu}\right) \tag{10}$$

or

$$\eta_1 p_{i+1,j}^{(k+0,5)} - \eta_2 p_{i,j}^{(k+0,5)} + \eta_3 p_{i-1,j}^{(k+0,5)} = p_{i+1,j}^{2(s)} - \left(2 + \frac{\gamma_1}{2\tilde{p}_{i,j}^{(k+0,5)}}\right) p_{i,j}^{2(s)} + p_{i-1,j}^{2(s)} +$$
$$+\left(L_1 \overline{p}_{i,j}^{2(k)} + \gamma_1 \frac{\overline{p}_{i,j}^{2(k)}}{2\tilde{p}_{i,j}^{(k+0,5)}} \pm \delta_{i,j} Q_{\nu}\right); \tag{10′}$$

where

$$\eta_1 = 2p_{i+1,j}^{(s)};\ \eta_2 = 2p_{i,j}^{(s)}\left(2 + \frac{\gamma_1}{2\tilde{p}_{i,j}^{(k+0,5)}}\right);\ \eta_3 = 2p_{i-1,j}^{(s)};\ \gamma_1 = \frac{\Delta x^2}{\Delta t};$$
$$L_1 \overline{p}_{i,j}^{2(k)} = \left(\overline{p}_{i,j+1}^{2(k)} - 2\overline{p}_{i,j}^{2(k)} + \overline{p}_{i,j-1}^{2(k)}\right);\ i = \overline{1\,\mathrm{N}},\ j = \overline{1\,\mathrm{J}}.$$

Grouping all the terms in (10′) we will obtain

$$a_i p_{i+1,j}^{(k+0,5)} - b_i p_{i,j}^{(k+0,5)} + c_i p_{i-1,j}^{(k+0,5)} = d_i. \tag{11}$$

Here

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Vol. 6, Issue 12 , December 2019

$$a_i = \eta_1;\ b_i = \eta_2;\ c_i = \eta_3;$$

$$d_i = \left(L_1 \overline{p}_{i,j}^{2(k)} + \gamma_1 \frac{\overline{p}_{i,j}^{2(k)}}{2\tilde{p}_{i,j}^{(k+0,5)}} \pm \delta_{i,j} Q_v \right) + p_{i+1,j}^{2(s)} - \left(2 + \frac{\gamma_1}{2\tilde{p}_{i,j}^{(k+0,5)}} \right) p_{i,j}^{2(s)} + p_{i-1,j}^{2(s)}.$$

After all we will get a SLAE mentioned below

$$A_1 x = \vec{b}_1, \tag{12}$$

where are A_1 - tridiagonal matrix with $(N * J)$ dimensionality and $\vec{b}_1$ - vector with J dimensionality:

$$A_1 = \begin{pmatrix} -b_0 & c_0 & 0 & 0 & 0 & . & . & . & 0 \\ a_1 & -b_1 & c_1 & 0 & 0 & . & . & . & 0 \\ 0 & a_2 & -b_2 & c_2 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & a_{N-1} & b_{N-1} & c_{N-1} \\ 0 & 0 & . & . & . & . & . & a_N & -b_N \end{pmatrix};\ \vec{b}_1 = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ . \\ . \\ . \\ . \\ d_N \end{pmatrix}.$$

Applying the above procedure by 0y direction, we will obtain similar system of algebraic equations:

$$A_2 x = \vec{b}_2. \tag{13}$$

Thus, to solve this two-dimensional problem of gas filtration in porous medium, we have to solve the SLAEs obtained by x and y.

Like most algorithms for solving complex problems, considered algorithm is a combination of fragments with various degrees of parallelism, that is alternately sequential-parallel. Inside the program code only the most resource-intensive sections undergoes for parallelization, in this case it is SLAE solving.

So, the next step is the Java implementation of Gaussian parallel algorithm for solving SLAE (12), (13). The FastMPJ [8], which corresponding to specification of mpiJava 1.2 [9] implemented for interprocessor communication.

The initial matrix of coefficients is distributed on p processes by cyclic horizontal strips with strip width in one row. In each process numbered pid are allocated strips with numbers pid, pid + p, pid + 2p etc. This line-cyclic scheme involves changing per one the number of process at passage from current to next line. Applying this scheme solves the problem of balancing the computational load. It reaches roughly the same amount of computation on each processor and reduced downtime in difference from line-sequential scheme, when at the same data distribution, each successive iteration increases the number of idle processes [11].

For operating over elements of original matrix (two-dimensional array), it transforms into one-dimensional array, that is related to Java runtime environment features. Positions of rows in this one-dimensional array are defined by a stack of offset values with interval equals to number of elements in row - n.

The parallel Gaussian algorithm for solving SLAE can be divided into two parts [11]. The first part – «forward elimination» makes use of elementary row operations and transforms a given matrix to upper triangular form. There are (n-1) iterations to eliminate unknown quantities. The second part – «back substitution» continues to use row operations until the solution is found. It puts the matrix into reduced row echelon form.

At the beginning of «forward elimination» with i, $0 \le i < n-1$ iteration is chosen leading row, which determined by principal-element method using the search of string with the largest value of elements in i column corresponding to eliminated variable xi. Since the rows are distributed across processes, to find the maximum value the process numbered pid>i makes exchange items they hold. After collecting all the data in each process it can be determined which of processes contains the leading row and what value is principal element.

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International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 12 , December 2019

To continue calculation the leading process sends its row of matrix and the appropriate part of vector b to all other processes numbered pid>i. After receiving the leading row, processes makes rows subtract using of elementary row operations, thus provide elimination of appropriate unknown xi.

At the «back substitution» the processes perform necessary calculations to find values of the unknowns. Once any process i, $0 \leq i < n-1$determined the value of its own variable xi, it sends this value to all other processes numbered pid<i. Then the processes makes substitute of received value and performs update of values for the elements of vector b.

V.DISCUSSION

The testing was carried out on a cluster built on the basis of PC connected through Ethernet.

Although considered algorithm assumes an optimal load balancing between nodes, the total time has been significantly influenced by the time required to make data transfer between processes. In the above line-sequential scheme of data distribution during the calculations the number of idle processes increases, but it respectively reduces the number of transfers, that partly compensates uneven computational load. In our case, the number of transfers is always kept on maximum. Therefore, the efficiency of considered algorithm is shown when order of the initial matrix above 400x400, i.e. when the time for data transfer becomes less significant compared to time spent straight for calculations.

This is leads to keep some compromise between desired speedup and efficiency [10], i.e., to preserve conformity between dimensionality of problem and number of involved processors. An undue increasing number of processes is slowing down the speed of calculations.

VI.CONCLUSION

Thus, it can be noted that parallelization of most resource-intensive stages in solving of problems of mass transfer in porous medium is significantly more efficient in general.

In the course of the calculation using considered algorithm there is efficiently to use the number of processes equals to number of CPUs or cores on each cluster node. For example, if a solution of n dimension problem is optimally to divide onto eight processes, so it should be used 4 CPUs (dual-core processors) to run the program.

Although the using of line-cyclic data distribution scheme is more appropriate for dense matrix cases, the efficiency of considered algorithm remains equally for tri- or five-diagonal matrices.

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ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 12 , December 2019

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