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Reliability of multi component standby system by using different distributions

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ABSTRACT: In this paper the reliability is obtained for the multi component standby stress- strength system. In this there are three cases have been taken i.e. stress and strength follow Burr-XII distribution, Inverse weibull distribution and generalized inverted exponential distribution. The general formula for marginal reliability is obtained for each case.

KEY WORDS: Reliability, burr-XII distribution, inverse weibull distribution, generalized inverted exponential distribution, stress- strength models.

I. INTRODUCTION

The reliability of a system is defined as the probability of a system will adequately perform its intended purpose for a given period of time under stated environmental conditions .In some cases system failures occur due to certain type of stresses acting on them. Thus system composed of random strengths will have its strength as random variable and the stress applied on it will also be a random variable. A system fails whenever an applied stress exceeds strength of the system. The reliability of an n-cascade system with stress attenuation was proposed by Pandit and Sriwastav Raghava char et al. studied the reliability of a cascade system with normal stress and strength distribution. In reliability theory, there are lots of real life situations where the concept of mixture distributions can be applied. For example, in life testing experiments, the systems will be failed due to different causes and the times to failure due to different reasons are likely to follow different distributions. Knowledge of these distributions is essential to eliminate cause of failures and thereby to improve the reliability. Gogoi and Borah have studied the estimation of reliability for multi component systems using exponential, gamma and lindley stress-strength distributions.

II.ASSUMPTIONS AND MODEL DESCRIPTION

The assumptions taken in this model are (i) The random variables X and Y are independent. (ii) The values of stress and strength are non-negative. If Y denotes the strength of the component and X is the stress imposed on it f(x), g(y) are the probability density functions of X & Y. Then the reliability of the component is given by

$$R = P(X < Y) = \int_{-\infty}^{\infty} \left[\int_{x}^{\infty} g(y)dy \right] f(x)dx$$

Consider a system of n- components, out of which only one is working under the impact of stresses and remaining (n-1) are standbys. Whenever the working component fails, one of standby components takes the place of a failed components and is subjected ti impact of stress then the system works.

Let Y be the strength of the n components arranged in order of activation in the system. Let $X_1, X_2 \dots\dots X_{n-1}, X_n$ are the stresses on the n component respectively when in operation. The (n-1) components as warm standby face (n-1) stresses $Z_1, Z_2 \dots\dots Z_{n-1}, Z_n$. Then the system reliability R_n of the standby is given by

$$R_n = R(1) + R(2) + \cdots \dots R(n) = \sum_{i=1}^{n} R(i)$$

Where the marginal reliability R(i), i=1,2, …n is the reliability of the system of the ith component

$$R(1) = P(Y \geq X_1)$$
$$R(2) = P[Y < X_1\{Y \geq Z_2, Y \geq X_2\}]$$
$$R(3) = P[Y < X_1\{Y \geq Z_2, Y < X_2, or\ Y < Z_2\}, \{Y \geq Z_3, Y \geq X_3\}]$$

In general

$$R(n) = P[Y < X_1\{Y \geq Z_2, Y < X_2, or\ Y < Z_2\}, \{Y \geq Z_3, Y < X_3\ or\ Y < Z_3\} \dots\dots\dots \{\{Y \geq X_{n-1}, Y < X_{n-1}\ or\ Y < Z_{n-1}\}\{\{Y \geq Z_n, Y \geq X_n\}]$$

Let $g(y), f_i(x), w_i(z)$ be the probability density functions of $Y, X_i\ and\ Z_i\quad for\ i = 1,2 \dots n$

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$$R(1) = \int_0^\infty \bar{G}(x) f_1(x) dx$$

$$R(2) = \left[\int_0^\infty G(x) f_1(x) dx\right]\left[\int_0^\infty \bar{G}(z) w_2(z) dz\right]\left[\int_0^\infty \bar{G}(x) f_2(x) dx\right]$$

$$R(3) = \left[\int_0^\infty G(x) f_1(x) dx\right]\left[\int_0^\infty \bar{G}(z) w_2(z) dz \int_0^\infty G(x) f_2(x) dx + \int_0^\infty G(z) w_2(z)\, dz\right]\left[\int_0^\infty \bar{G}(z) w_3(z) dz \int_0^\infty \bar{G}(x) f_3(x) dx\right]$$

In general

$$R(n) = \left[\int_0^\infty G(x) f_1(x) dx\right]\left[\int_0^\infty \bar{G}(z) w_2(z) dz \int_0^\infty G(x) f_2(x) dx + \int_0^\infty G(z) w_2(z)\, dz\right]\left[\int_0^\infty \bar{G}(z) w_3(z) dz \int_0^\infty \bar{G}(x) f_3(x) dx \int_0^\infty G(z) w_3(z) dz\right] + \cdots \ldots \ldots \left[\int_0^\infty \bar{G}(z) w_n(z) dz \int_0^\infty \bar{G}(x) f_n(x) dx\right]$$

III. RELIABILITY COMPUTATIONS

Case(i): Stress and strength follow Burr-XII distribution, then its probability density functions are

$$f(x,\alpha,\beta) = \alpha\beta x^{\beta-1}\left(1+x^\beta\right)^{-(\alpha+1)}, x \geq 0$$

$$g(y,\lambda,\beta) = \lambda\beta y^{\beta-1}\left(1+y^\beta\right)^{-(\lambda+1)}, y \geq 0$$

$$w(z,\mu,\beta) = \mu\beta z^{\beta-1}\left(1+z^\beta\right)^{-(\mu+1)}, z \geq 0$$

$$R(1) = \int_0^\infty \bar{G}(x) f_1(x) dx \quad where\ G(x) = \int_x^\infty g(y) dy$$

$$G(x) = \int_x^\infty \lambda\beta y^{\beta-1}\left(1+y^\beta\right)^{-(\lambda+1)} dy$$

$$= \left(1+x^\beta\right)^{-\lambda}$$

$$\bar{G}(x) = 1 = G(x) = 1 - \left(1+x^\beta\right)^{-\lambda}$$

$$R(1) = \int_0^\infty \left[1-\left(1+x^\beta\right)^{-\lambda}\right]\alpha_1\beta x^{\beta-1}\left(1+x^\beta\right)^{-(\alpha_1+1)} dx$$

$$R(1) = \frac{\lambda}{\lambda+\alpha_1}$$

$$R(2) = \left[\int_0^\infty G(x) f_1(x) dx\right]\left[\int_0^\infty \bar{G}(z) w_2(z) dz\right]\left[\int_0^\infty \bar{G}(x) f_2(x) dx\right]$$

$$= \left[\int_0^\infty \left(1+x^\beta\right)^{-\lambda}\alpha_1\beta x^{\beta-1}\left(1+x^\beta\right)^{-(\alpha_1+1)} dx\right]\left[\int_0^\infty \left[1-\left(1+z^\beta\right)^{-\lambda}\right]\mu_2\beta z^{\beta-1}\left(1+z^\beta\right)^{-(\mu_2+1)} dz\right]\left[\int_0^\infty \left[1 - \left(1+x^\beta\right)^{-\lambda}\right]\alpha_2\beta x^{\beta-1}\left(1+x^\beta\right)^{-(\alpha_2+1)} dx\right]$$

$$R(2) = \frac{\alpha_1\lambda^2}{(\lambda+\alpha_1)(\lambda+\alpha_2)(\lambda+\mu_2)}$$

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$$R(3)=\left[\int_0^\infty G(x)f_1(x)dx\right]\left[\int_0^\infty \bar{G}(z)w_2(z)dz\int_0^\infty G(x)f_2(x)dx+\int_0^\infty G(z)w_2(z)\,dz\right]\left[\int_0^\infty \bar{G}(z)w_3(z)dz\int_0^\infty \bar{G}(x)f_3(x)dx\right]$$

$$=\left[\int_0^\infty \left(1+x^\beta\right)^{-\lambda}\alpha_1\beta x^{\beta-1}\left(1+x^\beta\right)^{-(\alpha_1+1)}dx\right]\left[\int_0^\infty \left[1-\left(1+z^\beta\right)^{-\lambda}\right]\mu_2\beta z^{\beta-1}\left(1+z^\beta\right)^{-(\mu_2+1)}dz\left[\int_0^\infty \left(1+x^\beta\right)^{-\lambda}\alpha_2\beta x^{\beta-1}\left(1+x^\beta\right)^{-(\alpha_2+1)}dx\right]+\int_0^\infty \left(1+x^\beta\right)^{-\lambda}\mu_2\beta z^{\beta-1}\left(1+z^\beta\right)^{-(\mu_2+1)}dz\right]\left[\left[\int_0^\infty \left[1-\left(1+z^\beta\right)^{-\lambda}\right]\mu_3\beta z^{\beta-1}\left(1+z^\beta\right)^{-(\mu_3+1)}dz\right]\int_0^\infty \left[1-\left(1+x^\beta\right)^{-\lambda}\right]\alpha_3\beta x^{\beta-1}\left(1+x^\beta\right)^{-(\alpha_3+1)}dx\right]$$

$$R(3)=\frac{\alpha_1\lambda^2(\lambda\alpha_2+\lambda\mu_2+\alpha_2\mu_2)}{(\lambda+\alpha_1)(\lambda+\alpha_2)(\lambda+\alpha_3)(\lambda+\mu_2)(\lambda+\mu_3)}$$

In general

$$R(n)=\frac{\alpha_1\lambda^2\sum_{i=1}^{n-2}\lambda(\alpha_{n-1}+\mu_{n-1})+\alpha_{n-1}\mu_{n-1}}{\prod_{i=1}^{n}(\lambda+\alpha_i)\prod_{i=2}^{n-1}(\lambda+\mu_n)}$$

Case(ii): Stress and strength follow Inverse Weibull distribution, then its probability density functions are

$$f(x,\alpha,\beta)=\alpha\beta x^{-(\beta+1)}e^{-\alpha x^{-\beta}},x\geq 0$$
$$g(y,\lambda,\beta)=\lambda\beta x^{-(\beta+1)}e^{-\lambda y^{-\beta}},y\geq 0$$
$$w(z,\mu,\beta)=\mu\beta z^{-(\beta+1)}e^{-\mu z^{-\beta}},z\geq 0$$

$$R(1)=\int_0^\infty \bar{G}(x)f_1(x)dx\quad where\ G(x)=\int_x^\infty g(y)dy$$

$$G(x)=\int_x^\infty \lambda\beta x^{-(\beta+1)}e^{-\lambda y^{-\beta}}dy$$

$$G(x)=\left(1-e^{-\lambda x^{-\beta}}\right)$$

$$\bar{G}(x)=1-G(x)=e^{-\lambda x^{-\beta}}$$

$$R(1)=\int_0^\infty e^{-\lambda x^{-\beta}}\ \alpha_1\beta x^{-(\beta+1)}e^{-\alpha_1 x^{-\beta}}\,dx$$

$$R(1)=\frac{\alpha_1}{\lambda+\alpha_1}$$

$$R(2)=\left[\int_0^\infty G(x)f_1(x)dx\right]\left[\int_0^\infty \bar{G}(z)w_2(z)dz\right]\left[\int_0^\infty \bar{G}(x)f_2(x)dx\right]$$

$$=\left[\int_0^\infty \left(1-e^{-\lambda x^{-\beta}}\right)\alpha_1\beta x^{-(\beta+1)}e^{-\alpha_1 x^{-\beta}}dx\right]\left[\int_0^\infty e^{-\lambda z^{-\beta}}\mu_2\beta z^{-(\beta+1)}e^{-\mu_2 z^{-\beta}}dz\right]\left[\int_0^\infty e^{-\lambda x^{-\beta}}\ \alpha_2\beta x^{-(\beta+1)}e^{-\alpha_2 x^{-\beta}}\,dx\right]$$

$$R(2)=\frac{\lambda\mu_2\alpha_2}{(\lambda+\alpha_1)(\lambda+\alpha_2)(\lambda+\mu_2)}$$

$$R(3)=\left[\int_0^\infty G(x)f_1(x)dx\right]\left[\int_0^\infty \bar{G}(z)w_2(z)dz\int_0^\infty G(x)f_2(x)dx+\int_0^\infty G(z)w_2(z)\,dz\right]\left[\int_0^\infty \bar{G}(z)w_3(z)dz\int_0^\infty \bar{G}(x)f_3(x)dx\right]$$

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$$= \left[\int_0^\infty \left(1-e^{-\lambda x^{-\beta}}\right)\alpha_1\beta x^{-(\beta+1)}e^{-\alpha_1 x^{-\beta}}dx\right]\left[\left[\int_0^\infty e^{-\lambda z^{-\beta}}\mu_2\beta z^{-(\beta+1)}e^{-\mu_2 z^{-\beta}}dz\right]\left[\int_0^\infty \left(1 - e^{-\lambda x^{-\beta}}\right)\alpha_2\beta x^{-(\beta+1)}e^{-\alpha_2 x^{-\beta}}dx\right] + \left[\int_0^\infty \left(1-e^{-\lambda z^{-\beta}}\right)\mu_2\beta z^{-(\beta+1)}e^{-\mu_2 z^{-\beta}}dz\right]\left[\left[\int_0^\infty e^{-\lambda z^{-\beta}}\mu_3\beta z^{-(\beta+1)}e^{-\mu_3 z^{-\beta}}dz\right]\left[\int_0^\infty \left(1 - e^{-\lambda x^{-\beta}}\right)\alpha_3\beta x^{-(\beta+1)}e^{-\alpha_3 x^{-\beta}}dx\right]\right]\right]$$

$$R(3) = \frac{\lambda^2\mu_3\alpha_3(\lambda+\mu_2+\alpha_2)}{(\lambda+\alpha_1)(\lambda+\alpha_2)(\lambda+\alpha_3)(\lambda+\mu_2)(\lambda+\mu_3)}$$

In general

$$R(n) = \frac{\sum_{i=1}^{n-1}\lambda^{n-1}\mu_n\alpha_n(\mu_{n-2}+\alpha_{n-2}+\lambda)}{\prod_{i=1}^{n}(\lambda+\alpha_i)\prod_{i=2}^{n-1}(\lambda+\mu_n)}$$

Case(iii): Stress and strength follow Inverted exponential distribution, then its probability density functions are

$$f(x,\lambda,\alpha) = \lambda\frac{\alpha}{x^2}e^{-\alpha/x}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}, x \geq 0$$

$$g(y,\mu,\alpha) = \mu\frac{\alpha}{y^2}e^{-\alpha/y}\left(1-e^{-\frac{\alpha}{y}}\right)^{\mu-1}, y \geq 0$$

$$w(z,\theta,\alpha) = \theta\frac{\alpha}{z^2}e^{-\alpha/z}\left(1-e^{-\frac{\alpha}{z}}\right)^{\theta-1}, z \geq 0$$

$$R(1) = \int_0^\infty \bar{G}(x)f_1(x)dx \quad where\ G(x) = \int_x^\infty g(y)dy$$

$$G(x) = \int_x^\infty \mu\frac{\alpha}{y^2}e^{-\alpha/y}\left(1-e^{-\frac{\alpha}{y}}\right)^{\mu-1}dy$$

$$G(x) = \left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}$$

$$\bar{G}(x) = 1 - G(x) = 1 - \left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}$$

$$R(1) = \int_0^\infty \left[1-\left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}\right]\lambda_1\frac{\alpha}{x^2}e^{-\alpha/x}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda_1-1}dx$$

$$R(1) = \frac{\mu}{\lambda_1+\mu}$$

$$R(2) = \left[\int_0^\infty G(x)f_1(x)dx\right]\left[\int_0^\infty \bar{G}(z)w_2(z)dz\right]\left[\int_0^\infty \bar{G}(x)f_2(x)dx\right]$$

$$= \left[\int_0^\infty \left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}\lambda_1\frac{\alpha}{x^2}e^{-\alpha/x}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda_1-1}dx\right]\left[\int_0^\infty \left[1-\left(1-e^{-\frac{\alpha}{z}}\right)^{\mu}\right]\theta_2\frac{\alpha}{z^2}e^{-\alpha/z}\left(1-e^{-\frac{\alpha}{z}}\right)^{\theta_2-1}dz\right]\left[\int_0^\infty \left[1 - \left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}\right]\lambda_2\frac{\alpha}{x^2}e^{-\alpha/x}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda_2-1}dx\right]$$

$$R(2) = \frac{\lambda_1\mu^2}{(\lambda_1+\mu)(\lambda_2+\mu)(\theta_2+\mu)}$$

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$$R(3)=\left[\int_0^\infty G(x)f_1(x)dx\right]\left[\int_0^\infty \bar G(z)w_2(z)dz\int_0^\infty G(x)f_2(x)dx+\int_0^\infty G(z)w_2(z)\,dz\right]\left[\int_0^\infty \bar G(z)w_3(z)dz\int_0^\infty \bar G(x)f_3(x)dx\right]$$

$$=\left[\int_0^\infty \left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}\lambda_1\frac{\alpha}{x^2}e^{-\alpha/x}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda_1-1}dx\right]\left[\left[\int_0^\infty \left[1-\left(1-e^{-\frac{\alpha}{z}}\right)^{\mu}\right]\theta_2\frac{\alpha}{z^2}e^{-\alpha/z}\left(1-e^{-\frac{\alpha}{z}}\right)^{\theta_2-1}dz\right]\left[\int_0^\infty \left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}\lambda_2\frac{\alpha}{x^2}e^{-\alpha/x}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda_2-1}dx\right]+\left[\int_0^\infty \left[\left(1-e^{-\frac{\alpha}{z}}\right)^{\theta}\right]\theta_2\frac{\alpha}{z^2}e^{-\alpha/z}\left(1-e^{-\frac{\alpha}{z}}\right)^{\theta_2-1}dz\right]\right]\left[\int_0^\infty \left[1-\left(1-e^{-\frac{\alpha}{z}}\right)^{\mu}\right]\theta_3\frac{\alpha}{z^2}e^{-\alpha/z}\left(1-e^{-\frac{\alpha}{z}}\right)^{\theta_3-1}dz\right]\left[\int_0^\infty \left[1-\left(1-e^{-\frac{\alpha}{x}}\right)^{\mu}\right]\lambda_3\frac{\alpha}{x^2}e^{-\alpha/x}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda_3-1}dx\right]$$

$$R(3)=\frac{\lambda_1\mu^2(\mu\lambda_2+\mu\theta_2+\lambda_2\theta_2)}{(\lambda_1+\mu)(\lambda_2+\mu)(\lambda_3+\mu)(\theta_2+\mu)(\theta_3+\mu)}$$

In general

$$R(n)=\frac{\sum_{i=1}^{n-1}\lambda_1\mu^2(\mu\lambda_i+\mu\theta_{n-2}+\lambda_i\theta_{n-2})}{\prod_{i=1}^{n}(\lambda_i+\mu)\prod_{i=2}^{n-1}(\theta_i+\mu)}$$

IV.CONCLUSION

In the present work, the reliability of standby system is obtained for stress- strength system, When stress and strength follow Burr-XII distribution, Inverse weibull distribution and Generalized inverted exponential distribution. The general formula for marginal reliability is obtained for each case.

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