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Recurrent Algorithms for Estimating the Parameters of Regulators under Uncertain Input Perturbations in Adaptive Control Systems with the Reference Model

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ABSTRACT: The article presents an approach to the development of algorithms for the parametric synthesis of regulators in adaptive control systems of technological objects. The formalization of tasks and algorithms for the synthesis of adaptive control systems for dynamic objects based on the principle of reference models has been carried out, regular recurrent and adaptive algorithms for parametric synthesis of regulators in adaptive control systems with reference models have been developed. Algorithms for parametric synthesis of control devices have been developed, which allow effectively calculating the values of optimal regulator settings when the system is subject to indefinite input disturbances that are generated by effects of a mixed nature, both stochastic and a priori unknown, but limited.

KEYWORDS: control object, control device, adaptive system with reference model, dynamic filtering, regularization, regularization parameter.

I. INTRODUCTION

The intensification of technological modes of technological processes, aimed at improving the efficiency of existing and projected production of various industries, leads to the complication of technological processes as control objects. As a result, ordinary automatic control systems that are widely used in industry cannot provide the specified quality and reliability of control due to the lack of necessary a priori information about the system. In this regard, to improve the quality of management of technological objects, they use the principles of adaptation, or self-adjustment, which allow control systems to artificially adapt to changing conditions by obtaining, processing and analyzing the missing information about the controlled process using an adaptive controller. Using adaptive systems, it is possible to solve a very wide range of tasks, which include not only regulation tasks, but also the task of finding optimal conditions for the system as a whole, managing objects in rapidly changing conditions, control in the presence of interference, and other tasks. [2,6].

The principle of synthesis of adaptive control systems based on reference models has long attracted the attention of researchers [2,5]. In this case, the starting point of the synthesis is the assumption that the control object or, more precisely, its mathematical model and operating conditions are specified with a certain accuracy and the control goal is set. The selection of the structure of the regulator is carried out the control law is determined to ensure that the control goal is met. The synthesis of the regulator is conveniently accomplished with the help of an auxiliary control goal. So, in the stabilization problems, Lyapunov functions are used when the control of the controller is chosen from the condition of non-negativity and decrease of the control system's trajectory [2, 6]. In addition, the control target can be set using the reference model. The next step is the selection of customizable parameters. In the direct method of synthesis, the adjustable parameters (estimates) are the coefficients of the reference controller.

II. FORMULATION OF THE PROBLEM

Despite the considerable variety of methods of direct adaptive control, most of them are based on the following procedure for the synthesis of an adaptive regulator [1, 4].

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We will assume that the dynamics of a controlled process can be described using a linear vector difference equation of the form

$$A(q^{-1})y(t) = B(q^{-1})u(t-k) + d + (q^{-1})z(t), \tag{1}$$

where y is the m-dimensional vector of the output signal (measurement); u is the m-dimensional control vector (input signal); d is an m-dimensional vector consisting of constant elements, which characterizes the steady state output value of the response at a zero value of the input signal; z is a sequence of independent, equally distributed random vectors with zero mean values and covariance $E\{z(t)z^T(t)\} = r$; E - averaging operator; k is the delay time; q^{-1} - operator lag (reverse shift).

$$q^{-1}y(t) = y(t-1)$$

Polynomial matrices A, B and C are $m \times m$ determined by the dimension of the equations:

$$A(q^{-1}) = I + A_1 q^{-1} + ... + A_n q^{-n},$$

$$B(q^{-1}) = B_0 + B_1 q^{-1} + ... + B_n q^{-n},$$

$$C(q^{-1}) = I + C_1 q^{-1} + ... + C_n q^{-n},$$

where B_0 is a nondegenerate matrix; $\det C(\alpha)$ - the determinant of the matrix $C(\alpha)$, its roots lie outside the unit circle.

Assumptions about the system described by equation (1) are of the form: the number of outputs equals the number of inputs; B_0 - non-singular matrix; $\det C(\alpha)$ the roots lie outside the unit circle.

The quality of the functioning of the adaptive system is expressed in the form of the target functional of the form

$$J = E\left\{\left\|\sum_{i=0}^{k-1} p_i\, y(t-k-i) - \sum_{i=0}^{N} R_i\, w(t-i)\right\|^2 + \left\|\sum_{i=0}^{M} Q'_i\, u(t-i)\right\|^2\right\}, \tag{2}$$

where w is the m-dimensional vector of the master signal; p_i - real numbers; $p_0 = 1$; R_i и and Q'_i - $m \times m$-dimensional matrices; E is the averaging operator.

The values of the upper limits of the sums of the target functional N and M are noncritical. In the particular case, the possible values are $N = 0$, $M = 0$, or $M = 1$.

The optimal management strategy should minimize the accepted quality criterion (2) for all permissible control strategies. Control at time ***t*** depends on all monitored outputs $y(t),\ y(t-1), \ldots$ and all previously applied control signals $u(t-1),\ u(t-2), \ldots$

The target functional (2) can be represented in vector form

$$J = E\left\{\left\|P(q^{-1})y(t+k) - R(q^{-1})w(t)\right\|^2 + \left\|Q'(q^{-1})u(t)\right\|^2\right\}, \tag{3}$$

where P, R and Q are polynomial matrices.

The most common special cases of the criterion (3) are:

$$J_1 = E\left\{\left\|y(t+k) - w(t)\right\|^2 + u^T(t)\, diag[\lambda_1, \ldots, \lambda_m]u(t)\right\},$$

$$J_2 = E\left\{\left\|y(t+k) - w(t)\right\|^2 + [u(t) - u(t-1)]^T diag[\lambda_1, \ldots, \lambda_m][u(t) - u(t-1)]\right\}.$$

The first quality criterion contains components that penalize the deviation of the output signal y from the driver w, and also limits the control action. However, this quality criterion does not ensure equality between the averaged values of the output $E[y(t)]$ and the master $E[w(t)]$ signals. The second quality criterion is free from this drawback, since astatism is introduced into the control loop, as a result of which equality is ensured $E[y] = E[w(t)]$. The dynamic properties of the system deteriorate.

The structure of the adaptive regulator includes the optimal least-squares predictor obtained from the use of equation (1). The predictor extrapolates the output signal y at time t by k steps forward $y^*(t-k/t)$ based on information about

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previous values $y(t), y(t-1),\ldots;\ u(t), u(t-1),\ldots$ The predictor design procedure is discussed in detail in [2,3]. The predictor equation is

$$y^*(t+k/t)=\tilde{C}^{-1}(q^{-1})\left[\tilde{F}^{-1}(q^{-1})\,y(t)+\tilde{E}'(q^{-1})B(q^{-1})u(t)+\gamma\right]$$

The prediction error is determined by the ratio

$$e(t+k)=z(t+k)+\ldots+E'_{k-1}z(t+1). \tag{4}$$

The control law is chosen in the form

$$G_0u(t)=-\left[\sum_{i\geq 0}F_i y(t-i)+\sum_{i\geq 1}G_i u(t-1)+\sum_{i\geq 0}H_i w(t-i)+\gamma\right]. \tag{5}$$

To calculate the parameters of this control law, the evaluator is used. The evaluator's synthesis is based on the assumption that the parameter equation is approximated by

$$\theta_k=\theta_{k-1}+w_k\,, \tag{6}$$

and the measurement equation is

$$z_k=H_{k-\tau}\theta_k+\upsilon_k\,, \tag{7}$$

where

$$H_{k-\tau}=\left[y^T_{k-\tau},y^T_{k-\tau-1},\ldots;u^T_{k-\tau},u^T_{k-\tau-1},\ldots;y^{dT}_{k-\tau},y^{dT}_{k-\tau-1},\ldots;1\right],$$

$$\theta_k=\left[\theta_{1k},\ldots,\theta_{mk}\right]=\left[F_0,F_1,\ldots;G_0,G_1,\ldots;S_0,S_1,\ldots;\gamma\right]^T,$$

$$\theta_i^T=\left[f^0_{i1},\ldots,f^0_{im},f^1_{i1},\ldots,f^1_{im},\ldots;g^0_{i1},\ldots,g^0_{im},g^1_{i1},\ldots,g^1_{im},\ldots;s^0_{i1},\ldots,s^0_{im},s^1_{i1},\ldots,s^1_{im},\ldots;\gamma_i\right]$$

and w_k is the vector of perturbations; υ_k - a sequence of random vectors characterizing the errors of observations.

Analysis of the above computational scheme and various adaptation methods [1,2,5] shows that by now there are various algorithms for the synthesis of regulators in adaptive control systems with a reference model with complete information about the controlled process and statistical characteristics of object noise and measurement interference [2 4,5,7]. In these papers, effective methods are proposed and constructive conditions are investigated that guarantee the required asymptotic properties of tuning algorithms in the presence of information about the internal structure of perturbing processes. However, for many control systems of dynamic objects, there is a large uncertainty in their working conditions. Information about the actual values of the parameters of control objects are very inaccurate, and the laws of their possible changes are known very approximately, information about the initial state of the automatic control system is insufficient, information about possible input signals and disturbing influences are uncertain [1,2,8].

III. SOLUTION OF THE TASK

The practical implementation of efficient computational procedures for the synthesis of stable adaptive schemes for estimating controller settings leads to the need to apply regularization methods, the purpose of which is to provide an error of the solution of the same order as the accuracy of the initial data. Therefore, it is advisable to consider various possible approaches to solving problems of increasing the accuracy of calculating the controller settings and to identify the most promising for practical use methods and algorithms for solving ill-posed problems.

Equations (6), (7) should be parametrized as follows [9] .:

$$\theta(k+1)=\theta(k)+w(k),\ k=1,\ldots,n, \tag{8}$$

$$z(k)=H(k-l)\theta(k)+v(k),\ k=1,\ldots,n, \tag{9}$$

where $H(k-l)$ is the matrix of the corresponding dimension, $\theta(k)$ is the vector of parameters.

Suppose that are $\theta(0),w(0),\ldots,w(n-1),v(1),\ldots,v(n)$ Gaussian-independent random vectors with undefined vectors of means and known covariance matrices. We introduce the notation

$$\bar{\theta}(0)=E\{\theta(0)\},\ \ \bar{w}(k-1)=E\{w(k-1)\},\ \bar{v}(k)=E\{v(k)\},\ k=1,\ldots,n,$$

$$L(0)=E\{(\theta(0)-\bar{\theta}(0))(\theta(0)-\bar{\theta}(0))^T\},$$

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$$M(k-1)=E\{(w(k-1)-\overline{w}(k-1))(w(k-1)-\overline{w}(k-1))^T\},$$
$$N(k)=E\{(v(k)-\overline{v}(k))(v(k)-\overline{v}(k))^T\},$$

and

$$k(g,j)=(k(g),...,k(j)),\ g\le j,$$
$$k_{g,j}=(k_g,...,k_j),\ g\le j.$$

$\overline{\zeta}(\overline{g,j})$ - set of undefined parameters

$$\overline{\zeta}(\overline{g,j})=\{\overline{\theta}(g-1),\overline{w}[g-1,j-1],\overline{v}[g,j]\},$$

a $\overline{\zeta}_{\overline{g,j}}$ - set of fixed vectors

$$\overline{\zeta}_{\overline{g,j}}=\{\overline{\theta}_{g-1},\overline{w}_{g-1,j-1},\overline{v}_{g,j}\},\ 1\le g\le j\le n.$$

In accordance with [3], we consider the measure of parameter uncertainty, $\overline{\zeta}(\overline{1,n})$ i.e. function

$$\overline{\Phi}(\overline{\zeta}(\overline{1,n}))=\sum_{k=1}^{n}\left(\|\overline{w}(k-1)\|^2_{\overline{M}^{-1}(k-1)}+\|\overline{v}(k)\|^2_{\overline{N}^{-1}(k)}\right)+\|\overline{\theta}(0)-\hat{\theta}(0)\|^2_{\overline{L}^{-1}(0)},$$

Where $\overline{M}(k-1),\overline{N}(k),\overline{L}(0),\ k=1,...,n$, are given matrices, $\hat{\theta}(0)$ is a fixed vector.

We will assume that the following restriction holds on the measure $\overline{\Phi}(\overline{\zeta}(\overline{1,n}))$ of uncertainty of unknown parameters $\overline{\zeta}(\overline{1,n})$:

$$\overline{\Phi}(\overline{\zeta}(\overline{1,n}))\le 1. \qquad (10)$$

It is required to find $\theta^*(n)$ such a vector that

$$\theta^*(n)=\arg\min_p \max_{\overline{\zeta}(\overline{1,n})} E\{\|\theta(n)-p\|^2\}.$$

When solving this problem, it is advisable to construct a reachability set of $\Xi(n)$ for the vector $\theta(n)$, with constraint (10) on the measure of uncertainty, $\overline{\Phi}(\overline{\zeta}(\overline{1,n}))$, namely

$$\overline{\Xi}(n)=\cup\{\overline{\theta}(n)\,|\,\overline{\zeta}(1,n):\overline{\Phi}(\overline{\zeta}(\overline{1,n}))\le 1\}.$$

Following it can be shown that the solution of the problem in question is a vector $\theta^*(n)$ that minimizes the function $U(p)$:

$$U(p)=\max_{\overline{\zeta}(\overline{1,n})} E\{\|\theta(n)-p\|^2\},$$

There is equality

$$\max_{\overline{\zeta}(\overline{1,n})}\{\|\overline{\theta}(n)-p^*\|^2\}=\max_{\theta\in\overline{\Xi}(n)}\{\|\theta-p^*\|^2\}, \qquad (11)$$

where the set of $\overline{\Xi}(n)$ is determined by the equality

$$\overline{\Xi}(n)=\{\theta:\|\theta-\hat{\theta}(n)\|^2_{L^{-1}(n)}\le 1\}.$$

Indeed, equality (11) is valid, since solution $\overline{\Xi}(n)$ of the problem is the set of all vectors $\overline{\theta}(n)$ obtained using recurrence relations

$$\overline{\theta}(k)=\overline{\theta}(k-1)+\overline{w}(k-1)+L(k)H^T(k)N^{-1}(k)\times$$
$$\times(z(k)-\overline{v}(k)-H(k)\overline{\theta}(k-1)+\overline{w}(k-1)),\ k=1,...,n, \qquad (12)$$

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where $L(k)=((L(k-1)+M(k-1))^{-1}+H^T(k)N^{-1}(k)H(k))^{-1},\ k=1,...,n$.

When implementing (12), it is necessary to take into account condition (10). As the initial condition $\overline{\theta}(0)$, a fixed vector of averages is taken, namely θ_0.

In other words, $\Xi(n)$ is the reachable set for vectors $\overline{\theta}(n)$ associated with uncertainty $\zeta(\overline{1,n})$ using recurrence relations (12) with constraints (10) on the measure of uncertainty $\overline{\Phi}\left(\zeta\left(\overline{1,n}\right)\right)$.

Thus, function $U(p)$ can be represented as

$$U(p)=E\left\{\left\|\theta(n)-\overline{\theta}(n)\right\|^2\right\}+\max_{\theta\in\Xi(n)}\left\{\left\|\theta-p\right\|^2\right\}.$$

Then it suffices to find a vector $\theta^*(n)$ such that

$$\theta^*(n)=\arg\min_p\max_{\theta\in\Xi(n)}\left\{\left\|\theta-p\right\|^2\right\}$$

The vector $\theta^*(n)$ can be obtained using recurrence relations:

$$\theta^*(k)=\theta^*(k-1)+L(k)H^T(k)N^{-1}(k)(z(k)-H^T(k)\theta^*(k-1)),\ \ k=1,...,n, \tag{13}$$

under the initial condition $\theta^*(0)=\theta(0)$. Matrices $\overline{L}(k)$ can be obtained using recurrence relations of the form [10]:

$$L(k)=C(k)L(k-1)C^T(k)+S(k)T(k)S^T(k)=L(k)((L(k-1)+M(k-1))^{-1}(L(k-1)+M(k-1))\times$$
$$\times(L(k-1)+M(k-1))^{-1}+H^T(k)N^{-1}(k)N(k)N^{-1}(k)H(k))L(k),\ \ k=1,...,n,$$

Where

$$C(k)=L(k)(L(k-1)+M(k-1))^{-1},$$
$$K(k)=L(k)H^T(k)N^{-1}(k),$$
$$S(k)=C(k)-K(k).$$

The estimate (13) does not always exist. For its fairness, it is necessary that the matrix $N^{-1}(k)$ exists. For this, at least it is necessary that the matrices $N(ii)$ $(i=\overline{0,\ k})$ are nondegenerate. When the requirement of positive definiteness $N(ii)$ is removed, then we naturally have to admit its degeneracy. Sometimes it is possible to make some measurements accurately, i.e. no mistakes. In such cases $N(ii)=0$.

Suppose that the $(k+1)$-e observation is carried out without errors. Then

$$z(k+1)=H(k+1)\theta(k+1). \tag{14}$$

The relation (14) can be interpreted in two ways. In the first interpretation we can assume

$$z(k+1)=H(k+1)\theta(k+1)+v(k+1),$$

where $v(k+1)$ - is a random perturbation with a zero mean and zero covariance matrix.

In another interpretation, (14) is considered as a linear equality type constraint. The first interpretation is the most acceptable.

Then it can be shown [4, 6] that the optimal mean-square estimate of vector $\theta(k+1)$, based on observations $z(0),\ z(1),\ \ldots\ z(k+1)$, is determined by the formula

$$\theta(k+1)=\overline{\theta}(k+1)+M(k+1)H^T(k+1)\times$$
$$\times\left(H(k+1)M(k+1)H^T(k+1)\right)^{-1}\left(z(k+1)-H(k+1)\overline{\theta}(k+1)\right), \tag{15}$$

where

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$$\bar{\theta}(k+1)=\theta(k+1)+w(k),\ w(k)=E\{w(k)/z^0(k)\}, \quad (16)$$

$$M(k+1)=p(k)+M(k), \quad (17)$$

if a

$$K_{wv}(i,j)=E\{v_i w_j^T\},\ (i=\overline{0,k},\ j=\overline{0,k-1}),$$

$$K_{\theta v}(0,i)=E\{(\theta_0-\bar{\theta}_0)w_i^T\},\ (i=\overline{0,k-1}),\ K_{wv}(i,j)=0,\ K_{wv}(0,i)=0.$$

$$p(k+1)=M(k+1)-M(k+1)H(k+1)\left(H(k+1)M(k+1)H^T(k+1)\right)^{-1}H(k+1)M(k+1). \quad (18)$$

In deriving relations (15) - (18), the existence of a matrix was implied. If this matrix does not exist, it means that (14) includes some equations (namely, equations, - the rank of the matrix), which depend linearly on the others. Therefore, these equations can be excluded from (14). After elimination, instead we get an -dimensional vector, and now we can assume that a similar inverse matrix exists in the new problem.

In deriving relations (15) - (18), the existence of matrix $\left(H(k+1)M(k+1)H^T(k+1)\right)^{-1}$ was implied. If this matrix does not exist, it means that (14) includes some equations (namely, $l-r$ equations, r - the rank of matrix $H(k+1)$), which depend linearly on the r others. Therefore, these equations can be excluded from (14). After elimination, of $z(k+1)$ instead we get a $k+1$ – r - dimensional vector, and now we can assume that a similar inverse matrix exists in the new problem.

IV. CONCLUSION

The above regular algorithms allow estimating the parameters of an adaptive controller in the presence of a discrete dynamic system at the input of uncertain perturbations generated by the mixing of effects of different nature, i.e. both stochastic and a priori unknown, but limited.

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