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Some Properties of B^*c–Compact in Topological Spaces

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ABSTRACT: In this paper the ideas of B^*c- open set , B^*c-neighbourhood ,B^*c- converge , B^*c- cluster and B^*c- compact in topological spaces are discussed. Some basic properties of these ideas in topological spaces are studied. Finally some new results and theorems about them over topological spaces are investigated.

KEYWORDS:B^*c- open set ,B^*c- closed set , B^*c-neighborhood, B^*c- converge , B^*c- cluster , B^*c- compact .

I.INTRODUCTION

The basic idea of studying this sets is to generalize certain characteristics and to indicate their relations with other sets and to prove many of the hypotheses. In 1963, the concept of semi open sets was introduced by N. Levine [1]. In1982, the concept of pre-open setswere introduced by Mashhour , AbdElmonsef and El-Deeb [2] . In 1983, the class of $\beta-$ open sets and$\beta-$ closed sets were introduced by Abd El -Monsef M. and et. al. [3] ,some characteristics were also given in topological spaces that are related to these sets. In 1986, the concept of semi pre-open sets (which equivalence of the definition $\beta-$ open sets) were introduced by Andrijecivic D. [4]. Theorems and notions from semi pre-open sets proved by a lot of researchers. In 2018 , study about the concept B^*c- open set were introduced by karim. F. R . [5] and through it, introduced proof many of hypotheses and we introduced B^*c-neighborhood , B^*c- converge , B^*c- cluster and B^*c- compact as property of B^*c- open set.

II. B^*C-OPENSETS AND SOME PROPERTIES

Definition (2.1)[3]: Let (X,T) be atopological space .Then a subset C of X is said to be i)a $\beta-$ open set if$C \subseteq cl(int(cl\ C))$ .ii) a$\beta-$ closed set if $int(cl(int\ C)) \subseteq C$.The family of all $\beta-$ open (resp.$\beta-$ closed) set subsets of a space X will be as always denoted by $\beta o(X)$ (resp.$\beta c(X)\beta c(X)$).

Definition (2.2)[5]: Let (X,T) be a topological space and $C \subseteq X$. Then a$\beta-$ open set C is called a B^*c- open set if for all $x \in C$ their exists F_X closed set such that $x \in F_X \subseteq C$. C is a B^*c-closed set if C^c is aB^*c-open set the family of all B^*c- open (resp.B^*c- closed) set subset of a space X will be as always denoted by $B^*co(X)$ (resp.$B^*cc(X)$).The following example shows that aB^*c- open set need not be a closed set.

Example (2.3): Let (R,T) be a topological space (usual topology) , if $B = (0,1]$ such that $(0,1] = \bigcup_1^\infty \left[\frac{1}{n},1\right]$, then B is B^*c- open set , but it is not closed .The following example shows that the intersection of two B^*c- open sets need not be B^*c- open set.

Example (2.4): Let$X = \{a,b,c\}$, $T = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$.then the family of closed sets are :$\emptyset, X, \{c\}, \{a,c\}, \{b,c\}$,also, we can find two families $\beta o(X) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$ and $B^*co(X) = \{\emptyset, X, \{a,c\}, \{b,c\}\}$. Note that $\{a,c\} \in B^*co(X)$ and $\{a,c\} \in B^*co(X)$. But $\{a,c\} \cap \{b,c\} = \{c\} \notin B^*co(X)$.From the above example we notice that the family of all B^*c- open set subsets of a space X is a supra topology and need not be a topology in general .

Proposition (2.5) [5]: Let (X,T) be a topological space . Then i) the union family of B^*c-open set is a B^*c-open set .ii) the intersection family ofB^*c-closed set is a B^*c-closed set .The following theorem shows that the family of all B^*c-open sets will be a topology on X.

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Theorem (2.6): If the family of all β −open set of a space X is a topology on X, then the family of B^*c- open sets is also a topology on X .

Proof : It is clear that $\emptyset, X \in B^*co\ (X)$ and by proposition (2.5) the union of B^*c- open set is B^*c-open .To complete this proof , we need only to show that the finite intersection of B^*c-open sets is B^*c- open set . Let C and D be to B^*c- open sets then C and D are $\beta-$ open sets . since $\beta o(X)$ is a topology on X , so $C \cap D$ is β −open . Let $y \in C \cap D$, then $y \in C$and $y \in D$ so there exists E and H are$\beta-$ open sets such that $y \in E \subseteq C$ and $y \in H \subseteq D$ this implies that $y \in E \cap H \subseteq C \cap D$. since any intersection of closed sets is closed , then $E \cap H$ is closed set . thus $C \cap D$ is B^*c-open set . the proof is complete .

Proposition (2.7): The set C is B^*c-open in the space (X, T) if and only if for each $y \in C$,there exists a B^*c-open set D such that $y \in D \subseteq C$.

Proof : Assume that C is B^*c-open set in the space (X, T) , then for each $y \in C$, put $C = D$ is B^*c-open set containing y such that $y \in D \subseteq C$.Conversely, suppose that for each $y \in C$,there exists a B^*c- open set D such that$y \in D \subseteq C$,thus $C = \cup D_y$where $D_y \in B^*co\ (X)$ for each y, therefore C is B^*c-open set.In the following theorem, the family of $\beta-$ open sets is identical to the family of B^*c-open sets.

Theorem (2.8): If a space X is T_1 −space , then the families $\beta o(X) = B^*co\ (X)$.

Proof: Let C be any subset of a space X and $C \in \beta o(X)$, if $C = \emptyset$, then $C \in B^*co\ (X)$. If $C \neq \emptyset$, then for each $y \in C$ .Since a space X is $T_1 -$ space, then every singleton is closed set and hence $y \in \{y\} \subseteq C$, therefore $C \in B^*co\ (X)$.Hence $\beta o(X) \subset B^*co\ (X)$.but $B^*co\ (X) \subset \beta o(X)$ generally, therefore $\beta o(X) = B^*co\ (X)$.

Theorem (2.8)[6]:A space (X, T) be a regular if and only if for all U open subset of X and for all $Y \in U$, there exists V open subset of X such that $Y \in V \subseteq cl\ V \subseteq U$.

Theorem (2.9): Let (X, T) be a topological space , if X is regular , then $T \subseteq B^*co\ (X)$.

Proof: Let C be any subset of a space $X$and C is open set , if $C = \emptyset$, then $C \in B^*co\ (X)$. If $C \neq \emptyset$,since X is regular , so for each $y \in C \subseteq X$, there exists an open set V such that $y \in V \subseteq cl\ V \subseteq C$ (by theorem 2.8).Thus we have $y \in cl\ V \subseteq C$ .since $C \in T$ and hence $C \in \beta o(X)$, therefore $T \subseteq B^*co\ (X)$.

Proposition (2.10): A subset D of a space X is called B^*c- closed if and only if D is a β −closed set and it is an intersection of open sets .

Proof: Clear.

Example (2.11): In example (2.4) , the family of B^*c −closed subset of X is : $B^*cc(X) = \{\emptyset, X, \{a\}, \{b\}\}$. Note that $\{a\} \in B^*cc(X)$ and $\{b\} \in B^*cc(X)$, but $\{a\} \cup \{b\} = \{a, b\} \notin B^*cc(X)$ .The following theorem is true by using complement.

Theorem (2.12): If a space X is T_1 − space , then the families $\beta c(X) = B^*cc\ (X)$.

Definition (2.13): Let (X, T) be a topological space and $y \in X$, then a subset N of X is said to be B^*c − neighborhood of y, if there exists a B^*c −open set V in X such that $y \in V \subseteq N$.

Theorem (2.14): In a topological space (X, T), a subset C of X isB^*c −open set if and only if it is a B^*c −neighborhood of each of its points .

Proof: Let $C \subseteq X$ be a B^*c −open set ,since for every $y \in C$, $y \in C \subseteq C$ and C is B^*c − open. This implies that C is a B^*c −neighborhood of each of its points .Conversely, suppose that C is a B^*c − neighborhood of each of its

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points .then for each $y \in C$, there exists $D_y \in B^*co\ (X)$ such that $D_y \subseteq C$. Then $C = \cup \{D_y : y \in C\}$ .since each D_y is $B^*c -$ open set. It follows that C is $B^*c -$ open set.

Theorem (2.15): For any two subsets C, D of a topological space (X, T) and $C \subset D$, if C is a $B^*c -$ neighborhood of a point $y \in X$,then D is also $B^*c -$ neighborhood of the same point y.

Proof: Let C be a $B^*c -$ neighborhood of $y \in X$, and $C \subset D$, Then by definition (2.2), there exists a $B^*c -$ open set V such that $y \in V \subset C \subset D$, this implies that D is also a $B^*c -$neighborhood of a point y .

Remark (2.16): Every $B^*c -$neighborhood of a point is $\beta -$ neighborhood, it follows from every $B^*c -$open set is $\beta -$ open .

III. MAINRESULTS

Definition (3.1): Afilter base $\mathcal{E}$ in a topological space $(X, T) B^*c -$converges to a point $y \in X$ if for every $B^*c -$open set W containing y , there exists an $F \in \mathcal{E}$ such that $F \subset W$.

Definition (3.2): A filter base $\mathcal{E}$ in a topological space (X, T) is $B^*c -$cluster to a point $y \in X$ if $F \subset W \neq \emptyset$, for every $\mathcal{E}$ open set W containing (X, T) and every $F \in \mathcal{E}$.

Definition (3.3)[7]: Let (X, T) be a topological space . then a subset C of a space X is said to be a regular open if $C = int\,(cl\,(C)\,)$ .The complement of regular open set is said to be regular closed set .

Theorem (3.4): Every regular closed set is $B^*c -$open set .

Proof : It is clear from their definitions (2.2) ,(3.3) .

Theorem (3.5): Let $\mathcal{E}$ be a filter base in a topological space (X, T) . if $\mathcal{E} B^*c -$converges to a point $y \in X$, then $\mathcal{E} rc -$ converges to a point y.

Proof: Suppose that $\mathcal{E} B^*c -$ converges to a point $y \in X$. Let W be any regular closed set containing y, then $W \in B^*co\ (X)$. since $\mathcal{E} B^*c -$converges to a point $y \in X$, there exists an $F \in \mathcal{E}$ sueh that $F \subset W$. This shows that $\mathcal{E} rc -$ converges to a point y .The converse of above theorem is not true in general .As the following example shows .

Example (3.6): Let (R, U) be a usual topology and let $\mathcal{E} = \{R, [0 - \epsilon, 0 + \epsilon] : \epsilon > 0 \in R\}$ .Then $\mathcal{E} rc -$ converges to 0 ,but $\mathcal{E}$ does not $B^*c -$converges to 0 ,because the set $(0 - \epsilon, 0 + \epsilon)$ is $B^*c -$ open containing 0, there exists no $F \in \mathcal{E}$ such that $F \subset (0 - \epsilon, 0 + \epsilon)$.

Corollary (3.7): Let $\mathcal{E}$ be a filter base in a topological space (X, T). If $\mathcal{E} B^*c -$cluster to a point $y \in X$, then $\mathcal{E} rc -$cluster to a point y .

Proof: The proof similar to theorem (3.5).

Theorem (3.8): Let $\mathcal{E}$ be a filter base in a topological space (X, T) and H is any closed set containing Y. If there exists an $F \in \mathcal{E}$ such that $F \subset H$, then $\mathcal{E} B^*c -$converges to a point $y \in X$.

Proof: Let W be any $B^*c -$ open set containing y , then for each $y \in W$, there exists a closed set H such that $y \in H \subset W$. By hypothesis , there exists an $F \in \mathcal{E}$ such that $F \subset H \subset W$ which implies that $F \subset W$. Hence $\mathcal{E} B^*c -$converges to a point $y \in X$.

Theorem (3.9): Let $\mathcal{E}$ be a filter base in a topological space (X, T) and H is any closed set containing y such that $F \cap H \neq \emptyset$ for each $F \in \mathcal{E}$, then $\mathcal{E}$ is $B^*c -$cluster to a point $y \in X$.

Proof: The proof similar to Theorem (3.8).

Definition (3.10): We say that a topological space (X,T) is B^*c- compact if for every B^*c-open cover $\{W_\alpha : \alpha \in \Lambda\}$ of X, there exists a finite subset $\Lambda_\circ$ of Λ such that $X = \cup\{W_\alpha : \alpha \in \Lambda_\circ\}$.

Theorem (3.11): If every closed cover of a space X has a finite sub cover , then X is B^*c- compact.

Proof: Let $\{W_\alpha : \alpha \in \Lambda\}$ be any B^*c- open cover of X and $y \in X$, then for each $y \in W_\alpha(y)$, $\alpha \in \Lambda$, there exists a closed set $F_\alpha(y)$ such that $y \in F_\alpha(y) \subset W_\alpha(y)$.So the family $\{F_\alpha(y) : y \in X\}$ is a cover of X by closed set, then by hypothesis ,this family has a finite sub cover such that $X = \{F_\alpha(yi) : i = 1,2,\dots,n\} \subset \{W_\alpha(yi) : i = 1,2,\dots,n\}$ .Therefore $X = \{W_\alpha(yi) : i = 1,2,\dots,n\}$.Hence X is B^*c- compact.

Theorem (3.12): Every B^*c- compact T_1- space is $\beta-$ compact.

Proof: Suppose that X is T_1 and B^*c- compact space .Let $\{W_\alpha : \alpha \in \Lambda\}$ be any $\beta-$ open cover of X. Then for every $y \in X$, there exists $\alpha(y) \in \Lambda$ such that $y \in W_\alpha(y)$. Since X is T_1, by since X is B^*c- compact, so there exists a finite subset $\Lambda_\circ$ of Λ in X such that $X = \cup\{W_\alpha : \alpha \in \Lambda_\circ\}$. Hence X is $\beta-$ compact.

Corollary (3.13): Let X be a T_1-space .Then X is B^*c- compact if and only if X is $\beta-$ compact.

Proof: From the fact every $\beta-$ compact is B^*c- compact and the above theorem we can prove this corollary.

Theorem (3.14): If a topological space (X,T) is B^*c- compact, then it is $rc-$ compact .

Proof: Let $\{W_\alpha : \alpha \in \Lambda\}$ be any regular closed cover of X. Then $\{W_\alpha : \alpha \in \Lambda\}$ is a B^*c- open cover of X .Since X is B^*c- compact, there exists a finite subset $\Lambda_\circ$ of Λ such that $X = \cup\{W_\alpha : \alpha \in \Lambda_\circ\}$. Hence X is $rc-$ compact.

Theorem (3.15): Let a topological space (X,T) be a regular . If X is B^*c- compact , then it is compact.

Proof: Let $\{W_\alpha : \alpha \in \Lambda\}$ be any open cover of X .By theorem (2.9), $\{W_\alpha : \alpha \in \Lambda\}$ forms a B^*c- open cover of X. Since X is B^*c- compact, there exists a finite subset $\Lambda_\circ$ of Λ such that $X = \cup\{W_\alpha : \alpha \in \Lambda_\circ\}$. Hence X is compact .

Theorem (3.16): Let X be an almost regular space .If X is B^*c- compact , then it is nearly compact .

Proof: Let $\{W_\alpha : \alpha \in \Lambda\}$ be any regular open cover of X. Since X is almost regular space , then for each $y \in X$ and regular open $W_\alpha(y)$, there exists an open set G_y such that $y \in G_y \subseteq CL(G_y) \subseteq W_\alpha(y)$. But $CL(G_y)$ is regular closed for each $y \in X$, this implies that the family $\{CL(G_y) : y \in X\}$ is B^*c- open cover of X, since X is B^*c- compact , then there exists a subfamily $\{CL(G(yi)) : i = 1,2,\dots,n\}$ such that $X = \cup_{i=1}^{n} CL(G(yi)) \subseteq \cup_{i=1}^{n} W_\alpha(yi)$. Thus X is nearly compact.

REFERENCES

[1] Levine N, (1963), Semi open sets and semi continuous mapping in topological spaces. Amr Math Monthly, 70:36-41
[2]Mashhour AS, Abd El- Monsef ME, El-deeb SN., (1982), On pre continuous and weak pre continuous mapping. Proc Math and phys Soc., Egypt, 53:47-53.
[3] Abd El – Monsef M. E., El – Deeb S. N. Mahmoud R. A. , (1983), "β – open sets and β – continuous function" Bulletin of the Faculte of science, Assiut university, 12, p.p. 77– 90.
[4] Andijevic D, (1986), Semi pre Open Sets. Math. Vesink. 38:24 – 32.
[5] Karim. F. Radhi, (2018), On Minimal and Maximal B^*c – open sets , Msc. Thesis of Al-Qadisiyah university,3762,p.p.5-10.
[6] Willard. S, (1970), General Topology. Addison - wesley Inc. ,Mass .
[7] M. H. Stone, (1937), "Applications of the theory of Boolean Rings to topology" Transactions of the American Mathematical Society, Vol. 41, No. 3, pp.375-481.