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Some Properties of Cartesian product of Two Fuzzy Modular Spaces

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ABSTRACT: In this paper , we define the concept of the Cartesian product of two fuzzy modular spaces and prove some results related with it. Finally, we prove completeness of the Cartesian product of two complete fuzzy modular spaces.

KEYWORDS: modular space , Cartesian product , Cauchy sequence , Complete fuzzy modular space .

I.INTRODUCTION

The concept of fuzzy sets was introduction by Zadeh [6] in 1965 and there after several author applied it to different branches of pure and applied mathematics . The definition of fuzzy modular space was introduction by Young Shen and Wei Chen [5] in 2013 . We use this definition to prove that the Cartesian product of two fuzzy modular spaces is also fuzzy modular space.
In this effort, we introduce the Cartesian product of two fuzzy modular spaces and some theorem related with it.

II. MAIN RESULTS

Definition (1.1):[6]
A fuzzy set B in X (or a fuzzy subset in X) is a function from X to$\mathrm{I}$, i.e., $B \in \mathrm{I}^X$.
Definition(1.2): [5]
A fuzzy modular space is an ordered triple $(X, \mu, *)$ such that X is a vectorspace , $*$ is continuous t- norm and μ is a fuzzy set on $X \times (0, \infty)$satisfying the following condition , for all $x, y \in X$ and $\alpha , \beta \geq 0$ $with \alpha + \beta = 1$:
1. $\mu(x, t) > 0$.
2. $\mu(x, t) = 1$ for all$t > 0$ if and only if $x = 0$.
3. $\mu(x, t) = \mu(-x, t)$.
4. $\mu(\alpha x + \beta y, s + t) \geq \mu(x, s) * \mu(y, t)$.
5. $\mu(x, .) : (0, \infty) \to (0,1]$ is continuous.

Definition(1.3):
Let $(X, \mu_1, *)$ and $(Y, \mu_2, *)$ be two fuzzy modular spaces . the Cartesian product of $(X, M, *)$ and $(Y, M, *)$ is the product space $(X \times Y, M, *)$ where $X \times Y$ is the Cartesian product of the sets X and Y and μ is a function
$\mu: (X \times (0, \infty)) \times (Y \times (0, \infty)) \to [0,1]$ is given by :
$\mu((x, y), s + t) = \mu_1(x, s) * \mu_2(y, t)$ for all $(x, y) \in X \times Y$ and $t, s > 0$.
Theorem (1.4):
Let $(X, \mu_1, *)$ and $(Y, \mu_2, *)$ be two fuzzy modular spaces. Then $(X \times Y, \mu, *)$ is a fuzzy modularspace.
Proof :
Let $(x, y) \in X \times Y$, we have
(1) since $\mu_1(x, s) > 0$ $\forall s > 0$ and $\mu_2(y, t) > 0$ $\forall t > 0$, then
$\mu((x, y), s + t) = \mu_1(x, s) * \mu_2(y, t) > 0$.
(2)$\mu_1(x, s) = 1 \Leftrightarrow x = 0$, also $\mu_2(y, t) = 1 \Leftrightarrow y$. Together$\mu_1(x, s) * \mu_2(y, t) = 1$
$\Leftrightarrow (x, y) = 0$. Hence $\mu((x, y), s + t) = 1$ $\forall s, t > 0 \Leftrightarrow (x, y) = 0$.
(3)since $\mu_1(x, s) = \mu_1(-x, s)$ $\forall s > 0$ and $\mu_2(y, t) = \mu_2(-y, t)$ $\forall t > 0$, then
$\mu((x, y), s + t) = \mu_1(x, s) * \mu_2(y, t)$
$= \mu_1(-x, s) * \mu_2(-y, t) = \mu(-(x, y), s + t)$.

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(4) $\mu(\alpha(x_1,y_1)+\beta(x_2,y_2),s+t) \geq \mu((\alpha x_1+\beta x_2,\alpha y_1+\beta y_2),s+t)$
$\geq \mu_1(\alpha x_1+\beta x_2,s+t) * \mu_2(\alpha y_1+\beta y_2,s+t)$
$\geq \mu_1(x_1,s) * \mu_2(x_2,t) * \mu_2(y_1,s) * \mu_2(y_2,t)$
$\geq \mu_1(x_1,s) * \mu_2(y_1,s) * \mu_1(x_2,s) * \mu_2(y_2,t)$
$\geq \mu((x_1,y_1),s) * \mu((x_2,y_2),t)$
(5)since $\mu_1(x,s):(0,\infty) \to (0,1]$ is continuous and $\mu_2(y,t):(0,\infty) \to (0,1]$ is continuous
then $\mu((x,y),s+t):(0,\infty) \to (0,1]$ is continuous.

Theorem (1.5):

Let $\{x_n\}$ be a sequence in a fuzzy modular space $(X,\mu_1,*)$ converge to x in X and $\{y_n\}$ is a sequence in the fuzzy modular space $(Y,\mu_2,*)$ converge to y in Y. then $\{(x_n,y_n)\}$ is a sequence in a fuzzy modular space $(X\times Y,\mu,*)$ converge to (x,y) in $X\times Y$.

Proof:To prove that sequence $\{(x_n,y_n)\}$ in $X\times Y$ Converges to (x,y)

We show that $\lim_{n\to\infty} \mu((x_n,y_n)-(x,y),s+t)=1$

By Theorem (1.4) $(X\times Y,\mu,*)$ is fuzzy modular space

Since $\{x_n\}$ be a sequence in $(X,\mu_1,*)$ Convergence to x

Then $\lim_{n\to\infty} \mu_1(x_n-x,s)=1$

Since $\{y_n\}$ be a sequence in $(X,\mu_2,*)$ Convergence to y

Then $\lim_{n\to\infty} \mu_2(y_n-y,s)=1$

Then that $\lim_{n\to\infty} \mu((x_n,y_n)-(x,y),s+t)=\lim_{n\to\infty} \mu_1(x_n-x,s) * \lim_{n\to\infty} \mu_2(y_n-y,t)=1*1=1$

Thus $\{(x_n,y_n)\}$ Converges to (x,y).

Theorem (1.6) :
If $\{x_n\}$ be a Cauchy sequence in a fuzzy modular space $(X,\mu_1,*)$and $\{y_n\}$ is a Cauchy sequence in a fuzzy modular space $(Y,\mu_2,*)$then $\{(x_n,y_n)\}$ is a Cauchy in a fuzzy modular space $(X\times Y,\mu,*)$.

Proof :
By theorem (1.4)$(X\times Y,\mu\ *)$ is a fuzzy modular space .
Since $\{x_n\}$ be a Cauchy sequence in a fuzzy modular space $(X,\mu_1,*)$
Then $\lim_{n,m\to\infty} \mu_1(x_n-x_m,s)=1$
Since and $\{y_n\}$ is a Cauchy sequence in a fuzzy modular space $(Y,\mu_2,*)$
Then $\lim_{n,m\to\infty} \mu_2(y_n-y_m,t)=1$
Then$\lim_{n\to\infty} \mu((x_n,y_n)-(x_m,y_m),s+t)=[\lim_{n,m\to\infty} \mu_1(x_n-x_m,t)] * [\lim_{n,m\to\infty} \mu_2(y_n-y_m,s)=$
$1*1=1$
Thus $\{(x_n,y_n)\}$ is a Cauchy sequence in $(X\times Y,\mu,*)$.

Definition (1.7) :
A fuzzy modular space$(X,\mu,*)$is called Complete if every Cauchy sequence is convergent in $(X,\mu,*)$.

Theorem (1.8) :
If $(X,\mu_1,*)$ and $(Y,\mu_2,*)$ are Complete fuzzy modular space then the product $(X\times Y,\mu,*)$ is Complete fuzzy modular space .

Proof :
Let $\{(x_n,y_n)\}$ be a Cauchy sequence in $X\times Y$
Since $(X,\mu_1,*)$ and $(Y,\mu_2,*)$ are Complete fuzzy modular space
Then $\exists\ x$ in X and y in Y such that $\{x_n\}$ Convergent to x and $\{y_n\}$ Convergent to y
So $\lim_{n,m\to\infty} \mu_1(x_n-x,s)=1$ and $\lim_{n,m\to\infty} \mu_1(y_n-y,t)=1$

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Now Then $\lim_{n\to\infty} \mu\,((x_n\,,y_n) - (x,y), s+t) =$
$\left[\lim_{n,m\to\infty} \mu_1(x_n - x\,, s)\right] * [\lim_{n,m\to\infty} \mu_2(y_n - y\,,t) = 1 * 1 = 1$
Then $\{(x_n, y_n)\}$ Convergent to (x,y) in $X \times Y$.

Theorem (1.9):
If $(X \times Y, \mu, *)$ is a fuzzy modular space, then $(X, \mu_1, *)$ and $(Y, \mu_2, *)$ are fuzzy modular spaces by defining
$\mu_1(x,t) = \mu\big((x,0),t\big)$
and $\mu_2(y,t) = \mu\big((0,y),t\big)$ for all $x \in X$, y $\in Y$and $t > 0$.

Proof :

(1)$\mu_1(x,t) = \mu\big((x,0),t\big) > 0 \quad \forall x \in X$.
(2)For all $t > 0, 1 = \mu_1(x,t) = \mu\big((x,0),t\big) \Leftrightarrow x = 0$.
(3)For all $t > 0, \mu_1(x,t) = \mu_1(-x,t) = \mu(-(x,0),t)$.
(4)$\mu_1(\alpha x + \beta y, s+t) = \mu\big((\alpha x + \beta y, 0), s+t\big)$
$\geq \mu\big((x,0),s\big) * \mu\big((y,0),t\big) \geq \mu_1(x,s) * \mu_1(y,t)$.
(5)$\mu_1(x,.) = \mu\big((x,0),.\big)$ is continuous from $(0,\infty)$ to $(0,1]$for all $x \in X$. Then $(X, \mu_1, *)$ is fuzzy modular space
Similarly we can prove that $(Y, \mu_2, *)$.

Theorem (1.10):
If $(X \times Y, \mu, *)$ be a Complete fuzzy modular space , then $(X, \mu_1, *)$ and $(Y, \mu_2, *)$ are Complete fuzzy modular space .

Proof :
$(X, \mu_1, *)$ and $(Y, \mu_2, *)$ are fuzzy modular spaces by Theorem (1.9)
Let $\{x_n\}$ be a Cauchy sequence in $(X, \mu_1, *)$
Then $\{(x_n, 0)\}$ be a Cauchy sequence in $X \times Y$
But $X \times Y$ is complete fuzzy modular space
Then there is $(x,0)$ in $X \times Y$ such that $\{(x_n, 0)\}$ Convergent to $(x,0)$
Now , $\lim_{n\to\infty} \mu_1(x_n - x, t) = \lim_{n\to\infty} \mu((x_n - x, 0), t) = 1$
Then $(X, \mu_1, *)$ is Complete fuzzy modular space
Similarly we can prove that $(Y, \mu_2, *)$ is Complete fuzzy modular space .

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