ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 5 , May 2019

Modeling Pressure with Galleries Wells for Stationary Filtration Problems

GayratJuraev, ZakirTulaganov, ObidjonBozorov and EvgeniyShilnikov

National University of Uzbekistan named after MirzoUlugbek,City Tashkent, Uzbekistan
National University of Uzbekistan named after MirzoUlugbek,City Tashkent, Uzbekistan
National University of Uzbekistan named after MirzoUlugbek,City Tashkent, Uzbekistan
Keldysh Institute of Applied Mathematics RAS, City Moscow, Russia

ABSTRACT: This paper presents an analytical method for solving the filtration of a fluid in a semi-bounded porous medium in the presence of well galleries. Theressure distribution in stationary filtration is defined as combinations of elementary functions.

KEY WORDS: porous medium, galleries of chinks of vertical drainage, stationary filtration, pressure, pressure flow distributions.

I.INTRODUCTION

Calculation of complicated groundwater flow in the presence of various types of drainage facilities remains at present one of the important problems [3,4,7,8]. Such calculations have to be made in the tasks of migration of moisture and pollution, when predicting the hydrochemical regime of soil and groundwater, during irrigation, land drainage, development and operation of gas and oil fields, in assessing water reserves of pressure horizons, etc.

As practice shows, the intensive development of irrigated agriculture without proper substantiation and analysis of the interrelation of diverse natural conditions leads, as a rule, to both a change in the water-salt balance of soils and the environment as a whole [2,3,6,8].

Example for pressure filtrations flows is can serve as artesian groundwater, which when drilling wells give a fountain. An example of free-flow flows is the movement of water along the open channel of a river.

The purpose of this work is to obtain effective approximate analytical solutions for the evaluation of the pressure in a semi-bounded porous medium.

In this paper, planned pressure filtration in a semi-bounded environment is considered, when a well-permeable horizon is opened by a system of vertical drainage well galleries with perfect filters (Fig1). At given costs on the galleries of wells a constant pressure is provided at the point of their intersections (x_1, y_1). The considered horizon with a pressure $h(x,y)$ and filtration conductivity T is hydraulically connected through a low-permeable layer having a thickness m_n and filtration coefficient K_n with the underlying layer, the pressure in which is equal to $H = const$. Under the following assumptions, it is possible to express the function of reducing the pressure in elementary functions.

Here we consider the above problem with different boundary conditions.

As is known, in real situations, the pressures, velocities and costs in the study of groundwater flows, in general, filtration flows depend on four variables: spatial coordinates x, y, z and time t . However, the size of aquifers in area far exceeds their thickness (depth of the aquifer or reservoir).For this reason, for hydrogeological calculations it is sufficient to consider them in a flat plane. Then the above physical parameters are defined as functions of two spatial coordinates [2,3,7,8].

To achieve this goal, when modeling pressures in a semi-bounded porous medium with well galleries in both permeable and impermeable boundaries, a two-dimensional elliptical equation was used, subject to the general

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 5 , May 2019

conditions for both cases. Using the Fourier cosine transform with finite limits and the Fourier sine transform with infinite limits, we can obtain analytical solutions for determining pressures [1,6].

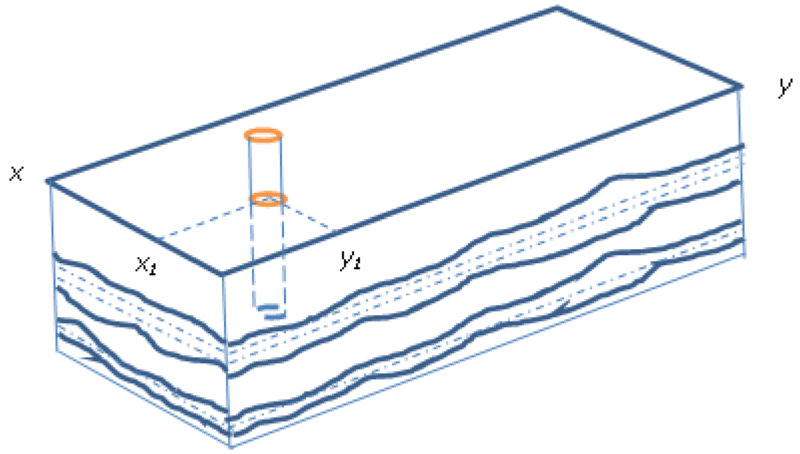

Fig1.

II. Modeling pressure in a semi-bounded porous medium with well galleries for permeable boundaries with $x = 0$, $x = l$ and $y = 0$

A problem with permeable boundaries is considered. In the pressure well-permeable layer there are galleries of vertical drainage wells with perfect filters in this horizon (Fig2). A constant level is maintained at the intersection points of these galleries.

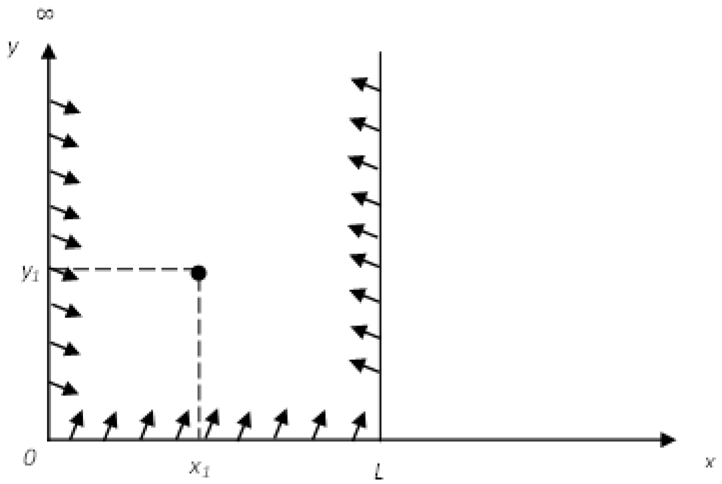

Fig2.

It is required to solve a two-dimensional elliptic equation

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{K_n}{m_n T}(H-h)=0. \qquad (0\le x\le L,\ \ 0\le y<\infty) \qquad (1)$$

satisfying condition

$$\left.\frac{\partial h}{\partial x}\right|_{x=0}=\left.\frac{\partial h}{\partial x}\right|_{x=L}=0, \qquad \lim_{y\to\infty}|h(x,y)|\le M=const, \qquad (2)$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 5 , May 2019

$$\left.\frac{\partial h}{\partial x}\right|_{x_1+0} - \left.\frac{\partial h}{\partial x}\right|_{x_1-0} = \frac{q_1(y)}{T}, \qquad \left.\frac{\partial h}{\partial y}\right|_{y_1+0} - \left.\frac{\partial h}{\partial y}\right|_{y_1-0} = \frac{q_2(x)}{T}, \tag{3}$$

$$\left.\frac{\partial h}{\partial y}\right|_{y=0} = 0. \tag{4}$$

Where

$$q_1(y) = BT\, sh(\omega L) \begin{cases} ch(\omega y) exp(-\omega y_1), \; 0 \le y \le y_1 \\ ch(\omega y_1) exp(-\omega y), y_1 \le y < \infty. \end{cases}$$

$$q_2(x) = BT \begin{cases} ch(\omega x) ch[\omega(L - x_1)], \; 0 \le x \le x_1 \\ ch(\omega x_1) ch[\omega(L - x)], x_1 \le x \le L. \end{cases}$$

$$B = \frac{\omega G(x_1, y_1)}{ch(\omega x_1) ch[\omega(L - x_1)] ch(\omega y_1) exp(-\omega y_1)},$$

$$G(x_1, y_1) = h(x_1, y_1) - H$$

$$c = \frac{K_n}{m_n T}, \qquad \omega = \sqrt{\frac{c}{2}}.$$

Here, the functions $q_1(y)$ and $q_2(x)$ are chosen so that a significant part of the flow falls on the well placed at the intersection point of the galleries (x_1, y_1) [1].

We introduce the function $U(x, y) = -G(x,y)$ then equation (1) and conditions (2) - (4) will be rewritten as

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - cU = 0 \qquad (0 \le x \le L, \; 0 \le y < \infty). \tag{5}$$

$$\left.\frac{\partial U}{\partial x}\right|_{x=0} = \left.\frac{\partial U}{\partial x}\right|_{x=L} = 0, \qquad \lim_{y \to \infty} |U(x, y)| \le M_1 = const, \tag{6}$$

$$\left.\frac{\partial U}{\partial x}\right|_{x_1+0} - \left.\frac{\partial U}{\partial x}\right|_{x_1-0} = \frac{q_1(y)}{T}, \qquad \left.\frac{\partial U}{\partial y}\right|_{y_1+0} - \left.\frac{\partial U}{\partial y}\right|_{y_1-0} = \frac{q_2(x)}{T}, \tag{7}$$

$$\left.\frac{\partial U}{\partial y}\right|_{y=0} = 0. \tag{8}$$

Applying to (5), (6) a Fourier cosine transform with finite limits

$$\bar{U}_c(n, y) = \int_0^L U(x, y) \cos\left(\frac{n\pi x}{L}\right) dx$$

and (5), (8) Fourier cosine transform with infinite limits

$$\bar{\bar{U}}_c(n, \xi) = \sqrt{2/\pi} \int_0^\infty \bar{U}_c(n, y) \cos(\xi y) dy$$

wewillreceive:

$$\bar{\bar{U}}_c(n, \xi) = \frac{\sqrt{2/\pi}\, \omega B sh(\omega L) \cos(n\pi x_1/L) \cos(\xi y_1)}{(\omega^2 + \xi^2)[\omega^2 + (n\pi/L)^2]}. \tag{9}$$

In the transition to the original in (9), the inverse cosine Fourier transform with infinite and finite limits was used:

$$\bar{U}_c(n, y) = \sqrt{2/\pi} \int_0^\infty \bar{\bar{U}}_c(n, \xi) \cos(\xi y) d\xi,$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 5 , May 2019

$$U(x,y)=\frac{1}{L}\bar{U}_c(0,y)+\frac{2}{L}\sum_{n=1}^{\infty}\bar{U}_c(n,y)cos\left(\frac{n\pi x}{L}\right)$$

where

$$\bar{U}_c(n,y)=\int_0^L U(n,y)\cos\left(\frac{n\pi x}{L}\right)dx$$

The values of the following sums of series and the value of improper integrals are taken from [5]:

$$\sum_{n=1}^{\infty}\frac{\cos\left(\frac{nx\pi}{L}\right)\cos\left(\frac{nx_1\pi}{L}\right)}{\omega^2+\left(\frac{nx\pi}{L}\right)^2}=-\frac{1}{2\omega^2}+$$

$$+\frac{L}{2\omega sh(\omega L)}\begin{cases}ch(\omega x)ch[\omega(L-x_1)], & 0\le x\le x_1\\ ch(\omega x_1)ch[\omega(L-x)], & x_1\le x\le L\end{cases},$$

$$\int_0^{\infty}\frac{\cos(y_1x)\cos(yx)}{\omega^2+x^2}dx=\frac{\pi}{2\omega}\begin{cases}\exp(-y_1\omega)\,ch(y\omega), & 0\le y\le y_1\\ \exp(-y\omega)\,ch(y_1\omega), & y_1\le y<\infty\end{cases}.$$

Passing from the function *U* to the function *h*, we finally obtain the solution of problem (1)-(4) in the form

$$h(x,y)=H+[h(x_1,y_1)-H]\times$$

$$\times\begin{cases}\dfrac{ch(\omega x)ch(\omega y)}{ch(\omega x_1)ch(\omega y_1)}, & 0\le x\le x_1, \quad 0\le y\le y_1;\\ \dfrac{ch(\omega x)exp(-\omega y)}{ch(\omega x_1)exp(-\omega y_1)}, & 0\le x\le x_1, \quad y_1\le y<\infty;\\ \dfrac{ch[\omega(L-x)]ch(\omega y)}{ch[\omega(L-x_1)]ch(\omega y_1)}, & x_1\le x\le L, \quad 0\le y\le y_1;\\ \dfrac{ch[\omega(L-x)]exp(-\omega y)}{ch[\omega(L-x_1)]exp(-\omega y_1)}, & 0\le x\le x_1, \quad y_1\le y<\infty.\end{cases} \qquad (10)$$

The task is easily generalized to the case of *N* galleries of wells whose lines of action are parallel to the boundaries of the area:

$$h(x,y)=H+\sum_{i=1}^{N}[h(x_i,y_i)-H]\times$$

$$\times\begin{cases}\dfrac{ch(\omega_i x)ch(\omega_i y)}{ch(\omega_i x_i)ch(\omega_i y_i)}, & 0\le x\le x_i, \quad 0\le y\le y_i;\\ \dfrac{ch(\omega_i x)exp(-\omega_i y)}{ch(\omega_i x_i)exp(-\omega_i y_i)}, & 0\le x\le x_i, \quad y_i\le y<\infty;\\ \dfrac{ch[\omega_i(L-x)]ch(\omega_i y)}{ch[\omega_i(L-x_i)]ch(\omega_i y_i)}, & x_i\le x\le L, \quad 0\le y\le y_i;\\ \dfrac{ch[\omega_i(L-x)]exp(-\omega_i y)}{ch[\omega_i(L-x_i)]exp(-\omega_i y_i)}, & 0\le x\le x_i, \quad y_i\le y<\infty.\end{cases} \qquad (11)$$

Figure 3 for two wells show the distributions of pressures $h(x,y)$ from different sides, calculated using the obtained calculation formulas (11) with the following parameters:

L=100 m; $h1$=35;x_1=60 m; y_1=50 m; $h(x_1,y_1,)$=5; c_1=10^{-2}; $\omega_1=\sqrt{c_1/2}$;
x_2=60 m; y_2=100 m; $h(x_2,y_2,)$=10; c_2=10^{-3}; $\omega_2=\sqrt{c_2/2}$.

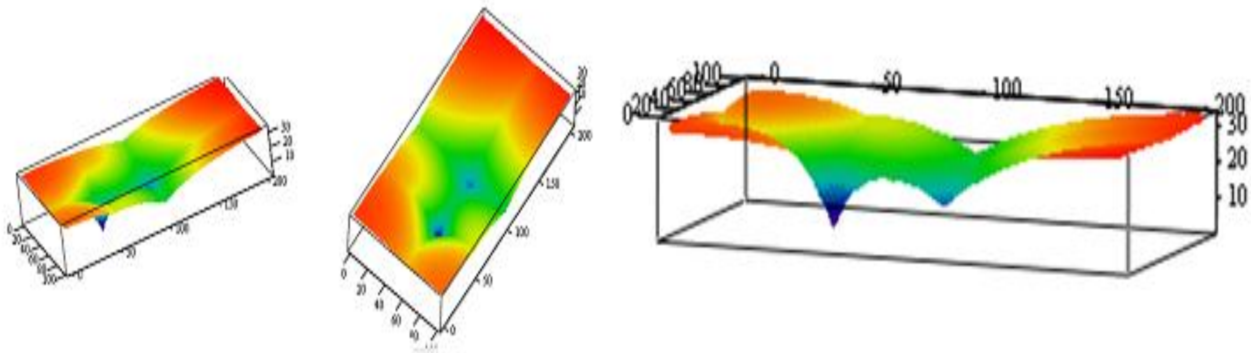

Fig3.

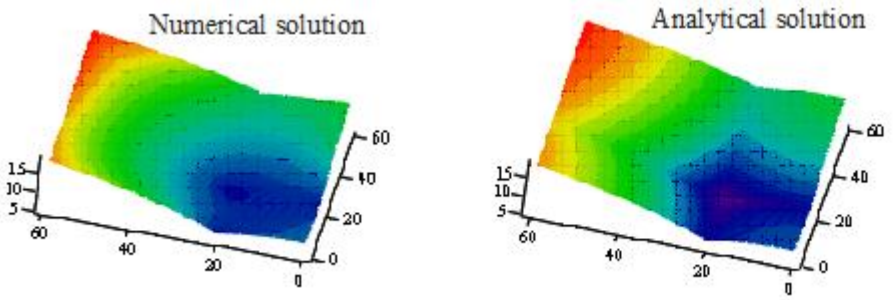

Fig4.

In fig. 4.for one well are given the surfaces of the pressure h (x, y) calculated by the obtained calculation formulas (10) and using the numerical method, where the following parameter values are used:

L = 60 m; $h1$ = 25;x_1 = 20 m; y_1 = 20 m; $h(x_1, y_1,)$=5; $c_1 = 10^{-3}$; $\omega_1 = \sqrt{c_1/2}$.

III. SIMULATION OF PRESSURES IN A SEMI-BOUNDED POROUS MEDIUM WITH WELL GALLERIES FOR PERMEABLE BOUNDARIES WITHx = 0 AND $x = L$, AS WELL AS AN IMPERMEABLE BOUNDARY WITH y = 0

Here we consider a problem that has both permeable and impenetrable boundaries (Fig5). In the pressure well-permeable layer there are galleries of vertical drainage wells with perfect filters in this horizon. At the intersection points of these galleries a constant level is maintained.

It is required to solve a two-dimensional elliptic equation

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{K_n}{m_n T}(H-h)=0 \qquad (0\le x\le L,\ \ 0\le y<\infty) \qquad (12)$$

satisfying condition

$$\left.\frac{\partial h}{\partial x}\right|_{x=0}=\left.\frac{\partial h}{\partial x}\right|_{x=L}=0, \qquad \lim_{y\to\infty}|h(x,y)|\le M=const, \qquad (13)$$

$$\left.\frac{\partial h}{\partial x}\right|_{x_1+0}-\left.\frac{\partial h}{\partial x}\right|_{x_1-0}=\frac{q_1(y)}{T}, \qquad \left.\frac{\partial h}{\partial y}\right|_{y_1+0}-\left.\frac{\partial h}{\partial y}\right|_{y_1-0}=\frac{q_2(x)}{T}, \qquad (14)$$

$$h(x,o)=h_1, \qquad (15)$$

where

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 5 , May 2019

$$q_1(y) = BT\, sh(\omega L) \begin{cases} sh(\omega y)exp(-\omega y_1), \; 0 \le y \le y_1 \\ sh(\omega y_1)exp(-\omega y), y_1 \le y < \infty. \end{cases}$$

$$q_2(x) = BT \begin{cases} ch(\omega x)ch[\omega(L - x_1)], \; 0 \le x \le x_1 \\ ch(\omega x_1)ch[\omega(L - x)], x_1 \le x \le L. \end{cases}$$

$$B = \frac{\omega G(x_1, y_1)}{ch(\omega x_1)ch[\omega(L - x_1)]sh(\omega y_1)exp(-\omega y_1)},$$

$$G(x_1, y_1) = h(x_1, y_1) - H - (h_1 - H)\exp(-\sqrt{c}y_1),$$

$$c = \frac{K_n}{m_n T}, \qquad \omega = \sqrt{\frac{c}{2}}, \qquad h_1 = const.$$

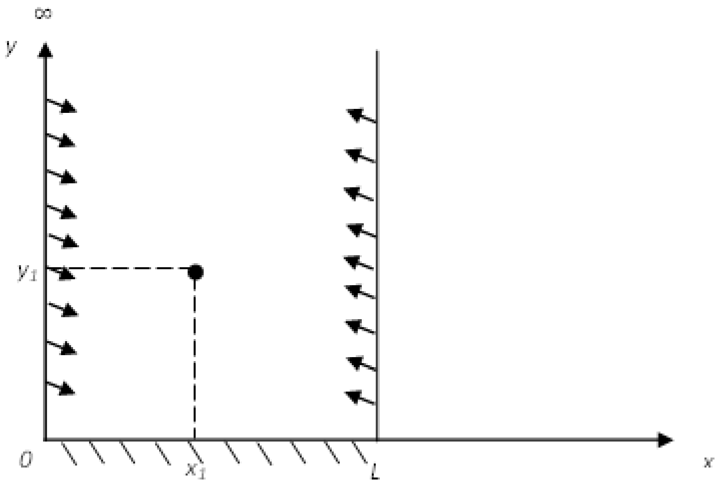

Fig5.

Here, the functions $q_1(y)$ and $q_2(x)$ are chosen so that a significant part of the flow falls on the well placed at the intersection point of the galleries (x_1, y_1) [1].

We introduce the function $U(x, y) = -G(x,y)$ then equation (12) and conditions (13)-(15) will be rewritten as

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - cU = 0. \qquad (0 \le x \le L, \; 0 \le y < \infty) \tag{16}$$

$$\left.\frac{\partial U}{\partial x}\right|_{x=0} = \left.\frac{\partial U}{\partial x}\right|_{x=L} = 0, \qquad \lim_{y\to\infty} |U(x, y)| \le M_1 = const, \tag{17}$$

$$\left.\frac{\partial U}{\partial x}\right|_{x_1+0} - \left.\frac{\partial U}{\partial x}\right|_{x_1-0} = \frac{q_1(y)}{T}, \qquad \left.\frac{\partial U}{\partial y}\right|_{y_1+0} - \left.\frac{\partial U}{\partial y}\right|_{y_1-0} = \frac{q_2(x)}{T}, \tag{18}$$

$$U(x, 0) = 0. \tag{19}$$

Applying to (16), (17) a Fourier cosine transform with finite limits

$$\overline{U}_c(n, y) = \int_0^L U(x, y)\cos\left(\frac{n\pi x}{L}\right) dx$$

and (16), (19) Fourier sine transform with infinite bounds

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 5 , May 2019

$$\bar{\bar{U}}_s(n,\xi) = \sqrt{2/\pi}\int_0^\infty \bar{U}_c(n,y)\sin(\xi y)dy$$

we will receive:

$$\bar{\bar{U}}_s(n,\xi) = \frac{\sqrt{2/\pi}\,\omega B sh(\omega L)\cos(n\pi x_1/L)\sin(\xi y_1)}{(\omega^2+\xi^2)[\omega^2+(n\pi/L)^2]}. \quad (20)$$

In the transition to the original in (20), the inverse sine transform of Fourier with infinite and cosine transform with finite limits was used:

$$\bar{U}_c(n,y) = \sqrt{2/\pi}\int_0^\infty \bar{\bar{U}}_s(n,\xi)\sin(\xi y)d\xi$$

$$U(x,y) = \frac{1}{L}\bar{U}_c(0,y) + \frac{2}{L}\sum_{n=1}^{\infty}\bar{U}_c(n,y)cos\left(\frac{n\pi x}{L}\right)$$

where

$$\bar{U}_c(n,y) = \int_0^L U(n,y)\cos\left(\frac{n\pi x}{L}\right)dx$$

and the values of the following sums of series and the value of improper integrals are taken from [5]:

$$\sum_{n=1}^{\infty}\frac{\cos\left(\frac{nx\pi}{L}\right)\cos\left(\frac{nx_1\pi}{L}\right)}{\omega^2+\left(\frac{nx\pi}{L}\right)^2} = -\frac{1}{2\omega^2}+$$

$$+\frac{L}{2\omega sh(\omega L)}\begin{cases} ch(\omega x)ch[\omega(L-x_1)], & 0\le x\le x_1 \\ ch(\omega x_1)ch[\omega(L-x)], & x_1\le x\le L \end{cases},$$

$$\int_0^\infty \frac{\sin(y_1 x)\sin(yx)}{\omega^2+x^2}dx = \frac{\pi}{2\omega}\begin{cases} \exp(-y_1\omega)\,sh(y\omega), & 0\le y\le y_1 \\ \exp(-y\omega)\,sh(y_1\omega), & y_1\le y<\infty \end{cases}.$$

Passing from the function U to the function h, we finally get the solution of the problem (16) - (19) in the form

$$h(x,y) = H + (h_1-H)\exp\left(-\sqrt{c}y\right) + \left[h(x_1,y_1) - H - (h_1-H)\exp(-\sqrt{c}y_1)\right]\times$$

$$\times\begin{cases} \frac{ch(\omega x)sh(\omega y)}{ch(\omega x_1)sh(\omega y_1)}, & 0\le x\le x_1, \quad 0\le y\le y_1; \\ \frac{ch(\omega x)exp(-\omega y)}{ch(\omega x_1)exp(-\omega y_1)}, & 0\le x\le x_1, \quad y_1\le y<\infty; \\ \frac{ch[\omega(L-x)]sh(\omega y)}{ch[\omega(L-x_1)]sh(\omega y_1)}, & x_1\le x\le L, \quad 0\le y\le y_1; \\ \frac{ch[\omega(L-x)]exp(-\omega y)}{ch[\omega(L-x_1)]exp(-\omega y_1)}, & 0\le x\le x_1, \quad y_1\le y<\infty. \end{cases} \quad (21)$$

In fig. 6 shows the surface of the pressure h (x, y), calculated by the obtained calculation formulas (21), where the following parameter values are used:

L=100 m; h1=35;x_1=25 m; y_1=40 m; $h(x_1,y_1)$=55; $c_1=10^{-2}$; $\omega_1 = \sqrt{c_1/2}$.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 5 , May 2019

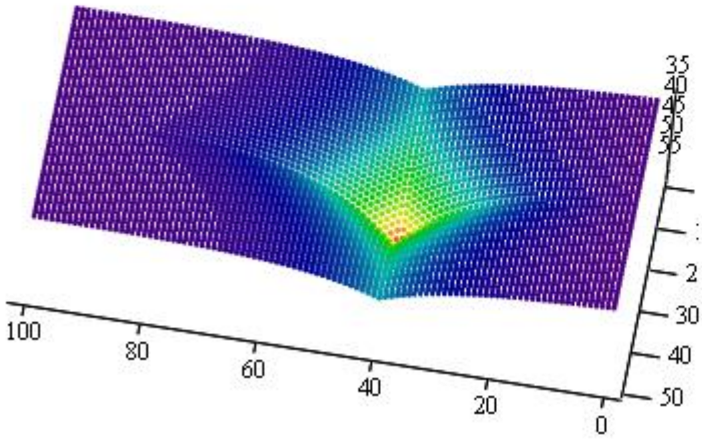

Fig6.

IV.CONCLUSION

As a result of the study, mathematical models of pressure filtration in a semi-bounded porous medium are proposed. Under certain conditions, efficient approximate analytical solutions have been built to determine the pressure in a semi-bounded porous medium. The results of the decision, as can be seen from the calculations carried out, agree quite well with the qualitative data of the process.

REFERENCES

[1] Baklushin M., Juraev G.U. The gas production process in multi-layer porous media, Matem. Mod.,2001, Volume 13, №9, 55–62.
[2] Bochever FM, Garmonov I.V., Lebedev A.V., Shestakov V.M. Basics of hydrogeological calculations. - M., Nedra, 1965. 309 p.
[3] Brekhuntsov A., Telkov A., Fedortsov V. The development of the theory of filtration of liquid and gas to horizontal wells. -Tyumen: SibNATS OJSC, 2004. - 290 pp .: 75 ill.
[4] Chetverushkin B.N. On the question of the lower bound on the scale in continuum mechanics / Time, chaos, mathematical problems, Vol.4. - M .: Publishing House of Moscow State University, 2009, p. 75-96.
[5] Gradstein I.S., Ryzhik I.M. Tables of integrals, sums of series and products. - M .: Gosizdat Phys.-Mat. Literature, 1962, -1100 p.
[6] Juraev G.U., Tulaganov Z.Sh., Tursunova Sh.H. Lowering the groundwater level to prevent soil salinization / Proceedings of the international conference "Mathematical analysis and its application in mathematical physics." September 17-20, 2018, Samarkand. Part 2 P.126.
[7] OleynikA.Ya. Geo-hydrodynamics of drainage. Kiev: NaukovaDumka, 1981, 281 p.
[8] Polubarinova-KochinaP.Ya. Theory of groundwater movement.- M .: Science, 1977, 664 p.