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# A New Similarity Measure of Pythagorean Fuzzy Set with Pythagorean Right Triangle Base according to The Centroid Point

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**ABSTRACT:** The aim of this paper is to extend the centroid point to survey the similarity measure between PFS in the case of transforms the Pythagorean right triangle. Method is according to Pythagorean fuzzy set (PFS) has a robust utility ability than an intuitionistic fuzzy set (IFS) to modify uncertainty in real-world problems. That launching a novel new similarity measure  $S_L^{PC}$  to numerical experiments and compare the candidate ranking based on the centroid point to measure the similarity between the PFS, which transforms the Pythagorean right triangle. Results verify the similarity measure  $S_L^{PC}$  gets a higher priority of the candidate ranking and ordering number 1. Conclusion proved the new similarity measure  $S_L^{PC}$  is usefulness and effectiveness.

**KEYWORDS :** the centroid point, Pythagorean fuzzy set, PFS, Intuitionistic fuzzy set, IFS.

## I. INTRODUCTION

A similarity measure is an instrument for survey the degree of similarity between two things. many researchers discuss applied in any applied in a wide variety of fields. Zadeh (1965) present a fuzzy set to solving uncertainly problem for real-life, and applied in all different scope of decision making. Atanassov (1986) defined intuitionistic fuzzy set (IFS) and applied extension real-life at 1994ab, 1995, 1999, separately. Gau and Buehrer (1993) were posed vague sets and Bustince and Burillo (1996) dished out that the vague sets were IFSs. They have been inclusive applied by Szmidt and Kacprzyk (1996) in decision-making problems. that Yager (2013) raised the pythagorean fuzzy sets (PFSs) is new method for treat with the membership grade are pairs  $(\mu, \nu)$  fulfill the status  $\mu^2 + \nu^2 \leq 1$ . PFSs has stronger utility than the intuitionistic fuzzy sets (IFSs) to modify ambiguity problems.

Yager (2013) raised Pythagorean fuzzy set (PFS) feature used the sum of membership and non-membership to one case fulfill criteria of results may be greater than 1, but sometimes its square sum is less than or equal 1. Thereby, Yager (2013, 2014) expansion Pythagorean fuzzy set (PFS) distinguishing feature by a membership and non-membership, that represent the status that the square sum of membership is less than or equal to 1. Yager (2014) supplied one instance illustrate status: decision-maker he prefers case is  $\frac{\sqrt{3}}{2}$  and he prefers non-membership is  $\frac{1}{2}$ . perhaps results sum of two values is greater than 1, it is inappropriate for IFS, but it is appropriate for PFS since  $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 \leq 1$ . Obviously, PFS better to modify the ambiguity problems in the actual case.

Recently, Shyi-Ming Chen (2016) proposed a concept of the centroid point to measure the similarity between two things, which inspired me to interesting in learning to discuss similarity measure of the centroid point extension two PFSs. based on



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transform Pythagorean right triangle. moreover Yager (2013) has been verified the pythagorean fuzzy sets (PFSs) have robust utility than the intuitionistic fuzzy sets (IFSs) to modify ambiguity problems.

In this program employed the numerical experiment was robust verification support this  $S_L^{PC}$  model was usefulness and effectiveness. therefore, proposed  $S_L^{PC}$  is new similarity measure.

Contents introduced as following, the first section is an introduction, brief introduced fuzzy logic simple background, the second section is preliminaries, introduced the basic concepts on PFS and existing the similarity measure, the third section is employing a new measure of similarity  $S_L^{PC}$  based on PFSs with Pythagorean right triangle for the centroid point. The fourth section is a numerical experiment. the last one is the conclusion.

## II. PRELIMINARIES

Here start to introduced previous many researchers discuss theories and concepts in PFS, and their methods. briefly recall the fundamental doctrine related to Pythagorean Fuzzy Set (PFS), as follows:

### II.1 THE BASIC CONCEPTS ON PFS

The Pythagorean theorem is derived from the plane geometry of Euclidean geometry.

The five axioms (public) of Euclidean plane geometry are:

You can draw a straight line from one point to another.

Any line segment can extend infinitely into a straight line.

Given an arbitrary line segment, you can use one of its endpoints as the center of the circle, which is used as a radius.

All right angles are equal.

If both lines intersect the third line and the sum of the inner angles on the same side is less than two right angles, the two lines must intersect on this side.

The fifth axiom is called parallel axiom (parallel public), and the proposition is as follows:

"Through a point that is no longer on the line, and there is only one line that does not intersect the line."

Manjil P Saikia (2015) Pythagorean theorem is a well-known result in triangular geometry. The square of the hypotenuse is equal to the sum of the squares of the other sides.

Therefore, if  $c$  denotes a right-angled bevel triangle,  $a$  and  $b$  indicate the other two sides, then the theorem says  $c^2 = a^2 + b^2$ . The triplet  $(a, b, c)$  that satisfies the theorem is called a Pythagorean triad.

Definition 1 Yager (2014)

$$p = \{(x, u_p(x), v_p(x)) | x \in X\}.$$

Definition 2 Zhang and Xu (2014)

$$s(x) = \mu^2 - v^2.$$

Where  $(x) \in [-1, 1]$ .



Definition 3 Zhang and Xu (2014)

$$a(x) = \mu^2 - v^2$$

Where  $a(x) \in [0,1]$

For any two PFSs  $x_1, x_2$ .

If  $s(x_1) > s(x_2)$ , then  $x_1 \succ x_2$ ;

If  $s(x_1) = s(x_2)$ , then

If  $a(x_1) > a(x_2)$ , then  $x_1 \succ x_2$ ;

If  $a(x_1) = a(x_2)$ , then  $x_1 \sim x_2$ ;

Definition 4 Zhang and Xu(2014),Peng and Yang(2016)

$$(1) A^c = \{(x, v_A(x), \mu_A(x)) | x \in X\};$$

$$(2) A \subseteq B \text{ iff } \forall x \in X, \mu_A(x) \leq \mu_B(x) \text{ and } v_A(x) \geq v_B(x);$$

$$(3) A = B \text{ iff } \forall x \in X \mu_A(x) = \mu_B(x) \text{ and } v_A(x) = v_B(x);$$

$$(4) \emptyset_A = \{(x, 1, 0) | x \in X\};$$

$$(5) \emptyset_A = \{(x, 0, 1) | x \in X\};$$

$$(6) A \cap B = \{(x, \mu_A(x) \wedge \mu_B(x), v_A(x) \vee v_B(x)) | x \in X\};$$

$$(7) A \cup B = \{(x, \mu_A(x) \vee \mu_B(x), v_A(x) \wedge v_B(x)) | x \in X\};$$

$$(8) A \oplus B = \{(x, \sqrt{\mu_A^2(x) + \mu_B^2(x) - \mu_A^2(x)})\}$$

$$(9) A \otimes B = \{(x, \mu_A(x)\mu_B(x), \sqrt{v_A^2(x) + v_B^2(x) - v_A^2(x)v_B^2(x)}) | x \in X\};$$

Definition 5 Peng, Yuangand Yang (2017)

$$(D1) 0 \leq D(A, B) \leq 1;$$

$$(D2) D(A, B) = D(B, A);$$

$$(D3) D(A, B) = 0 \text{ if and only if } A = B;$$

$$(D4) D(A, A^c) = 1 \text{ if and only if } A \text{ is a crisp set};$$

$$(D5) \text{ If } A \subseteq B \subseteq 0, \text{ then } D(A, B) \leq D(A, 0) \text{ and } (B, 0) \leq D(A, 0).$$

Definition 6 Peng, Yuangand Yang(2017)

$$(S1) 0 \leq S(A, B) \leq 1;$$

$$(S2) S(A, B) = S(B, A);$$

(S3)  $S(A, B) = 1$  if and only if  $A = B$ ;

(S4)  $S(A, B) = 0$  if and only if  $A$  is a crisp set;

(S5) If  $A \subseteq B \subseteq 0$  then  $S(A, 0) \leq S(A, B)$  and  $S(A, 0) \leq S(B, 0)$ .

Brief introduced some of the already exists Similarity Measures for PFSSs

(1) Peng, Harish Garg 's similarity measure  $S_{PH1}$  (2019)

$$S_{PH1}(M, N) = 1 - \sqrt[p]{\frac{1}{2nt_k^p} \sum_{i=1}^n \{ |(t_k - 1)(u_N^2(x_i)) - (v_M^2(x_i) - v_N^2(x_i))|^p + |(t_k - k)(u_N^2(x_i)) - (v_M^2(x_i) - v_N^2(x_i))|^p \}}$$

Peng's similarity measure  $S_p$  (2018)

$$S_p(M, N) = 1 - D(M, N) = 1 - \sqrt[p]{\frac{1}{2n(t+1)^p} \sum_{i=1}^n (|(t+1-a)((\mu_M^2(x_i) - \mu_N^2(x_i)) - a((v_N^2(x_i) - v_M^2(x_i)))|^p + |(t+1-b)((v_M^2(x_i) - v_N^2(x_i)) - b((\mu_N^2(x_i) - \mu_M^2(x_i)))|^p)}$$

Wei and Wei's similarity measure  $S_W$  (2018)

$$S_W(M, N) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_M^2(x_i)\mu_N^2(x_i) + v_N^2(x_i)v_M^2(x_i)}{\sqrt{\mu_M^4(x_i) + v_M^4(x_i)}\sqrt{\mu_N^4(x_i) + v_N^4(x_i)}}$$

Peng et al.'s similarity measure  $S_{p1\sim3}$  (2017)

$$S_{p1}(M, N) = 1 - \frac{\sum_{i=1}^n |\mu_M^2(x_i) - v_M^2(x_i) - (v_N^2(x_i) - \mu_N^2(x_i))|}{2n}$$

$$S_{p2}(M, N) = \frac{1}{n} \sum_{i=1}^n \frac{(\mu_M^2(x_i) \wedge \mu_N^2(x_i)) + (v_M^2(x_i) \wedge v_N^2(x_i))}{(\mu_M^2(x_i) \vee \mu_N^2(x_i)) + (v_M^2(x_i) \vee v_N^2(x_i))}$$

$$S_{p3}(M, N) = \frac{1}{n} \sum_{i=1}^n \frac{(\mu_M^2(x_i) \wedge \mu_N^2(x_i)) + (1 - v_M^2(x_i) \wedge (1 - v_N^2(x_i)))}{(\mu_M^2(x_i) \vee \mu_N^2(x_i)) + (1 + v_M^2(x_i) \vee (1 + v_N^2(x_i)))}$$

Zhang's similarity measure  $S_Z$  (2016)

$$S_Z(M, N) = \frac{1}{n} \sum_{i=1}^n \frac{|\mu_M^2(x_i) - v_N^2(x_i)| + |v_M^2(x_i) - \mu_N^2(x_i)| + |\pi_M^2(x_i) - \pi_N^2(x_i)|}{|\mu_M^2(x_i) - \mu_N^2(x_i)| + |v_M^2(x_i) - v_N^2(x_i)| + |\pi_M^2(x_i) - \pi_N^2(x_i)| + |\mu_M^2(x_i) - v_N^2(x_i)| + |v_M^2(x_i) - \mu_N^2(x_i)| + |\pi_M^2(x_i) - \pi_N^2(x_i)|}$$

### III.A NEW MEASURE OF SIMILARITY DUE TO PFSSs FOR THE CENTROID POINT

The concept of new similarity measure was constructed due to the pythagorean fuzzy sets (PFSS) theorem and the centroid point theorem, shown as following:

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|2(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i))|}{2} \times (1 - \pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i)) - \frac{|2(v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i)) - (\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i))|}{2} \times (\pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i))$$

Let  $\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i)) | 1 \leq i \leq n\}$  be Pythagorean fuzzysset X , where  $X = \{x_1, x_2, \dots, x_n\}$ .

Let  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  imply the Pythagorean fuzzy value of function  $x_i$  a part of the Pythagorean fuzzy set  $\tilde{A}$ , where  $1 \leq i \leq n$ . Initial, present conversion the Pythagorean fuzzy value  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and a fuzzy number  $A_{x_i}$ , which  $1 \leq i \leq n$ . Let  $A_{x_i}$  be the transformed the Pythagorean fuzzy number,  $X = [0,1]$

Through from the pythagorean fuzzy value  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  with three points  $T_{A_{x_i}}^{PFS} = (\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i) + \pi_{\tilde{A}}(x_i))$ ,  $U_{A_{x_i}}^{PFS} = (\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $V_{A_{x_i}}^{PFS} = (\mu_{\tilde{A}}(x_i) + \pi_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$ . as shown in Figure 1 . where the distance  $\frac{T_{A_{x_i}}^{PFS} U_{A_{x_i}}^{PFS}}$  between points  $T_{A_{x_i}}^{PFS}$  and  $U_{A_{x_i}}^{PFS}$  on Y-axis is equal to  $(v_{\tilde{A}}(x_i) + \pi_{\tilde{A}}(x_i)) - v_{\tilde{A}}(x_i) = \pi_{\tilde{A}}(x_i)$ , the distance  $\frac{U_{A_{x_i}}^{PFS} V_{A_{x_i}}^{PFS}}$  between points  $U_{A_{x_i}}$  and  $V_{A_{x_i}}$  on X - axis is equal to  $(\mu_{\tilde{A}}(x_i) + \pi_{\tilde{A}}(x_i)) - \mu_{\tilde{A}}(x_i) = \pi_{\tilde{A}}(x_i)$ , and the centroid point  $C_{A_{x_i}}^{PFS}$  of the conversion the Pythagorean fuzzy number  $A_{x_i}$ . Figure 1 shows the following:

$$C_{A_{x_i}}^{PFS} = ((\mu_{\tilde{A}}(x_i) + \frac{\sqrt{3}}{2}(\pi_{\tilde{A}}(x_i)), v_{\tilde{A}}(x_i) + \frac{1}{2}\pi_{\tilde{A}}(x_i)))$$

Because  $\pi_{\tilde{A}}(x_i) = 1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i)$ , we can get

$$\begin{aligned} C_{A_{x_i}}^{PFS} &= (\mu_{\tilde{A}}(x_i) + \frac{\sqrt{3}}{2}\pi_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i) + \frac{1}{2}\pi_{\tilde{A}}(x_i)) \\ &= (\mu_{\tilde{A}}(x_i) + \frac{\sqrt{3}}{2}(1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i)), v_{\tilde{A}}(x_i) + \frac{1}{2}(1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i))) \\ &= (2(\mu_{\tilde{A}}(x_i) + \frac{\sqrt{3}}{2}(1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i))), (2(v_{\tilde{A}}(x_i) + \frac{1}{2}(1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i)))) \\ &= (\frac{2(\mu_{\tilde{A}}(x_i) + \sqrt{3}(1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i))}{2}, \frac{2(v_{\tilde{A}}(x_i) + (1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i))}{2})) \\ &= (\frac{\sqrt{3} - (2 + \sqrt{3})\mu_{\tilde{A}}(x_i) - \sqrt{3}v_{\tilde{A}}(x_i)}{2}, \frac{1 - \mu_{\tilde{A}}(x_i) + v_{\tilde{A}}(x_i)}{2}) \end{aligned}$$

Also, we can get

$$C_{B_{x_i}}^{PFS} = ((\mu_{\tilde{B}}(x_i) + \frac{\sqrt{3}}{2}(\pi_{\tilde{B}}(x_i)), v_{\tilde{B}}(x_i) + \frac{1}{2}\pi_{\tilde{B}}(x_i)))$$

Because  $\pi_{\tilde{B}}(x_i) = 1 - \mu_{\tilde{B}}(x_i) - v_{\tilde{B}}(x_i)$ , we can get

$$\begin{aligned} C_{B_{x_i}}^{PFS} &= (\mu_{\tilde{B}}(x_i) + \frac{\sqrt{3}}{2}\pi_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i) + \frac{1}{2}\pi_{\tilde{B}}(x_i)) \\ &= (\mu_{\tilde{B}}(x_i) + \frac{\sqrt{3}}{2}(1 - \mu_{\tilde{B}}(x_i) - v_{\tilde{B}}(x_i)), v_{\tilde{B}}(x_i) + \frac{1}{2}(1 - \mu_{\tilde{B}}(x_i) - v_{\tilde{B}}(x_i))) \end{aligned}$$

$$\begin{aligned}
 &= 2 \left( \mu_{\tilde{B}}(x_i) + \frac{\sqrt{3}}{2} (1 - \mu_{\tilde{B}}(x_i) - v_{\tilde{B}}(x_i)) \right), \left( 2 \left( v_{\tilde{B}}(x_i) + \frac{1}{2} (1 - \mu_{\tilde{B}}(x_i) - v_{\tilde{B}}(x_i)) \right) \right) \\
 &= \left( \frac{2(\mu_{\tilde{B}}(x_i) + \sqrt{3}(1 - \mu_{\tilde{B}}(x_i) - v_{\tilde{B}}(x_i)))}{2}, \frac{2(v_{\tilde{B}}(x_i) + (1 - \mu_{\tilde{B}}(x_i) - v_{\tilde{B}}(x_i)))}{2} \right) \\
 &= \left( \frac{\sqrt{3} - (2 + \sqrt{3})\mu_{\tilde{B}}(x_i) - \sqrt{3}v_{\tilde{B}}(x_i)}{2}, \frac{1 - \mu_{\tilde{B}}(x_i) + v_{\tilde{B}}(x_i)}{2} \right)
 \end{aligned}$$

From Fig.1, we can see that  $\overline{T_{A_{x_i}}^{PFS} U_{A_{x_i}}^{PFS}} = \overline{U_{A_{x_i}}^{PFS} V_{A_{x_i}}^{PFS}} = \pi_{\tilde{A}}(x_i)$  where  $\overline{T_{A_{x_i}}^{PFS} U_{A_{x_i}}^{PFS}}$  denotes the distance between the point  $T_{A_{x_i}}^{PFS} = (\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i) + \pi_{\tilde{A}}(x_i))$  and the point  $U_{A_{x_i}}^{PFS} = (\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  on the Y-axis.  $\overline{U_{A_{x_i}}^{PFS} V_{A_{x_i}}^{PFS}}$  denotes the similarity measure between the point  $U_{A_{x_i}}^{PFS} = (\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $V_{A_{x_i}}^{PFS} = (\mu_{\tilde{A}}(x_i) + \pi_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$ , on the X-axis and  $\pi_{\tilde{A}}(x_i)$  imply the degree of indeterminacy of function  $x_i$  a part of the Pythagorean fuzzy set  $\tilde{A}$ , where  $\pi_{\tilde{A}}(x_i) = 1 - \mu_{\tilde{A}}(x_i) - v_{\tilde{A}}(x_i)$ ,  $\pi_{\tilde{A}}(x_i) \in [0,1]$  and  $1 \leq i \leq n$ .

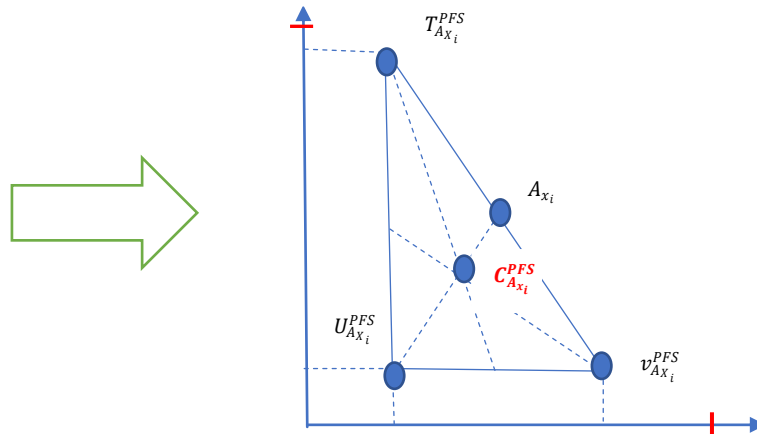


Fig 1: A New Similarity Measure of PFSs due to The Centroid Point

Let  $A_{x_i}$  and  $B_{x_i}$  be two the Pythagorean fuzzy numbers conversion from the Pythagorean fuzzy value  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $(\mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i))$  of function  $x_i$  a part of the Pythagorean fuzzy sets  $\tilde{A}$  and  $\tilde{B}$ , separately, as shown in Figure 2. the centroid points  $C_{A_{x_i}}$  and  $C_{B_{x_i}}$  of the conversion the Pythagorean fuzzy number  $A_{x_i}$  and  $B_{x_i}$ . Figure 2 shows in the following:

$$\begin{aligned}
 C_{A_{x_i}} &= \left( \frac{\sqrt{3} - (2 + \sqrt{3})\mu_{\tilde{A}}(x_i) - \sqrt{3}v_{\tilde{A}}(x_i)}{2}, \frac{1 - \mu_{\tilde{A}}(x_i) + v_{\tilde{A}}(x_i)}{2} \right) \\
 C_{B_{x_i}} &= \left( \frac{\sqrt{3} - (2 + \sqrt{3})\mu_{\tilde{B}}(x_i) - \sqrt{3}v_{\tilde{B}}(x_i)}{2}, \frac{1 - \mu_{\tilde{B}}(x_i) + v_{\tilde{B}}(x_i)}{2} \right)
 \end{aligned}$$

According to Figure 2, the degree of similarity measure  $S(A_{x_i}, B_{x_i})$  between the Pythagorean fuzzy value  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $(\mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i))$  is calculated as follows:

$$s(A_{x_i}, B_{x_i}) = 1 - \frac{|2(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i))|}{2} \times (1 - \pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i)) - \frac{|2(v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i)) - (\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i))|}{2} \times (\pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i))$$

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|\frac{\sqrt{3}}{2}(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i))|}{2} \times (1 - (\pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i))) - |\frac{\sqrt{3}}{2}(v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i)) - (\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i))| \times (\pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i)).$$

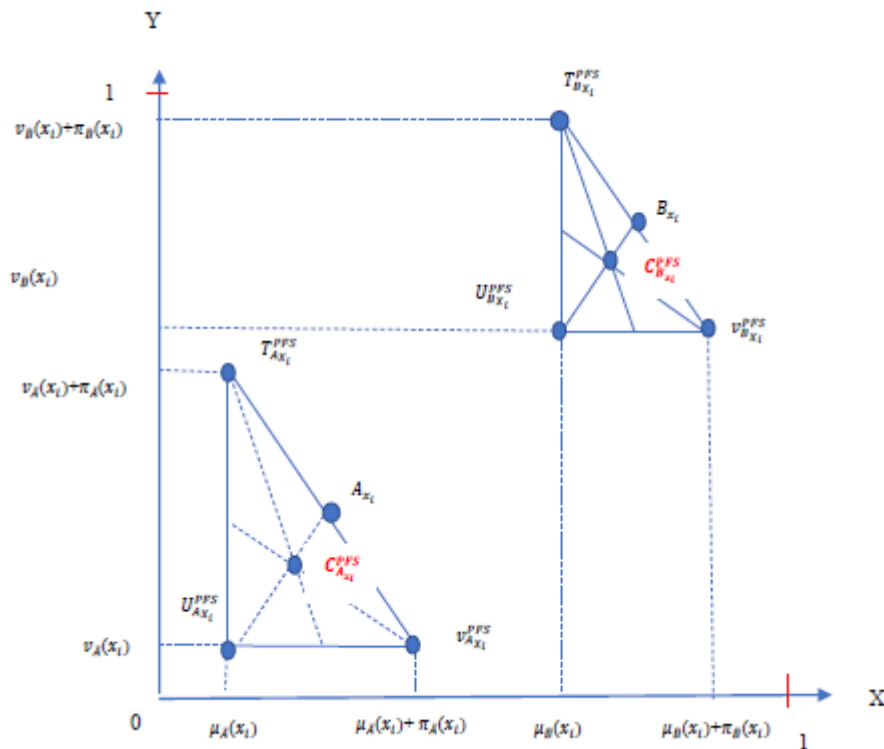


Fig 2: The transformed the Pythagorean fuzzy number  $A_{x_i}$  and  $B_{x_i}$  of the Pythagorean fuzzy values  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $(\mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i))$ , respectively.

Let  $A_{x_i}, B_{x_i}$  and  $C_{x_i}$  be three transformed Pythagorean fuzzy numbers of the Pythagorean fuzzy value  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i)), (\mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i))$  and  $(\mu_{\tilde{C}}(x_i), v_{\tilde{C}}(x_i))$  of function  $x_i$  a part of the Pythagorean fuzzy sets  $\tilde{A}, \tilde{B}$  and  $\tilde{C}$ , separately, where  $1 \leq i \leq n$ . Their properties are represented as following.

Here raised a new similarity measure according to Peng and Yuang and Yang (2017), definition 6, illustrated as proof as below properties, separately:

Property 3.1  $0 \leq S(M, N) \leq 1$

Proof:

Assume that  $\mu_A(x_i) = \mu_B(x_i) = 1$  and assume that  $v_A(x_i) = v_B(x_i) = 0$ , then  $\pi_A(x_i) = 1 - \mu_A(x_i) - v_A(x_i) = 1 - 1 - 0 = 0$  and  $\pi_B(x_i) = 1 - \mu_B(x_i) - v_B(x_i) = 1 - 1 - 0 = 0$ . Based on Eq.

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|\sqrt{3}(\mu_A(x_i) - \mu_B(x_i)) - (v_A(x_i) - v_B(x_i))|}{2} \times (1 - (\pi_A(x_i) - \pi_B(x_i))) - \frac{|\sqrt{3}(v_A(x_i) - v_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|}{2} \times (\pi_A(x_i) - \pi_B(x_i))$$

We can get

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|\sqrt{3}(1 - 0) - (0 - 1)|}{2} \times (1 - (0 - 0)) - \frac{|\sqrt{3}(0 - 1) - (1 - 0)|}{2} \times (0 - 0) = 0$$

Assume that  $\mu_A(x_i) = \mu_B(x_i) = 1$  and assume that  $v_A(x_i) = v_B(x_i) = 0$ , then based on eq.,

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|\sqrt{3}(\mu_A(x_i) - \mu_B(x_i)) - (v_A(x_i) - v_B(x_i))|}{2} \times (1 - (\pi_A(x_i) - \pi_B(x_i))) - \frac{|\sqrt{3}(v_A(x_i) - v_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|}{2} \times (\pi_A(x_i) - \pi_B(x_i))$$

We can get

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - |\sqrt{3}(1 - 1) - (0 - 0)| \times (1 - (0 - 0)) - |\sqrt{3}(0 - 0) - (1 - 1)| \times (0 - 0) = 1.$$

Property 3.2  $S(M, N) = S(N, M)$  ;

Proof:

Because  $1 - \frac{|\sqrt{3}(\mu_A(x_i) - \mu_B(x_i)) - (v_A(x_i) - v_B(x_i))|}{2} = 1 - \frac{|\sqrt{3}(v_A(x_i) - v_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|}{2}$ . Where from Eq.

$$\begin{aligned} S_L^{PC}(A_{x_i}, B_{x_i}) &= 1 - \frac{|\sqrt{3}(\mu_A(x_i) - \mu_B(x_i)) - (v_A(x_i) - v_B(x_i))|}{2} \times (1 - (\pi_A(x_i) - \pi_B(x_i))) - \frac{|\sqrt{3}(v_A(x_i) - v_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|}{2} \times (\pi_A(x_i) - \pi_B(x_i)) \\ &= \frac{|\sqrt{3}(v_A(x_i) - v_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|}{2} \times (\pi_A(x_i) - \pi_B(x_i)) - \frac{|1 - \sqrt{3}(\mu_A(x_i) - \mu_B(x_i)) - (v_A(x_i) - v_B(x_i))|}{2} \times (1 - (\pi_A(x_i) - \pi_B(x_i))) \\ &= S_L^{PC}(B_{x_i}, A_{x_i}) \end{aligned}$$





Consequently,  $S_L^{PC}(A_{x_i}, B_{x_i}) = S_L^{PC}(B_{x_i}, A_{x_i})$  is proved.

Property 3.3  $S(M, N) = 1$  if and only if  $M = N$ ;

Proof:

If  $S_L^{PC}(A_{x_i}, B_{x_i}) = 1$ , then based on Eq

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} \times (1 - (\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i))) - \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))|}{2} \times (\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i)) = 1$$

Then, we can get

$$\frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} \times (1 - (\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i))) + \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))|}{2} \times (\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i)) = 1$$

Then, we can get

$$\frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} + \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))|}{2} \times 1 = 1$$

Therefore, we can get

$$\frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))}{2} = \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))}{2} \text{ if } \frac{(\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i))}{2} \times \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} - \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} = 0$$

Then we can get that

$$\frac{\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i)}{2} = 0 \text{ or } \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} - \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} = 0$$

The proof is proved.

Property 3.4  $S(M, N) = 0$  if and only if  $M$  is a crisp set. (Omitted)

Property 3.5 If  $M \subseteq N \subseteq 0$  then  $S(M, 0) \leq S(M, N)$  and  $(M, 0) \leq S(N, 0)$ .

Proof:

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} \times (1 - (\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i))) - \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))|}{2} \times (\pi_{\bar{A}}(x_i) - \pi_{\bar{B}}(x_i))$$

$$\begin{aligned}
 &= 1 - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} \right) (1) - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} \right) (\pi_{\bar{A}}(x_i)) - \\
 &\left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} \right) (\pi_{\bar{B}}(x_i)) - \left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))|}{2} \right) (\pi_{\bar{A}}(x_i)) - \\
 &\left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))|}{2} \right) (\pi_{\bar{B}}(x_i)) \\
 &= 1 - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))|}{2} \right) - \\
 &\left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))|}{2} \right) - \\
 &\left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))|}{2} \right) \\
 &= 1 - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) - \\
 &\left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) - \\
 &\left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) \\
 &= 1 - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) - \\
 &\left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) - \\
 &\left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) \\
 &= 1 - \left( \left| \frac{(\sqrt{3}-1)((\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)))}{2} \right| \right) - \\
 &\left( \left| \frac{(\sqrt{3}-1)((\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i)))}{2} \right| \right) - \left( \left| \frac{(\sqrt{3}-1)((\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i)))}{2} \right| \right) - \left( \left| \frac{(\sqrt{3}-1)((v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i)))}{2} \right| \right) - \\
 &\left( \left| \frac{(\sqrt{3}-1)(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) \\
 &= 1 - \left( \frac{\sqrt{3}-1}{2} ( |((\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i)))| - |((\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i)))| - \right. \\
 &\left. |((\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i)))| - |((v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{A}}(x_i)))| - |(v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))(\pi_{\bar{B}}(x_i))| \right) \\
 &= 1 - \left( \frac{\sqrt{3}-1}{2} ( |(\mu_{\bar{A}}(x_i) - \mu_{\bar{B}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{B}}(x_i))| \right)
 \end{aligned}$$

Because  $\mu_{\bar{A}}(x_i) \leq \mu_C(x_i)$  and  $v_{\bar{A}}(x_i) \geq v_C(x_i)$ , we can see that  $\frac{\sqrt{3}}{2}(\mu_{\bar{A}}(x_i) - \mu_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i)) \leq 0$  and  $\frac{\sqrt{3}}{2}(v_{\bar{A}}(x_i) - v_C(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_C(x_i)) \geq 0$ . Thus, based on Eq.  $P_cS(A_{x_i}, C_{x_i})$ , we can get

$$\begin{aligned}
 S_L^{PC}(A_{x_i}, C_{x_i}) &= 1 - \left| \frac{\sqrt{3}}{2}(\mu_{\bar{A}}(x_i) - \mu_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i)) \right| \times \left( 1 - (\pi_{\bar{A}}(x_i) - \pi_C(x_i)) \right) - \left| \frac{\sqrt{3}}{2}(v_{\bar{A}}(x_i) - v_C(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_C(x_i)) \right| \times (\pi_{\bar{A}}(x_i) - \pi_C(x_i)) \\
 &= 1 - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i))|}{2} \right) (1) - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i))|}{2} \right) (\pi_{\bar{A}}(x_i) - \pi_C(x_i)) \\
 &\quad - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i))|}{2} \right) (\pi_C(x_i)) - \left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_C(x_i))|}{2} \right) (\pi_{\bar{A}}(x_i) - \pi_C(x_i)) \\
 &\quad - \left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_C(x_i))|}{2} \right) (\pi_C(x_i)) \\
 &= 1 - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i))(\pi_{\bar{A}}(x_i))|}{2} \right) \\
 &\quad - \left( \frac{|\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i))(\pi_C(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_{\bar{A}}(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i))|}{2} \right) \\
 &\quad - \left( \frac{|\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_C(x_i)) - (\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_C(x_i))|}{2} \right) \\
 &= 1 - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_C(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) \\
 &\quad - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_C(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_C(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) \\
 &\quad - \left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_C(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_C(x_i))}{2} \right| \right) \\
 &= 1 - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_C(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) \\
 &\quad - \left( \left| \frac{\sqrt{3}(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_C(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_C(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_{\bar{A}}(x_i))}{2} - \frac{(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) \\
 &\quad - \left( \left| \frac{\sqrt{3}(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_C(x_i))}{2} - \frac{(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_C(x_i))}{2} \right| \right) \\
 &= 1 - \left( \left| \frac{(\sqrt{3}-1)(\mu_{\bar{A}}(x_i) - \mu_C(x_i)) - (v_{\bar{A}}(x_i) - v_C(x_i))}{2} \right| \right) - \\
 &\quad \left( \left| \frac{(\sqrt{3}-1)(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) - \left( \left| \frac{(\sqrt{3}-1)(\mu_{\bar{A}}(x_i) - \mu_C(x_i))(\pi_C(x_i))}{2} \right| \right) - \left( \left| \frac{(\sqrt{3}-1)(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_{\bar{A}}(x_i))}{2} \right| \right) \\
 &\quad - \left( \left| \frac{(\sqrt{3}-1)(v_{\bar{A}}(x_i) - v_C(x_i))(\pi_C(x_i))}{2} \right| \right)
 \end{aligned}$$

$$= 1 - \left( \left| \frac{\sqrt{3}-1}{2} \left( |(\mu_{\bar{A}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{C}}(x_i))| - \left| ((\mu_{\bar{A}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{A}}(x_i))) \right| - \left| ((\mu_{\bar{A}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))) \right| - \left| ((v_{\bar{A}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{A}}(x_i))) \right| - \left| ((v_{\bar{A}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))) \right| \right) \right|$$

$$= 1 - \left| \frac{\sqrt{3}-1}{2} \left( |(\mu_{\bar{A}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{A}}(x_i) - v_{\bar{C}}(x_i))| \right) \right|$$

Moreover, because  $\mu_{\bar{B}}(x_i) \leq \mu_{\bar{C}}(x_i)$  and  $v_{\bar{B}}(x_i) \geq v_{\bar{C}}(x_i)$ . we can see that  $\frac{\sqrt{3}}{2}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i)) \leq 0$ . and  $32v_{\bar{B}}x_i - v_{\bar{C}}x_i - \mu_{\bar{B}}x_i - \mu_{\bar{C}}x_i \geq 0$ . Thus, based on Eq.  $PcSBxi, Cxi$ , we can get

$$S_L^{PC}(B_{x_i}, C_{x_i}) = 1 - \left| \frac{\sqrt{3}}{2} (\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i)) \right| \times \left( 1 - (\pi_{\bar{B}}(x_i) - \pi_{\bar{C}}(x_i)) \right) - \left| \frac{\sqrt{3}}{2} (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i)) - (\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i)) \right| \times (\pi_{\bar{B}}(x_i) - \pi_{\bar{C}}(x_i))$$

$$= 1 - \left( \frac{|\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))|}{2} \right) (1) - \left( \frac{|\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))|}{2} \right) (\pi_{\bar{A}}(x_i)) - \left( \frac{|\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))|}{2} \right) (\pi_{\bar{C}}(x_i)) - \left( \frac{|\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i)) - (\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))|}{2} \right) (\pi_{\bar{A}}(x_i)) - \left( \frac{|\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i)) - (\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))|}{2} \right) (\pi_{\bar{C}}(x_i))$$

$$= 1 - \left( \frac{|\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i)) - (v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i)) - (\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))|}{2} \right) - \left( \frac{|\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i)) - (\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))|}{2} \right)$$

$$= 1 - \left( \left| \frac{\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))}{2} - \frac{(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} - \frac{(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} - \frac{(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} \right| \right) 3$$

$$= 1 - \left( \left| \frac{\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))}{2} - \frac{(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} - \frac{(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} - \frac{(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{B}}(x_i))}{2} \right| \right) - \left( \left| \frac{\sqrt{3}(v_{\bar{B}}(x_i) - v_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} - \frac{(\mu_{\bar{B}}(x_i) - \mu_{\bar{C}}(x_i))(\pi_{\bar{C}}(x_i))}{2} \right| \right)$$



$$\begin{aligned}
 &= 1 - \left( \left| \frac{(\sqrt{3}-1)((\mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i)) - (v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i)))}{2} \right| \right) - \\
 &\left( \left| \frac{(\sqrt{3}-1)((\mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i))(\pi_{\tilde{B}}(x_i)))}{2} \right| \right) - \left( \left| \frac{(\sqrt{3}-1)((\mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i))(\pi_{\tilde{C}}(x_i)))}{2} \right| \right) - \left( \left| \frac{(\sqrt{3}-1)((v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i))(\pi_{\tilde{B}}(x_i)))}{2} \right| \right) - \\
 &\left( \left| \frac{(\sqrt{3}-1)((v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i))(\pi_{\tilde{C}}(x_i)))}{2} \right| \right) \\
 &= 1 - \left( \left| \frac{\sqrt{3}-1}{2} ( |((\mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i)) - (v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i)))| - |((\mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i))(\pi_{\tilde{B}}(x_i)))| - \right. \right. \\
 &\left. \left| ((\mu_{\tilde{A}}(x_i) - \mu_{\tilde{C}}(x_i))(\pi_{\tilde{C}}(x_i)))| - |((v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i))(\pi_{\tilde{B}}(x_i)))| - |(v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i))(\pi_{\tilde{C}}(x_i)))| \right) \right) \\
 &= 1 - \left| \frac{\sqrt{3}-1}{2} ( |(\mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i)) - (v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i))| \right)
 \end{aligned}$$

Then, we can get

$$\begin{aligned}
 &(i) S_L^{PC}(A_{x_i}, B_{x_i}) - S_L^{PC}(A_{x_i}, C_{x_i}) \\
 &= 1 - \left| \frac{\sqrt{3}-1}{2} ( |(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i))| \right) - \left| \frac{\sqrt{3}-1}{2} ( |(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{C}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{C}}(x_i))| \right) \\
 &= \left| \frac{\sqrt{3}-1}{2} (v_{\tilde{B}}(x_i) + v_{\tilde{C}}(x_i) - \mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i)) \right| \\
 &(ii) S_L^{PC}(B_{x_i}, C_{x_i}) - S_L^{PC}(A_{x_i}, C_{x_i}) \\
 &= 1 - \left| \frac{\sqrt{3}-1}{2} ( |(\mu_{\tilde{B}}(x_i) - \mu_{\tilde{C}}(x_i)) - (v_{\tilde{B}}(x_i) - v_{\tilde{C}}(x_i))| \right) - \left| \frac{\sqrt{3}-1}{2} ( |(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{C}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{C}}(x_i))| \right) \\
 &= \left| \frac{\sqrt{3}-1}{2} (\mu_{\tilde{B}}(x_i) - \mu_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i) - v_{\tilde{A}}(x_i)) \right|
 \end{aligned}$$

Because  $\mu_{\tilde{A}}(x_i) \leq \mu_{\tilde{B}}(x_i) \leq \mu_{\tilde{C}}(x_i)$  and  $v_{\tilde{A}}(x_i) \geq v_{\tilde{B}}(x_i) \geq v_{\tilde{C}}(x_i)$ . We can get  $\mu_{\tilde{B}}(x_i) - \mu_{\tilde{A}}(x_i) \geq 0$  and  $v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i) \geq 0$ . Because  $0 \leq \pi_{\tilde{A}}(x_i) \leq 1$  and  $0 \leq \pi_{\tilde{B}}(x_i) \leq 1$ . we can get  $0 \leq \pi_{\tilde{B}}(x_i) + \pi_{\tilde{A}}(x_i) \leq 2$ . Thus, we can get  $1 - \left| \frac{\sqrt{3}-1}{2} ( |(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i))| \right) \geq 0$ . Therefore, we can get  $S_L^{PC}(B_{x_i}, C_{x_i}) - S_L^{PC}(A_{x_i}, C_{x_i}) \geq 0$ . That is,  $S_L^{PC}(A_{x_i}, C_{x_i}) \leq S_L^{PC}(B_{x_i}, C_{x_i})$ . From (i) and (ii), we can state that if  $\tilde{A} \sqsubseteq \tilde{B} \sqsubseteq \tilde{C}$ , then  $S_L^{PC}(A_{x_i}, C_{x_i}) \leq S_L^{PC}(A_{x_i}, B_{x_i})$  and  $S_L^{PC}(A_{x_i}, C_{x_i}) \leq S_L^{PC}(B_{x_i}, C_{x_i})$ .

Let X be PFSs, where  $X = \{x_1, x_2 \dots x_n\}$ , and let  $\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i)) | 1 \leq i \leq n\}$  and  $\tilde{B} = \{(x_i, \mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i)) | 1 \leq i \leq n\}$ , be two PFSs X, where  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $(\mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i))$  are the Pythagorean fuzzy values of function  $x_i$ ; a part of the Pythagorean fuzzy sets  $\tilde{A}$  and  $\tilde{B}$  is shown as follows:

$$S_L^{PC}(\tilde{A}, \tilde{B}) = w_1 \times S_L^{PC}(A_{x_1}, B_{x_1}) + w_2 \times S_L^{PC}(A_{x_2}, B_{x_2}) + \dots + w_n \times S_L^{PC}(A_{x_n}, B_{x_n}) = \sum_{i=1}^n (w_i \times S_L^{PC}(A_{x_i}, B_{x_i}))$$

Where  $S_L^{PC}(\tilde{A}, \tilde{B}) \in \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \leq 1$ .  $w_i$  is the weight of function,  $\sum_{i=1}^n w_i = 1$ ,  $A_{x_i}$  and  $B_{x_i}$  are the corresponding conversion PFN of the Pythagorean fuzzy values  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $(\mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i))$  of function  $x_i$ ; a part of PFSs  $\tilde{A}$  and  $\tilde{B}$ , separately,  $S_L^{PC}(A_{x_1}, B_{x_1})$  is the degree of similarity measure between the Pythagorean fuzzy values  $(\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i))$  and  $(\mu_{\tilde{B}}(x_i), v_{\tilde{B}}(x_i))$ . Through by the raised similarity measure of the pythagorean fuzzy values describe  $S_L^{PC}(A_{x_1}, B_{x_1})$  and

$1 \leq i \leq n$ . The larger the value of  $S_L^{PC}(\tilde{A}, \tilde{B})$ , upper the degree of similarity measure between the Pythagorean fuzzy sets  $\tilde{A}$  and  $\tilde{B}$ . where  $S_L^{PC}(\tilde{A}, \tilde{B}) \in \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \leq 1$ .

**IV.COMPARISON NEW SIMILARITY MEASURE  $S_L^{PC}$  WITH ALREADY EXISTS SIMILARITY MEASURES FOR PFS**

In this work, proposed new similarity measure  $S_L^{PC}$  comparison with ones selected some similarity measures based on PFS form the already exists similarity measures, as Table 1. and as follows  $S_L^{PC}$ :

$$S_L^{PC}(A_{x_i}, B_{x_i}) = 1 - \frac{|2(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)) - (v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i))|}{2} \times (1 - \pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i)) - \frac{|2(v_{\tilde{A}}(x_i) - v_{\tilde{B}}(x_i)) - (\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i))|}{2} \times (\pi_{\tilde{A}}(x_i) - \pi_{\tilde{B}}(x_i))$$

**IV.1 Numerical experiments**

The numerical experiments are tested new similarity measure  $S_L^{PC}$  with already existssimilarity for PFS , in this paper, choose six cases and methods of  $S_p(2018), S_{pH1}(2019), S_{pH2}(2019), S_w(2018), S_{p1}(2017), S_{p2}(2017), S_{p3}(2017) S_z(2016)$  practical calculated, respectively. as Table 1 to3, shown in the following:

Table 1: The comparison outcome of the new similarity measure  $S_L^{PC}$  with some of the already existssimilarity measures for PFSs adopted from Peng and Harish Garg (2019)

	Case1	Case2	Case3	Case4	Case5	Case6
M	{(x, 0.5,0.5)}	{(x, 0.6,0.4)}	{(x, 0,0.87)}	{(x, 0.6,0.27)}	{(x, 0.125,0.075)}	{(x, 0.5,0.45)}
N	{(x, 0,0)}	{(x, 0,0)}	{(x, 0.28,0.55)}	{(x, 0.28,0.55)}	{(x, 0.175,0.025)}	{(x, 0.55,0.4)}
$S_p$ (2018)	0.9167	0.9	0.7336	0.7444	0.99	0.9525
$S_{pH1}$ (2019)	0.9167	0.9	0.7336	0.7444	0.99	0.9525
$S_w$ (2018)	N/A	N/A	0.968	0.438	0.9476	0.9812
$S_{p1}$ (2017)	1	0.9	0.7336	0.7444	0.99	0.9525
$S_{p2}$ (2017)	0	0	0.3621	0.2284	0.4483	0.8119
$S_{p3}$ (2017)	0.6	0.6176	0.3133	0.6028	0.9806	0.9168

$S_z$ (2016)	0.5	0.5	0.5989	0.1696	0.625	0.6557
$S_L^{PC}$	1	0.9	N/A	0.6919	0.75	1

Case1~case6 adopted from Peng and Harish Garg (2019).

See Table 1, obviously showing the similarity measure  $S_L^{PC}$  (Proposed) is comparison with decided candidates  $S_p$  (2018),  $S_{PH1}$  (2019),  $S_{PH2}$  (2019),  $S_w$  (2018),  $S_{p1}$  (2017),  $S_{p2}$  (2017),  $S_{p3}$  (2017),  $S_z$  (2016) ranking results as follows Table 2:

Table 2: The ranking and candidates

case	Case 1 to Case 6 comparison ranking	The best Candidates
1	$S_L^{PC} = S_{p1} > S_{PH1} > S_p > S_{p3} > S_z > S_{p2} > S_w$	$S_L^{PC} = S_{p1}$
2	$S_L^{PC} = S_p = S_{PH1} = S_{p1} > S_{p3} > S_z > S_{p2} > S_w$	$S_L^{PC}$
3	$S_w > S_p = S_{PH1} = S_{p1} > S_z > S_{p2} > S_{p3} > S_L^{PC}$	$S_w$
4	$S_p = S_{PH1} = S_{p1} > S_L^{PC} > S_{p3} > S_w > S_{p2} > S_z$	$S_p = S_{PH1} = S_{p1}$
5	$S_p = S_{PH1} = S_{p1} > S_{p3} > S_w > S_L^{PC} > S_z > S_{p2}$	$S_p = S_{PH1} = S_{p1}$
6	$S_L^{PC} > S_w > S_p = S_{PH1} = S_{p1} > S_{p3} > S_{p2} > S_z$	$S_L^{PC}$

It was got five candidates from Table 2, according to taking the ordering from the weights of the candidates shown in table 3.

Table 3: Comparison of ordering the weight candidates

The best Candidates	The total weights of case calculated	Order
$S_L^{PC}$	Case 1(1) + case 2 (0.9)+case 6 (1) = 2.9	1
$S_p$	Case 4 (0.7444) + case 5 (0.99) = 1.7344	3
$S_{p1}$	Case1(1)+case 4(0.7444) + case 5(0.99)= 2.7344	2
$S_w$	Case 3 (0.968) =0.968	5
$S_{PH1}$	Case 5 (0.99) =0.99	4

**V. CONCLUSION**

The result was  $S_L^{PC}$  compared with some of the existing similarity measures of  $PFSS_W, S_P, S_{PH1}, S_{P1}, S_{P2}, S_{P3}, S_Z$ , shown as Tables 1 to 3 to illustrate the candidates ordering outcomes were. Obviously,  $S_L^{PC}$  priority order is first, which means this research method is usefulness and effectiveness.

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