ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

Fractional Calculus of a class of Univalent Functions with Some Geometric Properties with Operator AMEEN

Karrar khudhair obayes, Aseel Ameen Harbi,Nagham kamil hadi ,Maryam najeh khaleel, Jumana jamal kadhim

Department of Mathematics, College of Computer Science and. Mathematics, University of Al-Qadisiya,Diwaniya, Iraq

ABSTRACT: In this paper we will study a class $M(0,\beta,b,\lambda,\mu)$,Which is composed of analytic and univalent functions with negative coefficients in the open unit disk U={z∈C:|z|<1}defined by Hadamard product (or convolution) with AMEEN - Operator, we obtain coefficient bounds and extreme points for this class. Also distortion theorem using fractional calculus techniques and some results for this classare obtained.

2000 Mathematics Subject Classifications: 30C45

KEY WORDS AND PHRASES: Univalent Function, Fractional Calculus, Hadamard Product, Distortion TheoremAMEEN-Operator,,Extreme Point..

I.INTRODUCTION

The integral AMEEN-operator of $f \in S$ for $\lambda > -1, \mu \geq 0$ is denoted byM_λ^μ and defined as following:

$$Mk(z) = \frac{(\lambda+1)^\mu}{\Gamma(\mu)} \int_0^1 t^\lambda \left(\log \frac{1}{t}\right)^{\mu-1} \frac{K(zt)}{t}\, dt$$

$$= z - \sum_{n=2}^{\infty} \left(\frac{\lambda+1}{\lambda+n}\right)^\mu a_n z^n (\lambda > -1, \mu \geq 0, K \in S) \qquad (1)$$

The operator is known as the Komatuoperator[2].A function $K \in S$, $z \in U$ is said to be in the class $M(0,\beta,b,\lambda,\mu)$ if and only if it satisfies the inequality

$$\operatorname{Re}\left\{\beta \frac{M_\lambda^\mu K(z)}{z} + (1-\beta)(M_\lambda^\mu K(z))' + \alpha z (M_\lambda^\mu K(z))''\right\} > 1 - |b| \quad (2)$$

For some $\alpha(\alpha \geq 0), -1 \leq \beta \leq 0, b \in \mathbb{C}, \lambda > -1$ and $\mu \geq 0$, for all $z \in U$.

The class $M(0,0,1-\gamma,\lambda,0)$ was introduced b Altintas[1] who obtained several results concerning this class .The class $M(0,0,b,\lambda,0)$ was introduced by Srivastava and Owa[3].

The class $M(0,\beta,b,\lambda,0)$ was introduced by Atshan and Kulkarni[1].

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

Definition (1): We say that the function f of complex variable is analytic in a domain D if is differentiable at every point in that domain D.

Definition (2): A function f analytic in a domain D is said to be univalent there if it does not take the same value twice that is $K(z_1) \neq K(z_2)$ for all pairs of distinct points z_1 and z_2 in D.

Definition (3): A function $f \in A$ is said to be convex function of order α if and only if

$$\operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\alpha,(0\le\alpha<1;z\in U)$$

We denote the class of all convex functions of order α in U by $C(\alpha)$.

Note that $S^*(0) = S^*$, $C(0) = C$ and $C \subset S^* \subset A$, and the Koebe function is starlike but not convex, where the Koebe function given by

$$K(z) = \frac{z}{(1-z)^2} = \sum_{n=1}^{\infty} nz^n$$

is the most famous function in the class A , which maps U onto **C** minus a slit along the negative real axis from $-\frac{1}{4}$ to $-\infty$

Theorem (1): Let $f \in S$ .Then f is in the class $M(0,\beta,b,\lambda,\mu)$ if and only if

$$\sum_{n=2}^{\infty}\left[\beta+n(1-\beta)\right]\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_n\le|b| \quad (.3)$$

The result (3) is sharp.

Proof: Assume that $K \in M(O,\beta,b,\lambda,\mu)$ .Then, we find from (.2) that

$$\operatorname{Re}\left\{\beta\left[1-\sum_{n=2}^{\infty}a_n\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}z^{n-1}\right]+(1-\beta)\left[1-\sum_{n=2}^{\infty}na_n\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}z^{n-1}\right]\right.$$

$$\left.+\alpha z\left[-\sum_{n=2}^{\infty}n(n-1)a_n\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}z^{n-2}\right]\right\}>1-|b|.$$

If we choose z to be the real and let $z\to1$,

$1-\sum_{n=2}^{\infty}[\beta+n(1-\beta+0n-0)]\,(\frac{\lambda+1}{\lambda+n})^{\mu}a_n \geq 1-|b|$,we get

$$1-\sum_{n=2}^{\infty}[\beta+n(1-\beta)]\,(\frac{\lambda+1}{\lambda+n})^{\mu}a_n \geq 1-|b|,$$

Which is equivalent to (3).conversely, assume that (3) is true. Then, we have

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

$$\left|\beta\frac{M_\lambda^\mu K(z)}{z}-(1-\beta)(M_\lambda^\mu K(z))'-\alpha z(M_\lambda^\mu K(z))''-1\right|$$

$$\leq\sum_{n=2}^{\infty}\left[\beta+n(1-\beta)\right]\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_n\leq|b|.$$

The implies that $K\in M(0,\beta,b,\lambda,\mu)$. The result (3) is sharp for the function

$$K(z)=z-\frac{|b|}{\left[\beta+n(1-\beta\right]\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}}z^n,n\geq 2 \qquad (4)$$

In the following theorem, we obtain interesting properties of the class $K\in M(0,\beta,b,\lambda,\mu)$.

Theorem (2): Let $K\in M(0,\beta,b,\lambda,\mu)$.Then

$$|z|-\frac{|b|}{2-\beta}|z|^2\leq\left|Q_\lambda^\mu K(z)\right|\leq|z|+\frac{|b|}{2-\beta}|z|^2 \qquad (5)$$

Proof: Easy to see it;for $K\in M(0,\beta,b,\lambda,\mu)$,

$$(2-\beta+2(0))\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_n\leq\sum_{n=2}^{\infty}\left[\beta+n(1-\beta+0n-0)\right]\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_n\leq|b|$$

.

Hence

$$\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_n\leq\frac{|b|}{(2-\beta)},2-\beta\leq\beta+n(1-\beta)$$

Now,

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

$$\left|M_{\lambda}^{\mu}K(z)\right|=\left|z-\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_{n}z^{n}\right|\leq|z|+\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_{n}|z|^{n}$$

$$\leq|z|+|z|^{2}\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_{n}\leq|z|+|z|^{2}\frac{|b|}{(2-\beta+2(0)},$$

and

$$\left|M_{\lambda}^{\mu}K(z)\right|=\left|z-\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_{n}z^{n}\right|\geq|z|-\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_{n}|z|^{n}$$

$$\geq|z|-|z|^{2}\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{\lambda+n}\right)^{\mu}a_{n}\geq|z|-|z|^{2}\frac{|b|}{(2-\beta)}.$$

Theorem(3): Let $K(z)=z-\sum_{n=2}^{\infty}a_{n,i}z^{n}(a_{n},i\geq0,i=1,2,\ldots,m)$

be in the class $M(0,\beta,b,\lambda,\mu)$.Then the function

$$K(z)=\sum_{i=1}^{m}d_{i}f_{i}(z)\ ,(\sum_{i=1}^{m}d_{i}=1)$$

is in the class $M(0,\beta,b,\lambda,\mu)$.

Proof: By definition of K(z), we have

$$K(z)=z-\sum_{n=2}^{\infty}\left[\sum_{i=1}^{m}d_{i}a_{n,i}\right]z^{n}$$

Thus, we have from Theorem(.1)

$$\sum_{n=2}^{\infty}[\beta+n(1-\beta)](\frac{\lambda+1}{\lambda+n})^{\mu}\left[\sum_{i=1}^{m}d_{i}a_{n,i}\right]$$

$$=\sum_{i=1}^{m}d_{i}\left[\sum_{n=2}^{\infty}[\beta+n(1-\beta)](\frac{\lambda+1}{\lambda+n})^{\mu}a_{n,i}\right]\leq\sum_{i=1}^{m}d_{i}|b|=|b|,$$

Which completes the proof of Theorem(.3)

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

REFERENCES

[1 W.G. Atshan and S. R. Kulkarni, On a new class of analytic functions with negative coefficients, AnaleleUniversitatii Oradea Fasc. Matematica, TomXVI (2009), 43-51.
[2] Y. Komatu, On analytic prolongation of a family of operators ,Mathematica (Cluj) 39 (55) (1990) , 141-145.
[3]H.M. Srivastava and S. Owa (Eds.), Current Topics in Analytic Function Theory , World Scientific , Singapore , 1992.