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Equations of motion and difference schemes for calculating rods under spatially variable loading

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ABSTRACT: In this article is given the work and, based on the refined theory of rods of V.K. Kabulov, systems of differential equations of motion of thin-walled elastoplastic rods of arbitrary cross section under spatially variable loading in current coordinates, taking into account damage accumulation, are presented and are presented in vector form. To solve the boundary value problem, the finite difference method and the elastic solution method are used. As an example, a diagram of the implementation of the calculation of the rods for the elastic case is shown.

KEYWORDS: mathematical model, algorithmization, variational principle, finite difference method, iteration method, elastoplastic core.

I.INTRODUCTION

The paper considers the equations of motion and difference schemes for calculating rods under cyclic loading based on the theory of small elastoplastic deformations by A. A. Ilyushin [1] and the refined theory of rods proposed by V. Z. Vlasov and V. K. Kabulov [2-3].

The questions of algorithmization and automation of solving problems of the theory of elasticity and plasticity were first posed by V.K. Kabulov and developed by his students and followers. The works of T. Buriev developed numerical methods for solving boundary value problems, examined computer implementation issues, and the construction of an algorithmic system for calculating structural elements within and beyond elasticity under variable loads and unloadings at current values, taking into account damage accumulation. T. Yuldashev developed ideas of algorithmization of solving problems of mechanics of a deformable solid in some directions. In particular, the creation of an algorithmic system for processing symbolic information in Deformed Solid Mechanics, algorithms for solving problems of shell structures, mathematical models and an algorithm for calculating thin-walled structures taking into account physical and geometric nonlinearity have been developed.

To study the effect of joint longitudinal, transverse and torsional forces on thin-walled structures of the type of rods under spatially variable loading, the applied theory of rods is used. The development of modern theory of rods gave impetus to the creation of V.Z. Vlasov of the theory of constrained torsion of thin-walled open-profile rods and A.A. Umansky - closed-profile rods. As is known, under spatial loading, the distributions of displacements, strains, and stresses in the cross sections of the rod are quite complex, therefore, the refined theory is based on a number of static hypotheses [2-3].

II.STATEMENT OF BOUNDARYVALUEPROBLEMS.

Consider a thin-walled rod of arbitrary section (rectangular, round, annular) when exposed to external variable forces. The ox axis is directed along the length of the rod, and the and axes are directed along the cross section.

Following [4], we introduce the differences

$$\bar{u}_i^{(n)} = (-1)^n (u_i^{(n-1)} - u_i^{(n)}), \ \bar{e}_{ij}^{(n)} = (-1)^n (e_{ij}^{(n-1)} - e_{ij}^{(n)}), \qquad (1)$$
$$\bar{\sigma}_{ij}^{(n)} = (-1)^n (\sigma_{ij}^{(n-1)} - \sigma_{ij}^{(n)})$$

Based on the assumptions and hypotheses [2, 3], we represent the general displacements of the structure in the form (we omit the dash sign):

In Cartesian coordinates

$$u_1^{(n)}(\mathrm{x,y,z,t}) = u^{(n)} - y\alpha_1^{(n)} - z\alpha_2^{(n)} + \varphi v_1^{(n)} + a_1\beta_1^{(n)} + a_2\beta_2^{(n)},$$
$$u_2^{(n)}(\mathrm{x,y,z,t}) = v^{(n)} - z\theta^{(n)}, \qquad (2)$$
$$u_3^{(n)}(\mathrm{x,y,z,t}) = w^{(n)} + y\theta^{(n)}$$

In cylindrical coordinates($x = x, \quad y = r\cos\gamma, \quad z = r\sin\gamma$):

$$u_1^{(n)}(\mathrm{x,r,}\gamma\mathrm{,t}) = u^{(\mathrm{n})} - \alpha_1^{(\mathrm{n})} r\cos\gamma - \alpha_2^{(\mathrm{n})} r\sin\gamma + \varphi(r,\gamma)v_1^{(\mathrm{n})} + a_1(r,\gamma)\beta_1^{(\mathrm{n})} + a_2(r,\gamma)\beta_2^{(\mathrm{n})},$$
$$u_2^{(n)}(\mathrm{x,r,}\gamma\mathrm{,t}) = v^{(n)} - \theta^{(n)} r\sin\gamma,$$
$$u_3^{(n)}(\mathrm{x,r,}\gamma\mathrm{,t}) = w^{(n)} + \theta^{(n)} r\cos\gamma \qquad (3)$$

where $\alpha_1^{(n)}, \alpha_2^{(n)}$ are the angles of rotation of the cross section under pure bending under n-th loading; $\beta_1^{(n)}$, $\beta_2^{(n)}$ - angles of transverse shear, $\theta^{(n)}$ - angle of torsion, $v_1^{(n)}$ - linear twist under loading, φ - torsion function of Saint-Venant. Here, the sought quantities $u^{(n)}, v^{(n)}, w^{(n)}, \alpha_1^{(n)}, \alpha_2^{(n)}, \theta^{(n)}, v_1^{(n)}, \beta_1^{(n)}, \beta_2^{(n)}$ are functions with respect to the spatial variable x and time t.

According to (3), we determine the components of deformation under n-th loading:

$$\varepsilon_{11}^{(n)} = \frac{\partial u^{(n)}}{\partial x} - r\cos\gamma\frac{\partial \alpha_1^{(n)}}{\partial x} - r\sin\gamma\frac{\partial \alpha_2^{(n)}}{\partial x} + \phi(r,\gamma)\frac{\partial v^{(n)}}{\partial x} + a_1(r,\gamma)\frac{\partial \beta_1^{(n)}}{\partial x} + a_2(r,\gamma)\frac{\partial \beta_2^{(n)}}{\partial x},$$
$$\varepsilon_{13}^{(n)} = \frac{\partial w^{(n)}}{\partial x} + r\cos\gamma\frac{\partial \theta^{(n)}}{\partial x} - \alpha_2^{(n)} + \left(\sin\gamma\frac{\partial \varphi}{\partial r} + \frac{\cos\gamma}{r}\frac{\partial \varphi}{\partial \gamma}\right)v^{(n)} + \left(\sin\gamma\frac{\partial a_1}{\partial r} + \frac{\cos\gamma}{r}\frac{\partial a_1}{\partial \gamma}\right)\beta_1^{(n)} +$$
$$+\left(\sin\gamma\frac{\partial a_2}{\partial r} + \frac{\cos\gamma}{r}\frac{\partial a_2}{\partial \gamma}\right)\beta_2^{(n)},$$
$$\varepsilon_{12}^{(n)} = \frac{\partial v^{(n)}}{\partial x} - r\sin\gamma\frac{\partial \theta^{(n)}}{\partial x} - \alpha_1 + \left(\cos\gamma\frac{\partial \varphi}{\partial r} - \frac{\sin\gamma}{r}\frac{\partial \varphi}{\partial \gamma}\right)v^{(n)} + \left(\cos\gamma\frac{\partial a_1}{\partial r} - \frac{\sin\gamma}{r}\frac{\partial a_1}{\partial \gamma}\right)\beta_1^{(n)} +$$
$$+\left(\cos\gamma\frac{\partial a_2}{\partial r} - \frac{\sin\gamma}{r}\frac{\partial a_2}{\partial \gamma}\right)\beta_2^{(n)} \qquad (4)$$

To derive the equations of motion of the rods under spatial loading, taking into account elastoplastic deformations, we use the Hamilton-Ostrogradsky variational principle [3]:

$$\delta\int_t (T - \Pi + A)dt = 0 \qquad (5)$$

We calculate the kinetic energy variations, using the relation

$$\delta\int_t Tdt = \int_t\int_v \rho\sum_{i=1}^{3}\left(\frac{\partial u_i^{(n)}}{\partial t}\cdot\delta\frac{\partial u_i^{(n)}}{\partial t}\right)dvdt. \qquad (6)$$

Performing integration operations in parts, we obtain:

$$\delta\int_t Tdt = \int_V \rho\sum_{i=1}^{3}\left[\frac{\partial u_i^{(n)}}{\partial t}\cdot\delta u_i^{(n)}\right]dv\Big|_t - \int_t\int_V \rho\sum_{i=1}^{3}\left[\frac{\partial^2 u_i^{(n)}}{\partial t^2}\cdot\delta u_i^{(n)}\right]dvdt. \qquad (7)$$

Substituting expressions u_i from (3) into the kinetic energy variations (7) and performing integration operations over the sections of the rod, we have:

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$$\delta\int_t T dt = \int_x \tilde{A}\frac{\partial Y^{(n)}}{\partial t} E\delta Y^{(n)}\, dx\Big|_t - \int_t\int_x \tilde{A}\frac{\partial^2 Y}{\partial t^2} E\delta Y\ dxdt \qquad (8)$$

where $Y^{(n)} = \{u^{(n)}, v^{(n)}, w^{(n)}, \alpha_1^{(n)}, \alpha_2^{(n)}, \theta^{(n)}, v_1^{(n)}, \beta_1^{(n)}, \beta_2^{(n)}\}$ is the displacement vector, $\tilde{A}$ is the matrix of the ninth order, E is the identity matrix.

Variations of potential energy in this formulation have the form:

$$\delta\int_t \Pi dt = \int_t\int_v\left(\sum_{i=1}^{3}\sigma_{i1}^{(n)}\delta e_{i1}^{(n)}\right)dv = \int_t\int_V\Bigg\{\sigma_{11}^{(n)}\delta\left(\frac{\partial u^{(n)}}{\partial x} - r\cos\gamma\frac{\partial\alpha_1^{(n)}}{\partial x} - r\sin\gamma\frac{\partial\alpha_2^{(n)}}{\partial x} + \varphi\frac{\partial v_1^{(n)}}{\partial x} +$$

$$a_1\frac{\partial\beta_1^{(n)}}{\partial x} + \; + a_2\frac{\partial\beta_2^{(n)}}{\partial x}\right) + \sigma_{13}^{(n)}\delta\left[\frac{\partial w^{(n)}}{\partial x} + r\cos\gamma\frac{\partial\theta^{(n)}}{\partial x} - \alpha_2 + \left(\sin\gamma\frac{\partial\varphi}{\partial r} + \frac{\cos\gamma}{r}\frac{\partial\varphi}{\partial\gamma}\right)v_1^{(n)} +$$

$$+\left(\sin\gamma\frac{\partial a_1}{\partial r} + \frac{\cos\gamma}{r}\frac{\partial a_1}{\partial\gamma}\right)\beta_1^{(n)} + \left(\sin\gamma\frac{\partial a_2}{\partial r} + \frac{\cos\gamma}{r}\frac{\partial a_2}{\partial\gamma}\right)\beta_2^{(n)}\right] +$$

$$+\sigma_{12}^{(n)}\delta\left[\frac{\partial v^{(n)}}{\partial x} - r\sin\gamma\frac{\partial\theta^{(n)}}{\partial x} - \alpha_1^{(n)} + \left(\cos\gamma\frac{\partial\varphi}{\partial r} - \frac{\sin\gamma}{r}\frac{\partial\varphi}{\partial\gamma}\right)v^{(n)} +$$

$$+\left(\cos\gamma\frac{\partial a_1}{\partial r} - \frac{\sin\gamma}{r}\frac{\partial a_1}{\partial\gamma}\right)\beta_1^{(n)} + \left(\cos\gamma\frac{\partial a_2}{\partial r} - \frac{\sin\gamma}{r}\frac{\partial a_2}{\partial\gamma}\right)\beta_2^{(n)}\right]\Bigg\}dVdt. \qquad (9)$$

According to the deformation theory of plasticity, the stress components are connec-ted through deformations under alternating loading in the current coordinates [5]:

$$\sigma_{11}^{(k)} = 3G\left\{e_{11}^{(k)} - \left[\omega^{(k)}e_{11}^{(k)} + \sum_{m=1}^{k-1}\omega^{0(k-m)}\bar{\varepsilon}_{11}^{0(k-m)}\right]\right\},$$

$$\sigma_{13}^{(k)} = G\left\{e_{13}^{(k)} - \omega^{(k)}\bar{\varepsilon}_{13}^{(k)} - \sum_{m=1}^{k-1}\omega^{0(k-m)}\bar{\varepsilon}_{13}^{0(k-m)}\right\},$$

$$\sigma_{12}^{(k)} = G\left\{e_{12}^{(k)} - \omega^{(k)}\bar{\varepsilon}_{12}^{(k)} - \sum_{m=1}^{k-1}\omega^{0(k-m)}\bar{\varepsilon}_{12}^{0(k-m)}\right\}. \qquad (10)$$

Here

$$\omega^{(n)} = \begin{cases} 0, & \text{при } \bar{\varepsilon}_u^{(n)} \le \bar{\varepsilon}_s^{(n)}(\eta) \\ \lambda_n\left[1 - \dfrac{\bar{\varepsilon}_s^{(n)}(\eta)}{\bar{\varepsilon}_u^{(n)}}\right], & \text{при } \bar{\varepsilon}_u^{(n)} > \bar{\varepsilon}_s^{(n)}(\eta) \end{cases}$$

In the case of the generalized Masing principle $\lambda_n = \lambda,\ \bar{\varepsilon}_u^{(n)} = \alpha_n\varepsilon_s$, when using the Gusenkov-Schneiderovich strain diagrams $\bar{\varepsilon}_u^{(n)} = 2\varepsilon_s,\ \lambda_n = 1 - g_n$, where g_n is determined experimentally, and when damage accumulation is taken into account

$$\bar{\varepsilon}_s^{(n)}(\eta) = \alpha_1^{n-z}(1+\alpha_1)\varepsilon_s + (3G)^{-1}B^{1/\alpha}\cdot\left[1 - 0{,}5(1+\alpha_1)\alpha_1^{n-2}\right]\left[1-(1-\eta)^{1+\alpha}\right]^{1/\alpha}(n-1)^{-1/\alpha}$$

The damage function η is determined from the kinetic equation [4]:

$$\frac{d\eta}{dn} = A\frac{\left(\bar{\sigma}_u^{(n)}\right)^{\alpha}}{\left(1-\gamma\eta^{r}\right)^{\beta}}, \qquad (11)$$

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under the condition η (0) = 0, η (N) = 1, where N is the number of half-cycles before the onset of the limiting state (destruction).

Now we transform the variations of potential energy. To do this, open the brackets and select the integral over the cross section of the bar. After some calculations and notation from (9), we have:

$$\delta\int_t \Pi dt = \int_t \left\{ N_x^{(n)}\delta u^{(n)} - M_z^{(n)}\delta\alpha_1^{(n)} - M_y^{(n)}\delta\alpha_2^{(n)} + Q_y^{(n)}\delta v^{(n)} + Q_z^{(n)}\delta w^{(n)} + M_x^{(n)}\delta\theta^{(n)} + \right.$$

$$\left. + M_\varphi^{(n)}\delta v_1^{(n)} + M_{\alpha_1}^{(n)}\delta\beta_1^{(n)} + M_{\alpha_2}^{(n)}\delta\beta_2^{(n)} \right\} dt\Big|_x - \int_t\int_x \left\{ \frac{\partial N_x^{(n)}}{\partial x}\delta u^{(n)} + \frac{\partial Q_y^{(n)}}{\partial x}\delta v^{(n)} + \frac{\partial Q_z^{(n)}}{\partial x}\delta w^{(n)} + \right.$$

$$+ \left(Q_y^{(n)} - \frac{\partial M_z^{(n)}}{\partial x} \right)\delta\alpha_1^{(n)} + \left(Q_z^{(n)} - \frac{\partial M_y^{(n)}}{\partial x} \right)\delta\alpha_2^{(n)} + \frac{\partial M_x^{(n)}}{\partial x}\delta\theta^{(n)} + \left(\frac{\partial M_\varphi^{(n)}}{\partial x} - Q_{v_1}^{(n)} \right)\delta v_1^{(n)} +$$

$$\left. + \left(\frac{\partial M_{\alpha_1}^{(n)}}{\partial x} - Q_{\beta_1}^{(n)} \right)\delta\beta_1^{(n)} + \left(\frac{\partial M_{\alpha_2}^{(n)}}{\partial x} - Q_{\beta_2}^{(n)} \right)\delta\beta_2^{(n)} \right\} dxdt \qquad (12)$$

The following notation is introduced here:

$$\int_F r\sigma_{11}^{(n)} drd\gamma = N_x^{(n)}, \quad \int_F r\sigma_{12}^{(n)} drd\gamma = Q_y^{(n)}, \quad \int_F r^2\cos\gamma\sigma_{11}^{(n)} drd\gamma = M_z^{(n)},$$

$$\int_F \left(r^2\cos\gamma\sigma_{13}^{(n)} - r^2\sin\gamma\sigma_{12}^{(n)} \right) drd\gamma = M_x^{(n)}, \quad \int_F r^2\sin\gamma\sigma_{11}^{(n)} drd\gamma = M_y^{(n)},$$

$$\int_F r\sigma_{13}^{(n)} drd\gamma = Q_z^{(n)}, \int_F r\varphi\sigma_{11}^{(n)} drd\gamma = M_\varphi^{(n)}, \quad \int_F ra_1\sigma_{11}^{(n)} drd\gamma = M_{\alpha_1}^{(n)}, \quad \int_F ra_2\sigma_{11}^{(n)} drd\gamma = M_{\alpha_2}^{(n)},$$

$$\int_F \left[\left(r\sin\gamma\frac{\partial\varphi}{\partial r} + \cos\gamma\frac{\partial\varphi}{\partial\gamma} \right)\sigma_{13}^{(n)} + \left(r\cos\gamma\frac{\partial\varphi}{\partial r} - \sin\gamma\frac{\partial\varphi}{\partial\gamma} \right)\sigma_{12}^{(n)} \right] drd\gamma = Q_v^{(n)},$$

$$\int_F \left[\left(r\sin\gamma\frac{\partial a_1}{\partial r} + \cos\gamma\frac{\partial a_1}{\partial\gamma} \right)\sigma_{13}^{(n)} + \left(r\cos\gamma\frac{\partial a_1}{\partial r} - \sin\gamma\frac{\partial a_1}{\partial\gamma} \right)\sigma_{12}^{(n)} \right] drd\gamma = Q_{\beta_1}^{(n)},$$

$$\int_F \left[\left(r\sin\gamma\frac{\partial a_2}{\partial r} + \cos\gamma\frac{\partial a_2}{\partial\gamma} \right)\sigma_{13}^{(n)} + \left(r\cos\gamma\frac{\partial a_2}{\partial r} - \sin\gamma\frac{\partial a_2}{\partial\gamma} \right)\sigma_{12}^{(n)} \right] drd\gamma = Q_{\beta_2}^{(n)}. \qquad (13)$$

Taking into account the introduced notation, expressions for internal efforts and moments, for example N_x and M_{a_2} can be represented as:

$$N_x^{(k)}(x,t) = 3G\left\{ \left(\tilde{F} - \tilde{F}_\omega^{(k)} \right)\frac{\partial u^{(k)}}{\partial x} - \left(S_z - S_{z\omega}^{(k)} \right)\frac{\partial \alpha_1^{(k)}}{\partial x} - \left(S_y - S_{y\omega}^{(k)} \right)\frac{\partial \alpha_2^{(k)}}{\partial x} + \right.$$

$$+ \left(S_\varphi - S_{\varphi\omega}^{(k)} \right)\frac{\partial v_1^{(k)}}{\partial x} + \left(S_{a_1} - S_{a_1\omega}^{(k)} \right)\frac{\partial \beta_1^{(k)}}{\partial x} + \left(S_{a_2} - S_{a_2\omega}^{(k)} \right)\frac{\partial \beta_2^{(k)}}{\partial x} -$$

$$-F_\omega^{(k)}\frac{\partial u^{o(k-1)}}{\partial x} + S_{z\omega}^{(k)}\frac{\partial \alpha_1^{o(k-1)}}{\partial x} + S_{y\omega}^{(k)}\frac{\partial \alpha_2^{o(k-1)}}{\partial x} - S_{\varphi\omega}^{(k)}\frac{\partial v_1^{o(k-1)}}{\partial x} - S_{a_1\omega}^{(k)}\frac{\partial \beta_1^{o(k-1)}}{\partial x} - S_{a_2\omega}^{(k)}\frac{\partial \beta_2^{o(k-1)}}{\partial x} +$$

$$+\sum_{m=1}^{k-1}\left[F_\omega^{o(k-m)}\frac{\partial}{\partial x}\left(u^{o(k-m)} - u^{o(k-m-1)} \right) - S_{z\omega}^{o(k-m)}\frac{\partial}{\partial x}\left(\alpha_1^{o(k-m)} - \alpha_1^{o(k-m-1)} \right) - \right.$$

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$$-S_{y\omega}^{o(k-m)}\frac{\partial}{\partial x}\left(\alpha_2{}^{o(k-m)}-\alpha_2{}^{o(k-m-1)}\right)+S_{\varphi\omega}^{o(k-m)}\frac{\partial}{\partial x}\left(\nu_1{}^{o(k-m)}-\nu_1{}^{o(k-m-1)}\right)+$$

$$+S_{a_1\omega}^{o(k-m)}\frac{\partial}{\partial x}\left(\beta_1{}^{o(k-m)}-\beta_1{}^{o(k-m-1)}\right)+S_{a_2\omega}^{o(k-m)}\frac{\partial}{\partial x}\left(\beta_2{}^{o(k-m)}-\beta_2{}^{o(k-m-1)}\right)\Big]\Big\};$$

$$M_{a_2}^{(k)}(x,t)=3G\left\{\left[\left(S_{a_2}-S_{a_2\omega}^{(k)}\right)\frac{\partial u^{(k)}}{\partial x}+\left(I_{a_2y}-I_{a_2y}^{\omega(k)}\right)\frac{\partial\alpha_1{}^{(k)}}{\partial x}+\left(I_{a_1z}-I_{a_1z}^{\omega(k)}\right)\frac{\partial\alpha_2{}^{(k)}}{\partial x}\right.\right.$$

$$+\left(I_{a_2\varphi}-I_{a_2\varphi}^{\omega(k)}\right)\frac{\partial\nu_1{}^{(k)}}{\partial x}+\left(I_{a_1a_2}-I_{a_1a_2}^{\omega(k)}\right)\frac{\partial\beta_1^{(k)}}{\partial x}+\left(I_{a_2}-I_{a_2\omega}^{(k)}\right)\frac{\partial\beta_2^{(k)}}{\partial x}-$$

$$-S_{a_2\omega}^{(k)}\frac{\partial u^{0(k-1)}}{\partial x}+I_{a_2y}^{\omega(k)}\frac{\partial\alpha_1{}^{0(k-1)}}{\partial x}+I_{a_1z}^{\omega(k)}\frac{\partial\alpha_2{}^{0(k-1)}}{\partial x}-I_{a_2\varphi}^{\omega(k)}\frac{\partial\nu_1{}^{0(k-1)}}{\partial x}-I_{a_1a_2}^{\omega(k)}\frac{\partial\beta_1^{0(k-1)}}{\partial x}-I_{a_2\omega}^{(k)}\frac{\partial\beta_2^{0(k-1)}}{\partial x}+$$

$$+\sum_{m=1}^{k-1}\left[S_{a_2}^{0(k-m)}\frac{\partial}{\partial x}\left(u^{0(k-m)}-u^{0(k-m-1)}\right)-I_{a_2y}^{0(k-m)}\frac{\partial}{\partial x}\left(\alpha_1{}^{0(k-m)}-\alpha_1{}^{0(k-m-1)}\right)-\right.$$

$$-I_{a_1z}^{0(k-m)}\frac{\partial}{\partial x}\left(\alpha_2{}^{0(k-m)}-\alpha_2{}^{0(k-m-1)}\right)+I_{a_2\varphi}^{0(k-m)}\frac{\partial}{\partial x}\left(\nu_1{}^{0(k-m)}-\nu_1{}^{0(k-m-1)}\right)+$$

$$\left.+I_{a_1a_2}^{(k)}\frac{\partial}{\partial x}\left(\beta_1^{0(k-m)}-\beta_1^{0(k-m-1)}\right)+I_{a_2}^{(k)}\frac{\partial}{\partial x}\left(\beta_2^{0(k-m)}-\beta_2^{0(k-m-1)}\right)\right] \tag{14}$$

where

$$\tilde{F}=\int_F rdrd\gamma,\quad S_z=\int_F r^2\cos\gamma\, drd\gamma,\quad S_y=\int_F r^2\sin\gamma\, drd\gamma,\quad S_\varphi=\int_F r\varphi(r,\gamma)drd\gamma$$

$$S_{a_1}=\int_F ra_1(r,\gamma)drd\gamma,\quad S_{a_2}=\int_F ra_2(r,\gamma)drd\gamma,\quad J_{a_2y}=\int_F r^2a_2(r,\gamma)\cos\gamma\, drd\gamma,$$

$$J_{a_2z}=\int_F r^2a_2(r,\gamma)\sin\gamma\, drd\gamma,\quad J_{a_2\phi}=\int_F r\phi a_2(r,\gamma)drd\gamma,\quad J_{a_1a_2}=\int_F ra_1a_2drd\gamma,$$

$$J_{a_2}=\int_F ra_2^2(r,\gamma)drd\gamma,$$

In a similar way, integrals $F_\omega,\ldots,J_{a_2}^{\omega}$ containing plasticity functions are defined, for example,

$$\tilde{F}_\omega=\int_F \omega rdrd\gamma,\ldots,\ J_{a_2}^{\omega}=\int_F \omega a_2^2(r,\gamma)rdrd\gamma.$$

Substituting expressions of type (14) on the potential energy variations, we obtain:

$$\delta\int_t \Pi dt=\int_t\left\{\left(A^{yn}-A^{n\pi}\right)\frac{\partial Y^{(n)}}{\partial x}+\left(B^{yn}-B^{n\pi}\right)Y^{(n)}\right\}E\delta Y^{(n)}\,dt\Big|_t+\int_t\int_x\left\{\frac{\partial}{\partial x}\left(\left(A^{yn}-A^{n\pi}\right)\frac{\partial Y^{(n)}}{\partial x}+\right.\right.$$

$$\left.+\left(B^{yn}-B^{n\pi}\right)Y^{(n)}\right)+\left(C^{yn}-C^{n\pi}\right)\frac{\partial Y^{(n)}}{\partial x}+\left(D^{yn}-D^{n\pi}\right)Y^{(n)}\Big\}E\delta Y^{(n)}dxdt \tag{15}$$

Variations in the work of external forces are taken in the form of:

$$\delta A=\int_V\sum_{i=1}^{3}p_i^{(n)}\delta u_i^{(n)}d\upsilon+\int_s\sum_{i=1}^{3}q_i^{(n)}\delta u_i^{(n)}ds+\int_{s_1}\sum_{i=1}^{3}f_i^{(n)}\delta u_i^{(n)}ds_1\Big|_x \tag{16}$$

where p_i is volume forces, q_i is surface forces, f_i is end forces.

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In the variation of the work of external forces (16), we substitute the expressions of displacements (3) and, integrating over the cross sections of the rod, we have:

$$\delta\int_t A dt = \int_t Q^{гр}\delta y dt\Big|_x + \int_t\int_x Q^{н} dy dx dt \tag{17}$$

Derivation of the equation of motion. Substituting vector expressions of the variation of kinetic (8), potential (15) energies and work of external forces (17) into the variational principle (5) we obtain:

$$\int_t\int_x \left\{ \tilde{A}\frac{\partial^2 Y}{\partial t^2} + \frac{\partial}{\partial x}\left[\left(A^{уп} - A^{пл}\right)\frac{\partial Y}{\partial x} + \left(B^{уп} - B^{пл}\right)Y\right] + \right.$$
$$\left. +\left(C^{уп} - C^{пл}\right)\frac{\partial Y}{\partial x} + \left(D^{уп} - D^{пл}\right)Y + Q\right\} E\,\delta Y\, dx\, dt +$$
$$+\int_t \left\{\left(A^{уп} - A^{пл}\right)\frac{\partial Y}{\partial x} + \left(B^{уп} - B^{пл}\right)Y + Q^{гр}\right\} E\,\delta Y\, dt\Big|_x + \int_x \tilde{A}\frac{dY}{dt} E\delta Y\, dx\Big|_t = 0 \tag{18}$$

From this variational equation we obtain the following boundary-value problem for the kth loading in vector form: equations of motion

$$\tilde{A}\frac{\partial^2 Y}{\partial t^2} + \frac{\partial}{\partial x}\left[\left(A^{уп} - A^{пл(k)}\right)\frac{\partial Y^{(k)}}{\partial x} + \left(B^{уп} - B^{пл(k)}\right)Y^{(k)}\right] + \left(C^{уп} - C^{пл(k)}\right)\frac{\partial Y^{(k)}}{\partial x} +$$
$$+\left(D^{уп} - D^{пл(k)}\right)Y^{(k)} = Q_n^{(k)} + \frac{\partial}{\partial x}\left(A^{пл(k)}\frac{\partial Y^{0(k-1)}}{\partial x} + B^{пл(k)}Y^{0(k-1)}\right) + C^{пл(k)}\frac{\partial Y^{0(k-1)}}{\partial x} +$$
$$+D^{пл(k)}Y^{0(k-1)} + \sum_{m=1}^{k-1}\left\{\frac{\partial}{\partial x}\left[A^{пло(k-m)}\frac{\partial}{\partial x}\left(Y^{0(k-m)} - Y^{0(k-m-1)}\right) + B^{пло(k-m)}\left(Y^{0(k-m)} - Y^{0(k-m-1)}\right)\right] + \right.$$
$$\left. +C^{пло(k-m)}\frac{\partial}{\partial x}\left(Y^{0(k-m)} - Y^{0(k-m-1)}\right) + D^{пло(k-m)}\left(Y^{0(k-m)} - Y^{0(k-m-1)}\right)\right\}; \tag{19}$$

border conditions

$$\left\{\left(A^{уп} - A^{пл(k)}\right)\frac{\partial Y^{(k)}}{\partial x} + \left(B^{уп} - B^{пл(k)}\right)Y^{(k)} - \bar{Q}_{гр}^{(k)} - B^{пло(k)}Y^{0(k-1)} - A^{пло(k)}\frac{\partial Y^{0(k-1)}}{\partial x} - \right.$$
$$\left. -\sum_{m=1}^{k-1}\left[A^{пло(k-m)}\frac{\partial}{\partial x}\left(Y^{0(k-m)} - Y^{0(k-m-1)}\right) + B^{пло(k-m)}\left(Y^{0(k-m)} - Y^{0(k-m-1)}\right)\right]\right\}\delta Y^{(k)}\Big|_x = 0 \tag{20}$$

initial conditions

$$\tilde{A}\frac{dY^{(n)}}{dt}E\delta\, Y^{(n)}\Big|_t = 0 \tag{21}$$

Here the matrices A, B, C, D are quadratic matrices of the ninth order, $Q^{н}$ and $Q^{гр}$ the vectors of external forces of the ninth order and have the form:

$(a_{ij} = a_{ij}^{уп} - a_{ij}^{пл(n)}, b_{ij} = b_{ij}^{уп} - b_{ij}^{пл(n)}, c_{ij} = c_{ij}^{уп} - c_{ij}^{пл(n)}, d_{ij} = d_{ij}^{уп} - d_{ij}^{пл(n)})$

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$$A=\begin{pmatrix} a_{11} & 0 & 0 & a_{14} & a_{15} & 0 & a_{17} & a_{18} & a_{19} \\ 0 & a_{22} & 0 & 0 & 0 & a_{26} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & a_{36} & 0 & 0 & 0 \\ a_{41} & 0 & 0 & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} \\ a_{51} & 0 & 0 & a_{54} & a_{55} & 0 & a_{57} & a_{58} & a_{59} \\ 0 & a_{62} & a_{63} & 0 & 0 & a_{66} & 0 & 0 & 0 \\ a_{71} & 0 & 0 & a_{74} & a_{75} & 0 & a_{77} & a_{78} & a_{79} \\ a_{81} & 0 & 0 & a_{84} & a_{85} & 0 & a_{87} & a_{88} & a_{89} \\ a_{91} & 0 & 0 & a_{94} & a_{95} & 0 & a_{97} & a_{98} & a_{99} \end{pmatrix}, \quad B=\begin{pmatrix} b_{11} & 0 & 0 & b_{14} & b_{15} & 0 & b_{17} & b_{18} & b_{19} \\ 0 & b_{22} & 0 & 0 & 0 & b_{26} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 & b_{36} & 0 & 0 & 0 \\ b_{41} & 0 & 0 & b_{44} & b_{45} & 0 & b_{47} & b_{48} & b_{49} \\ b_{51} & 0 & 0 & b_{54} & b_{55} & 0 & b_{57} & b_{58} & b_{59} \\ 0 & b_{62} & b_{63} & 0 & 0 & b_{66} & 0 & 0 & 0 \\ b_{71} & 0 & 0 & b_{74} & b_{75} & 0 & b_{77} & b_{78} & b_{79} \\ b_{81} & 0 & 0 & b_{84} & b_{85} & 0 & b_{87} & b_{88} & b_{89} \\ b_{91} & 0 & 0 & b_{94} & b_{95} & 0 & b_{97} & b_{98} & b_{99} \end{pmatrix},$$

$$C=\begin{pmatrix} c_{11} & 0 & 0 & c_{14} & c_{15} & 0 & c_{17} & c_{18} & c_{19} \\ 0 & c_{22} & 0 & 0 & 0 & c_{26} & c_{27} & c_{28} & c_{29} \\ 0 & 0 & c_{33} & 0 & 0 & c_{36} & c_{37} & c_{38} & c_{39} \\ c_{41} & c_{42} & 0 & c_{44} & c_{45} & c_{46} & c_{47} & c_{48} & c_{49} \\ c_{51} & 0 & c_{53} & c_{54} & c_{55} & c_{56} & c_{57} & c_{58} & c_{59} \\ 0 & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & c_{67} & c_{68} & c_{69} \\ c_{71} & c_{72} & c_{73} & c_{74} & c_{75} & c_{76} & c_{77} & c_{78} & c_{79} \\ c_{81} & c_{82} & c_{83} & c_{84} & c_{85} & c_{86} & c_{87} & c_{88} & c_{89} \\ c_{91} & c_{92} & c_{93} & c_{94} & c_{95} & c_{96} & c_{97} & c_{98} & c_{99} \end{pmatrix}, \quad D_n=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 & d_{27} & d_{28} & d_{29} \\ 0 & 0 & 0 & 0 & d_{35} & 0 & d_{37} & d_{38} & d_{39} \\ 0 & 0 & 0 & d_{44} & 0 & 0 & d_{47} & d_{48} & d_{49} \\ 0 & 0 & 0 & 0 & d_{55} & 0 & d_{57} & d_{58} & d_{59} \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{67} & d_{68} & d_{69} \\ 0 & 0 & 0 & d_{74} & d_{75} & 0 & d_{77} & d_{78} & d_{79} \\ 0 & 0 & 0 & d_{84} & d_{85} & 0 & d_{87} & d_{88} & d_{89} \\ 0 & 0 & 0 & d_{94} & d_{95} & 0 & d_{97} & d_{98} & d_{99} \end{pmatrix},$$

where

$$a_{11}=\tilde{F},\ a_{14}=-S_z, a_{15}=-S_y, a_{17}=S_\varphi, a_{18}=S_{a_1},\ a_{19}=S_{a_2};\quad a_{22}=\tilde{F},\ a_{26}=-S_y; a_{33}=\tilde{F}_1\ ,$$

$$a_{36}=S_z;\ a_{41}=-S_z,\ a_{44}=J_z,\ a_{45}=J_{yz}, a_{47}=-J_{y\varphi},\ a_{48}=-J_{ya_1},\ a_{49}=-J_{ya_2};\ a_{51}=-S_y,$$

$$a_{54}=J_{yz}, \mathrm{a}_{55}=J_y, a_{57}=J_{z\varphi}, a_{58}=-J_{za_1}, a_{59}=-J_{za_2};\ a_{62}=-S_y, a_{63}=S_z,\ \mathrm{a}_{66}=J_\rho;$$

$$a_{71}=S_\varphi, a_{74}=-J_{y\varphi},\ a_{74}=-J_{z\varphi}, a_{77}=J_{\varphi_2},\ a_{78}=J_{a_1\varphi},\ a_{79}=J_{a_2\varphi};\ a_{81}=S_{a_1}, a_{84}=-J_{ya_1}, a_{85}=-J_{za_1},$$

$$a_{87}=J_{a_1\varphi}, a_{88}=J_{a_1},\ a_{89}=J_{a_1a_2}; a_{91}=S_{a_2}, a_{94}=-J_{ya_2},\ a_{95}=-J_{za_2},\ a_{97}=J_{a_2\varphi},\ a_{98}=J_{a_1a_2},\ a_{99}=J_{a_2}$$

Similarly, matrix elements b_{ij}, c_{ij}, d_{ij} and integrals $F_\omega^{(n)},\ldots,S_{y\omega}^{(n)}$ containing plasticity functions $\omega^{(n)}$ in matrix coefficients are determined, for example:

$$F_\omega^{(n)}=\int_F \omega^{(n)}dF,\ldots I_\omega^{a_2\varphi}=\int_F \omega^{(n)}a_2\varphi dF$$

Expressions of internal efforts and moments in vector form can be represented as:

$$P^{(n)}(x,t)=\frac{3Gh_0I_0}{l^3}\left\{(\tilde{A}^{yn}-\tilde{A}^{пл(n)})\frac{\partial u^{(n)}}{\partial x}+(\tilde{B}^{yn}-\tilde{B}^{пл(n)})u^{(n)}\right\}. \tag{22}$$

where

$$P^{(n)}(x,t)=\left\{N_x^{(n)},M_y^{(n)},M_z^{(n)},M_\phi^{(n)},M_{a_1}^{(n)},M_{a_2}^{(n)},Q_1^{(n)},M_x^{(n)},Q_2^{(n)},Q_{\alpha_1}^{(n)},Q_{\alpha_2}^{(n)},M_\varphi^{(n)}\right\}$$

Here $\tilde{A}^{yn}, \tilde{A}^{пл(n)}, \tilde{B}^{yn}, \tilde{B}^{пл(n)}$ are twelfth-order square matrices and the elements are described as follows:

$$\tilde{a}_{ij}=a_{ij}^{yn},\quad \tilde{a}_{10,s}=b_{s,5};\quad \tilde{a}_{11,s}=b_{s,6};\quad \tilde{a}_{12,s}=b_{s,4};\quad \tilde{b}_{ij}=b_{ij},$$

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$$\tilde{b}_{10,r}=d_{2,s};\ \tilde{b}_{11,r}=d_{r,6};\ \tilde{b}_{12,r}=d_{r,4};\ (i,j=1,2,..9;s=7,8,9;r=2,3,4,5,6;).$$

III.CONSTRACTING ADIFFERENCE SCHEME.

To solve the boundary value problem, the finite difference method and the method of elastic solutions by A.A. Ilyushin are used.

Following [5], we write equations (19) in the following form:

$$\tilde{A}\frac{\partial^2}{\partial t^2}\overline{Y}^{(k)}+\frac{\partial}{\partial x}\left[\left(A^{yn}-A^{n\pi(k)}\right)\frac{\partial}{\partial x}\overline{Y}^{(k)}+\left(B^{yn}-B^{n\pi(k)}\right)\overline{Y}^{(k)}\right]+\left(C^{yn}-C^{n\pi(k)}\right)\frac{\partial}{\partial x}\overline{Y}^{(k)}+\left(D^{yn}-D^{n\pi(k)}\right)\overline{Y}^{(k)}=\overline{F}^{(k)}\quad(23)$$

where

$$\vec{F}^{(k)}=\vec{Q}^{(k)}+\vec{Q}^{n\pi(k)}+\vec{Q}^{n\pi 0(k)};$$

$$\vec{Q}^{n\pi(k)}=\frac{\partial}{\partial x}\left(A^{n\pi(k)}\frac{\partial}{\partial x}\overline{Y}^{0(k-1)}+B^{n\pi(k)}\overline{Y}^{0(k-1)}\right)+C^{n\pi(k)}\frac{\partial}{\partial x}\overline{Y}^{0(k-1)}+D^{n\pi(k)}\overline{Y}^{0(k-1)};$$

$$\vec{Q}^{n\pi 0(k)}=\sum_{m=1}^{k-1}\left\{\frac{\partial}{\partial x}\left[A^{n\pi(k-m)}\frac{\partial}{\partial x}\left(\overline{Y}^{0(k-m)}-\overline{Y}^{0(k-m-1)}\right)+B^{n\pi(k-m)}\left(\overline{Y}^{0(k-m)}-\overline{Y}^{0(k-m-1)}\right)\right]+\right.$$

$$\left.+C^{n\pi o(k-m)}\frac{\partial}{\partial x}\left(\vec{Y}^{0(k-m)}-\vec{Y}^{0(k-m-1)}\right)+D^{n\pi o(k-m)}\left(\vec{Y}^{0(k-m)}-\vec{Y}^{0(k-m-1)}\right)\right.,$$

The boundary conditions (20) can be rewritten in the following form

$$\left[\left(A^{yn}-A^{n\pi(k)}\right)\frac{\partial\overline{Y}^{(k)}}{\partial x}+\left(B^{yn}-B^{n\pi(k)}\right)\overline{Y}^{(k)}\right]\Bigg|_{\Gamma}=\hat{\vec{F}}^{(k)}\quad(24)$$

where

$$\hat{\vec{F}}^{(k)}=\hat{\vec{Q}}_{\varepsilon p}^{(k)}+\hat{\vec{Q}}^{n\pi(k)}+\hat{\vec{Q}}^{n\pi 0(k)}$$

The notation introduced here

$$\hat{\vec{Q}}^{n\pi(\kappa)}=A^{n\pi(\kappa)}\frac{\partial}{\partial x}\vec{Y}^{0(k-1)}+B^{n\pi(\kappa)}\vec{Y}^{0(k-1)},$$

$$\hat{\vec{Q}}^{n\pi o(\kappa)}=\sum_{m=1}^{k-1}\left[A^{n\pi o(k-m)}\frac{\partial}{\partial x}\left(\overline{Y}^{0(k-m)}-\overline{Y}^{0(k-m-1)}\right)+B^{n\pi o(k-m)}\left(\overline{Y}^{0(k-m)}-\overline{Y}^{0(k-m-1)}\right)\right]$$

When constructing solutions to the system of differential equations (23) with boundary (25) and initial (21) conditions, the centraldifference scheme of the second order of accuracy [6]:

$$\frac{\partial Y}{\partial t}=\frac{1}{2\tau}\left(Y_{i,j+1}-Y_{i,j-1}\right),\quad \frac{\partial^2 Y}{\partial t^2}=\frac{1}{\tau^2}\left(Y_{i,j+1}-2Y_{i,j}+Y_{i,j-1}\right),$$

$$\frac{\partial Y}{\partial x}=\frac{1}{2h}\left(Y_{i+1,j}-Y_{i-1,j}\right),\quad \frac{\partial^2 Y}{\partial x^2}=\frac{1}{h^2}\left(Y_{i+1,j}-2Y_{i,j}+Y_{i-1,j}\right)\quad(25)$$

where $t=j\tau,\ \ x=ih$

Using (25), we approximate the terms of the differential expressions of system (23). As a result, we obtain (at $\omega=0$):

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$$Y_{i,j+1} = \tilde{A}Y_{i-1,j} + \tilde{B}Y_{i,j} + \tilde{C}Y_{i+1,j} + \tilde{F}_{i,j} - Y_{i,j-1}; \tag{26}$$

where

$$\tilde{A} = \tau^2 A^{-1}\left(\frac{B}{h^2} - \frac{C}{2h}\right), \quad \tilde{B} = \tau^2 A^{-1}\left(\frac{2A}{\tau^2} - \frac{2B}{h^2} + D_n\right), \tilde{C} = \tau^2 A^{-1}\left(\frac{B}{h^2} + \frac{C}{2h}\right), \quad \tilde{F}_{i,j} = \tau^2 A^{-1} F_{i,j}$$

We now approximate the boundary conditions (24); for this, we take the approximation with a step forward for i = 0 and the approximation with a step back for i = N

$$i = 0, \quad \frac{\partial Y}{\partial x} = \frac{1}{2h}\left(-3Y_{0,j} + 4Y_{1,j} - Y_{2,j}\right), \tag{27}$$

$$i = N, \quad \frac{\partial Y}{\partial x} = \frac{1}{2h}\left(3Y_{N,j} - 4Y_{N-1,j} + Y_{N-2,j}\right), \tag{28}$$

Then from (24), taking into account (27) $i = 0$ and (28) $i = N$, we have

$$Y_{0,j} = \overline{D}^{-1}\left(\frac{2\overline{B}}{h}Y_{1,j} - \frac{\overline{B}}{2h}Y_{2,j} + \overline{C}_A Y^0_{0,j} - P^{\text{гр}}_{0,j}\right), \quad \overline{D} = \frac{3\overline{B}}{2h} + \overline{C}, \tag{29}$$

$$Y_{N,j} = \tilde{\overline{D}}^{-1}\left(-\frac{2\overline{B}}{h}Y_{N-1,j} + \frac{\overline{B}}{2h}Y_{N-2,j} + \overline{C}_A Y^0_{N,j} - P^{\text{гр}}_{N,j}\right), \quad \tilde{\overline{D}} = -\frac{3\overline{B}}{2h} + \overline{C}. \tag{30}$$

When $i = 1$, it follows from (26) that

$$Y_{1,j+1} = \tilde{A}Y_{0,j} + \tilde{B}Y_{1,j} + \tilde{C}Y_{2,j} + \tilde{F}_{1,j} - Y_{1,j-1};$$

from here, taking into account (29), after reducing such terms, we obtain

$$Y_{1,j+1} = \tilde{\tilde{B}}Y_{1,j} + \tilde{\tilde{C}}Y_{2,j} + \tilde{\tilde{F}}_{1,j} - Y_{1,j-1}; \tag{31}$$

where

$$\tilde{\tilde{B}} = \tilde{A}\overline{D}^{-1}\frac{2\overline{B}}{h} + \tilde{B}, \quad \tilde{\tilde{C}} = -\tilde{A}\overline{D}^{-1}\frac{2\overline{B}}{h} + \tilde{C}, \quad \tilde{\tilde{F}}_{1,j} = -\tilde{A}\overline{D}^{-1}P^{\text{гр}}_{0,j} + \tilde{F}_{1,j}.$$

For $i = N-1$ from (26) we have

$$Y_{N-1,j+1} = \tilde{A}Y_{N-2,j} + \tilde{B}Y_{N-1,j} + \tilde{C}Y_{N,j} + \tilde{F}_{N-1,j} - Y_{N-1,j-1}; \tag{32}$$

We substitute (30) into (32), after the reduction of such terms we have

$$Y_{N-1,j+1} = \tilde{\tilde{A}}Y_{N-2,j} + \tilde{\tilde{B}}_1 Y_{N-1,j} + \tilde{\tilde{F}}_{N-1,j} - Y_{N-1,j-1}; \tag{33}$$

where, $\tilde{\tilde{A}} = \tilde{A} + \tilde{C}\tilde{\overline{D}}^{-1}\frac{\overline{B}}{2h}$, $\quad \tilde{\tilde{B}}_1 = \tilde{B} - \tilde{C}\tilde{\overline{D}}^{-1}\frac{2\overline{B}}{h}$, $\quad \tilde{\tilde{F}}_{N-1,j} = -\tilde{C}\tilde{\overline{D}}^{-1}P^{\text{гр}}_{N,j} + \tilde{F}_{N-1,j}$.

In equations (31), (26) and (33), respectively, the functions $Y_{1,j-1}, Y_{i,j-1}, Y_{N-1,j-1}$. are participated. These functions at t = 0 or j = 0 are not yet known. They are determined from the initial conditions (21).

From the initial conditions (21) we have

$$A\frac{\partial Y}{\partial t}\Big|_{t=0} = \dot{Y}^0_{i,0}, \quad Y\Big|_{t=0} = Y^0_{i,0}, \tag{34}$$

since $t = 0$ corresponds to $j = 0$, hence

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$$\frac{A}{2\tau}\left(Y_{i,j+1}-Y_{i,j-1}\right)=\dot{Y}^0_{i,0},\quad Y_{i,0}=Y^0_{i,0}, \tag{35}$$

for $j=0$ from relation (35) we have

$$Y_{i,-1}=Y_{i,1}-2\tau\, A^{-1}\,\dot{Y}^0_{i,0}, Y_{i,0}=Y^0_{i,0}. \tag{36}$$

The functions $Y_{1,1}$ from (31), $Y_{i,j}$ from (26), $Y_{i-1,j}$ from (33) after using relation (36) are expressed in terms of the initial conditions.

From (31) with $i=1,\ j=0$ we have

$$Y_{1,1}=\tilde{\tilde{B}}Y^0_{1,0}+\tilde{\tilde{C}}Y^0_{2,0}+\tilde{\tilde{F}}_{1,0}-Y_{1,-1}.$$

Hence, given (36)

$$Y_{1,1}=\frac{1}{2}\left(\tilde{\tilde{B}}Y^0_{1,0}+\tilde{\tilde{C}}Y^0_{2,0}-2\tau\, A^{-1}\,\dot{Y}^0_{i,0}+\tilde{\tilde{F}}_{1,0}\right), \tag{37}$$

at $i=i,\ j=0$ from (26)

$$Y_{i,1}=\tilde{A}Y_{i-1,0}+\tilde{B}Y_{i,0}+\tilde{C}Y_{i+1,0}+\tilde{F}_{i,0}-Y_{i,-1}.$$

In view of (34) - (36)

$$Y_{i,1}=\frac{1}{2}\left(\tilde{A}Y_{i-1,0}+\tilde{B}Y_{i,0}+\tilde{C}Y_{i+1,0}\right)+\tau\, A^{-1}\,\dot{Y}^0_{i,0}. \tag{38}$$

From (33) with $i=N-1,\ j=0$, taking into account (36), we have

$$Y_{N-1,1}=\frac{1}{2}\left(\tilde{\tilde{A}}Y^0_{N-2,0}+\tilde{\tilde{B}}Y^0_{N-1,0}+\tilde{\tilde{F}}_{N-1,0}\right)+\tau\, A^{-1}\,\dot{Y}^0_{N-i,0}\ . \tag{39}$$

As a result, we obtain the following system of finite difference equations:

with $i=1,\ j=0$ we have;

$$Y_{1,1}=\frac{1}{2}\left(\tilde{\tilde{B}}Y^0_{1,0}+\tilde{\tilde{C}}Y^0_{2,0}+\tilde{\tilde{F}}_{1,0}-2\tau\, A^{-1}\,\dot{Y}^0_{i,0}\right),$$

with $i=1,\ j=0$;

$$Y_{i,1}=\frac{1}{2}\left(\tilde{A}Y_{i-1,0}+\tilde{B}Y_{i,0}+\tilde{C}Y_{i+1,0}+\tilde{F}_{i,0}\right)+\tau\, A^{-1}\,\dot{Y}^0_{i,0},$$

with $i=N-1,\ j=0$;

$$Y_{N-1,1}=\frac{1}{2}\left(\tilde{\tilde{A}}Y^0_{N-2,0}+\tilde{\tilde{B}}Y^0_{N-1,0}+\tilde{\tilde{F}}_{N-1,0}\right)+\tau\, A^{-1}\,\dot{Y}^0_{N-i,0}\ ,$$

with $i=1,\ j=1$;

$$Y_{1,2}=\tilde{\tilde{B}}Y_{1,1}+\tilde{\tilde{C}}Y_{2,1}+\tilde{\tilde{F}}_{1,1}-Y^0_{1,0}\ ,$$

with $i=1,\ j=1$;

$$Y_{i,2}=\tilde{\tilde{A}}Y_{i-1,1}+\tilde{\tilde{B}}Y_{i,1}+\tilde{\tilde{F}}_{i,1}-Y^0_{i,0}, \tag{40}$$

with $i=N-1,\ j=1$;

$$Y_{N-1,2}=\tilde{A}Y_{N-2,1}+\tilde{B}Y_{N-1,1}+\tilde{C}Y_{N,1}+\tilde{\tilde{F}}_{N-1,1}-Y^0_{N-1,0},$$

with $i=1,\ j\geq 2$;

$Y_{1,j+1} = \tilde{\tilde{B}}Y_{1,j} + \tilde{\tilde{C}}Y_{2,j} + \tilde{\tilde{F}}_{1,j} - Y_{1,j-1}$,
with $i = i,\ j \geq 2$;
$Y_{i,j+1} = \tilde{A}Y_{i-1,j} + \tilde{B}Y_{i,j} + \tilde{C}Y_{i+1,j} + \tilde{F}_{i,j} - Y_{i,j-1}$,
with $i = N-1\ j \geq 2$;

$$Y_{N-1,j+1} = \tilde{\tilde{A}}Y_{N-2,j} + \tilde{\tilde{B}}Y_{N-1,j} + \tilde{\tilde{F}}_{N-1,j} - Y_{N-1,j-1}.$$

Thus, the Cauchy problem was formed in the form of algebraic equations (40). Here, the inner loop is parameter i, and the outer loop is parameter j.

As an example, we consider the elastic-plastic calculation of thin-walled rods based on a generalized diagram of cyclic deformation under repeated static loading [4]. Calculation of rods of rectangular cross section, pinched at the ends with the following initial data: geometric and mechanical characteristics of the rod: $l = 2{,}5\text{м};\ h = 0{,}1\text{м};$ $b_0 = 0{,}1\text{м};\ E = 2*10^5\ МПа\ \varepsilon_s = 0.0015$

uniformly distributed external loads: $f_0^+ = 25$; $f_0^- = 50$; $\bar{f}_0^+ = 10$; $\bar{f}_0^- = 5$ $(кг/см^2)$; $\bar{\gamma} = \frac{\pi}{4}; \alpha = \frac{\pi}{3}; \gamma* = \frac{\pi}{6}; \alpha* = \frac{\pi}{2}; q^{(k)} = \delta(-1)^{k+1}$ $(\delta = 1;1.5;2)$.

Table 1 shows the numerical values of the calculated quantities $W^{(k)}$, $\alpha_1^{(k)}$, $\beta_1^{(k)}$ along the length of the rod under cyclic loading $(k = 1,2,5,6)$.

Table 1

	x	k=1	k=2	k=5	k=6
$W^{(k)}$	0,1	-0,038498	0,038514	-0,038502	0,038515
	0,2	-0,124965	0,125019	-0,124982	0,125024
	0,4	-0,284147	0,284276	-0,284190	0,284286
	0,6	-0,284124	0,284251	-0,284166	0,284262
	0,8	-0,124919	0,124971	-0,124934	0,124976
	0,9	-0,038463	0,038478	-0,038467	0,038480
$\alpha_1^{(k)}$	0,1	-0,724490	0,724800	-0,724583	0,724827
	0,2	-0,965903	0,966339	-0,966050	0,966376
	0,4	-0,482706	0,482927	-0,482782	0,482945
	0,6	0,483426	-0,483655	0,483511	-0,483673
	0,8	0,966394	-0,966825	0,966536	-0,966861
	0,9	0,724772	-0,725073	0,724856	-0,725100
$\beta_1^{(k)}$	0,1	-0,021403	0,021411	-0,021405	0,021412
	0,2	-0,016126	0,016134	-0,016129	0,016135
	0,4	-0,005407	0,005411	-0,005409	0,005411
	0,6	0,005309	-0,005312	0,005311	-0,005313
	0,8	0,016028	-0,016036	0,016031	-0,016037
	0,9	0,021306	-0,021314	0,021307	-0,021314

Changes in the components of displacements along the length of the bar are shown in Figure 1.

Figure 1.

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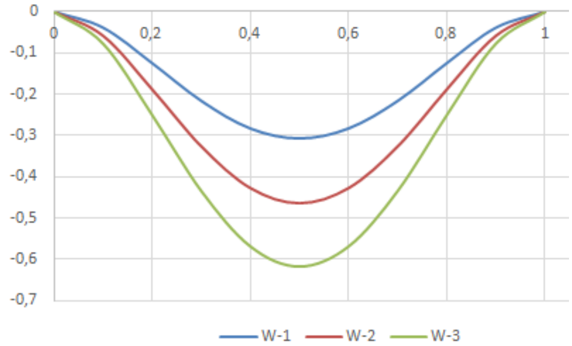

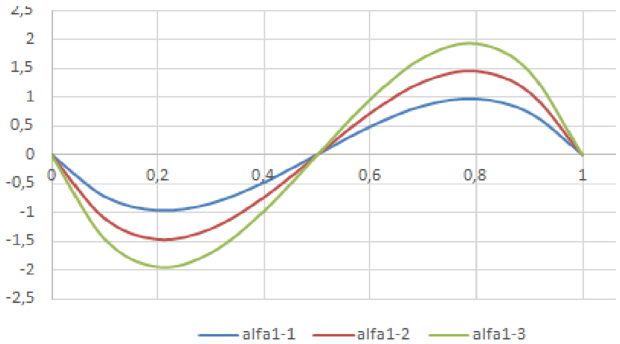

Figure 2 shows the change in the ductility zone with respect to loading cycles at $k = 2,3,4,5,6$.

Figure 2.

k=2	k=3	k=4	k=5	k=6
				
0-0,2	0,2-0,4	0,4-0,6	0,6-0,8	0,8-1

IV.CONCLUSION AND FUTURE WORK

Findings. Based on the Hamilton-Ostrogradsky variational principle and refined theory of rods, systems of differential equations of motion of thin-walled elastoplastic rods of arbitrary cross section under spatially variable loading in current coordinates are derived.

To solve the boundary value problem, the finite difference method and the elastic solution method are used. As an illustration, a diagram of the implementation of the calculation of the rods under re-static loading is shown.

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