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SIMPSON'S Rule to Solve Fuzzy Differential Equations

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ABSTRACT: Fuzzy differential equations are used in modeling problems in science and engineering. In this paper we define the fuzzy differential equation of the first order and solve this equation by numerical solution in simpson's rule . We showed the approximate solution by using this method, and applied in matlab computer software.

KEYWORDS: Fuzzy differential equation, fuzzy numbers,*simpson'sruleMethod*

I. INTRODUCTION

Zadeh [1] was the first introduced The concept of fuzzy set.. The fuzzy differential equation and fuzzy initial value problems are studied by Kaleva [2], [3] and Seikkala [9]. In the last few years, many researchers have worked on theoretical and numerical Solution of FDEs [5–10]. In this paper we introduced the solve of fuzzy differential equation and using the ***simpson's rule Method*** to solve some examples by using computer software to find the approximation solution.

II. PRELIMINARIES

A definition of fuzzy numbers can found in [4]. Fuzzy numbers may be triangular or triangular shaped fuzzy numbers. A triangular fuzzy number N is defined by three real numbers a < b < c, where the base of the triangle is the interval [a, c] and its vertex is at

x = b, it is written as N= (a/b/c). The membership function for the triangular fuzzy number N = (a/b/c) is defined as the following:

$$N(x) = \{\frac{x-a}{b-a} a \le x \le b \frac{x-c}{b-c} b \le x \le c$$

we require the graph of the corresponding membership function to be continuous and:

[1] Monotonically increasing on [a, b].

[2] Monotonically decreasing on [b, c].

The core of a fuzzy number is the set of values where the membership value equals one.

If N = (a/b/c) or N ≈ (a/b/c) then the core of N is the single point b. Let T be the set of all triangular or triangular shaped fuzzy numbers and η∈ T.

We define the r-level set

$$[\eta]_r = \{\eta(x) \ge r\}, \quad 0 \le r \le$$

Which is a closed bounded interval and denoted by

$$[\eta]_r = [\underline{\eta}(r), \underline{\eta}(r)]$$

It is clear that the following statements are true.

- $\underline{\eta}(r)$ Is a bounded left continuous non decreasing function over [0, 1].
- $\underline{\eta}(r)$ Is a bounded right continuous non increasing function over [0,1].
- $\underline{\eta}(r) \le \underline{\eta}(r)$for all r∈ [0, 1]. For more details, see [4].

III. FIRST ORDER FUZZY DIFFERENTIAL EQUATION

A first order fuzzy initial value differential equation is given by:

$$\{y' = f(t, y(t)) t \in [t_o, T] y(t_o) = y_o$$

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Such that y is a fuzzy function of t, f (t, y) is a fuzzy function of the scrips variable t and the fuzzy variable y. y_o is the fuzzy derivative of y and $y(t_o) = y_o$ is a triangular or a triangular shaped fuzzy number. We denote the fuzzy function y by y = [$\underline{y}$,$\underline{y}$]. It means that the r-level set of y(t) for $t \in [t_o, T]$ is

$$[\, y(t)]_r = [\underline{y}(t;r), \underline{y}(t;r)]$$

Also:

$$[y'(t)]_r = [\underline{y}'(t;r), \underline{y}'(t;r)]$$
$$[\, f(t, y(t))]_r = [\underline{f}(t, y(t);r), \underline{f}(t, y(t);r)]$$

We write:

$$f(t,y) = [\underline{f}(t,y), \underline{f}(t,y)]$$

We have:

$$\underline{y}'(t;r) = \underline{f}(t, y(t);r) = F[t, \underline{y}(t;r), \underline{y}(t;r)]$$
$$\underline{y}'(t;r) = \underline{f}(t, y(t);r) = G$$

Also we write

$$[\, y(t_o)]_r = [\underline{y}(t_o;r), \underline{y}(t_o;r)]$$
$$[y_o]_r = [\left(\underline{y}_o(r)\right), \left(\underline{y}_o(r)\right)]$$
$$\underline{y}(t_o;r) = \underline{y}_o(r) \quad , \quad \underline{y}(t_o;r) = \underline{y_o}$$

By using the extension principle, we have the membership function:

$$f(t, y(t))(s) = sup\ sup\{s = f(t,\tau)\}, \quad s$$

So Fuzzy number

$$f\big(t, y(t)\big)$$
$$[f(t, y(t))]_r = \left[\underline{f}(t, y(t);r), \underline{f}(t, y(t);r)\right], \qquad r \in [0,1]$$

Where

$$\underline{f}(t, y(t);r) = \{u \in [y(t)]_r\}$$
$$\underline{f}(t, y(t);r) = \{u \in [y(t)]_r\}$$

IV. *SIMPSON'S RULE METHOD* IN FUZZY DIFFERENTIAL EQUATION

The form first order fuzzy differential equation as

$$y'(t) = f(t,y)$$
$$y(t_o) = y_o$$

The exact solution be

$$[Y(t_n)]_r = [\underline{Y}(t_n;r), \underline{Y}(t_n;r)]$$

The approximation solution is given by

$$[y(t_n)]_r = [\underline{y}(t_n;r), \underline{y}(t_n;r)]$$

By using ***simpson's rule Method*** have

$$[y(t_n)]_r = [\underline{y}(t_n;r), \underline{y}(t_n;r)$$

$$\underline{y}(t_{n+1};r) = \underline{y}(t_o) + \int_0^t \underline{f}(t_n, y(t_n,r)dt$$

$$\underline{y}(t_{n+1};r) = \underline{y}(t_o) + \int_0^t \underline{f}(t_n, y(t_n,r)dt$$

$$\int_0^t \underline{f}(t_n, y(t_n,r)dt \cong \frac{h}{6}(k_{1,1}(t_n, y(t_n;r)) + 4\left(\frac{k_{2,1}(t_n, y(t_n;r) + 2\,k_{3,1}(t_n, y(t_n;r)}{2}\right) + k_{4,1}(t_n, y(t_n;r))$$

$$\int_0^t \underline{f}(t_n, y(t_n,r)dt \cong \frac{h}{6}(k_{1,2}(t_n, y(t_n;r)) + 4\left(\frac{k_{2,2}(t_n, y(t_n;r) + 2\,k_{3,2}(t_n, y(t_n;r)}{2}\right) + k_{4,2}(t_n, y(t_n;r))$$

Where $k_{j,1}, k_{j,2}$ define as follow:

$$k_{1,1}(t_n, y(t_n;r)) = h\,\{y(t_n,u)|u \in \left(\underline{y}(t_n;r), \underline{y}(t_n;r)\right)\}$$

$$k_{1,2}(t_n, y(t_n;r)) = h\,\{y(t_n,u)|u \in \left(\underline{y}(t_n;r), \underline{y}(t_n;r)\right)\}$$

$$k_{2,1}(t_n, y(t_n;r)) = h\,\{y(t_n + \frac{h}{2}, u)|u \in \left(q_{1,1}(t_n; y(t_n,r)), q_{1,2}(t_n; y(t_n,r))\right)\}$$

$$k_{2,2}(t_n, y(t_n;r)) = h\,\{y(t_n + \frac{h}{2}, u)|u \in \left(q_{1,1}(t_n; y(t_n,r)), q_{1,2}(t_n; y(t_n,r))\right)\}$$

$$k_{3,1}(t_n, y(t_n;r)) = h\,\{y(t_n + \frac{h}{2}, u)|u \in \left(q_{2,1}(t_n; y(t_n,r)), q_{2,2}(t_n; y(t_n,r))\right)\}$$

$$k_{3,2}(t_n, y(t_n;r)) = h\,\{y(t_n + \frac{h}{2}, u)|u \in \left(q_{2,1}(t_n; y(t_n,r)), q_{2,2}(t_n; y(t_n,r))\right)\}$$

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$$k_{4,1}\big(t_n, y(t_n;r)\big) = h\,\{y(t_n + \frac{h}{2}, u) | u \in \Big(q_{3,1}\big(t_n; y(t_n,r)\big), q_{3,2}\big(t_n; y(t_n,r)\big)\Big)\}$$

$$k_{4,2}\big(t_n, y(t_n;r)\big) = h\,\{y(t_n + \frac{h}{2}, u) | u \in \Big(q_{3,1}\big(t_n; y(t_n,r)\big), q_{3,2}\big(t_n; y(t_n,r)\big)\Big)\}$$

Where:

$$q_{1,1}\big(t_n; y(t_n,r)\big) = \underline{y}(t_n,r) + \frac{h}{2} k_{1,1}\big(t_n, y(t_n;r)\big)$$

$$q_{1,2}\big(t_n; y(t_n,r)\big) = \underline{y}(t_n,r) + \frac{h}{2} k_{1,2}\big(t_n, y(t_n;r)\big)$$

$$q_{2,1}\big(t_n; y(t_n,r)\big) = \underline{y}(t_n,r) + \frac{h}{2} k_{2,1}\big(t_n, y(t_n;r)\big)$$

$$q_{2,2}\big(t_n; y(t_n,r)\big) = \underline{y}(t_n,r) + \frac{h}{2} k_{2,2}\big(t_n, y(t_n;r)\big)$$

$$q_{3,1}\big(t_n; y(t_n,r)\big) = \underline{y}(t_n,r) + \frac{h}{2} k_{3,1}\big(t_n, y(t_n;r)\big)$$

$$q_{3,2}\big(t_n; y(t_n,r)\big) = \underline{y}(t_n,r) + \frac{h}{2} k_{3,2}\big(t_n, y(t_n;r)\big)$$

Now using the initial condition, we compute:

$$\underline{y}(t_{n+1};r) = \underline{y}(t_n;r) + \frac{h}{6}\Big(k_{1,1}\big(t_n, y(t_n;r)\big) + 4\left(\frac{k_{2,1}(t_n, y(t_n;r) + 2\,k_{3,1}(t_n, y(t_n;r)}{2}\right) + k_{4,1}\big(t_n, y(t_n;r)\big)$$

$$\underline{y}(t_{n+1};r) = \underline{y}(t_n;r) + \frac{h}{6}\Big(k_{1,2}\big(t_n, y(t_n;r)\big) + 4\left(\frac{k_{2,2}(t_n, y(t_n;r) + 2\,k_{3,2}(t_n, y(t_n;r)}{2}\right) + k_{4,2}\big(t_n, y(t_n;r)\big)$$

The solution at t_n

$$0 \le n \le N \quad and\ a = t_o \le t_1 \le t_2 \le \cdots \le t_n = b, \text{ and } h = \frac{b-a}{N} = t_{n+1} - t_n,$$

$$\underline{Y}(t_{n+1};r) = \underline{Y}(t_n;r) + F[t_n, y(t_n;r)]$$

$$\underline{Y}(t_{n+1};r) = \underline{Y}(t_n;r) + G[t_n, y(t_n;r)]$$

Example (1):

Let the fuzzy initial value problem

$$y'(t) = y(t), \quad t \in [0,1]$$

$$y(0) = (0.75 + 0.25r, \qquad 1.125 - 0.125\,r), \qquad 0 < r \le 1$$

Solve:

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The exact solution is obtained as:

$$\underline{Y}(t;r) = \underline{y}(t;r)e^{t}$$
$$\underline{Y}(t;r) = \underline{y}(t;r)e^{t}$$

At t=1:

$$Y(1;r) = [(0.75+0.25r)e, \qquad (1.125-0.125\,r)e]$$

The approximation solution is at h=0.01 and a=0, b=1, then we have:

$$0 \le n \le N \quad and \quad 0 = t_o \le t_1 \le t_2 \le \cdots \le t_n = 1, \text{ and } 0.01 = \frac{1-0}{N} = t_{n+1} - t_n$$

$$k_{1,1}\big(t_n, y(t_n;r)\big) = (0.01)\,\{y(t_n,u)|u \in (0.75+0.25r, 1.125-0.125\,r)\}$$
$$k_{1,2}\big(t_n, y(t_n;r)\big) = (0.01)\,\{y(t_n,u)|u \in (0.75+0.25r, 1.125-0.125\,r)\}$$
$$q_{1,1}\big(t_n; y(t_n,r)\big) = 0.75+0.25r+\frac{0.01}{2}k_{1,1}\big(t_n, y(t_n;r)\big)$$
$$q_{1,2}\big(t_n; y(t_n,r)\big) = 1.125-0.125\ +\frac{0.01}{2}k_{1,2}\big(t_n, y(t_n;r)\big)$$
$$k_{2,1}\big(t_n, y(t_n;r)\big) = h\,\{y(t_n+\frac{h}{2}, u)|$$
$$u \in \Big(q_{1,1}\big(t_n; y(t_n,r)\big), q_{1,2}\big(t_n; y(t_n,r)\big)\Big)\}$$
$$k_{2,1}\big(t_n, y(t_n;r)\big) = 0.01\,\{y(t_n+\frac{0.01}{2}, u)|$$
$$u \in \Big(q_{1,1}\big(t_n; y(t_n,r)\big), q_{1,2}\big(t_n; y(t_n,r)\big)\Big)\}$$
$$k_{2,2}\big(t_n, y(t_n;r)\big) = 0.01\,\{y(t_n+\frac{0.01}{2}, u)|u$$
$$\in \Big(q_{1,1}\big(t_n; y(t_n,r)\big), q_{1,2}\big(t_n; y(t_n,r)\big)\Big)\}$$
$$q_{2,1}\big(t_n; y(t_n,r)\big) = 0.75+0.25r+\frac{0.01}{2}k_{2,1}\big(t_n, y(t_n;r)\big)$$
$$q_{2,2}\big(t_n; y(t_n,r)\big) = 1.125-0.125\,r+\frac{0.01}{2}k_{2,2}\big(t_n, y(t_n;r)\big)$$
$$k_{3,1}\big(t_n, y(t_n;r)\big) = \ 0.01\,\{y(t_n+\frac{h}{2}, u)|$$
$$u \in \Big(q_{2,1}\big(t_n; y(t_n,r)\big), q_{2,2}\big(t_n; y(t_n,r)\big)\Big)\}$$
$$k_{3,2}\big(t_n, y(t_n;r)\big) = \ 0.01\,\{y(t_n+\frac{h}{2}, u)|$$
$$u \in \Big(q_{2,1}\big(t_n; y(t_n,r)\big), q_{2,2}\big(t_n; y(t_n,r)\big)\Big)\}$$

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$$q_{3,1}\left(t_n;y(t_n,r)\right) = 0.75 + 0.25r + \frac{0.01}{2}k_{3,1}\left(t_n,y(t_n;r)\right)$$
$$q_{3,2}\left(t_n;y(t_n,r)\right) = 1.125 - 0.125 \ + \frac{0.01}{2}k_{3,2}\left(t_n,y(t_n;r)\right)$$
$$k_{4,1}\left(t_n,y(t_n;r)\right) = 0.01\ \{y(t_n+\frac{h}{2},u)|u \in \left(q_{3,1}\left(t_n;y(t_n,r)\right),q_{3,2}\left(t_n;y(t_n,r)\right)\right)\}$$
$$k_{4,2}\left(t_n,y(t_n;r)\right) = 0.01\ \{y(t_n+\frac{h}{2},u)|u \in \left(q_{3,1}\left(t_n;y(t_n,r)\right),q_{3,2}\left(t_n;y(t_n,r)\right)\right)\}$$

Then:

$$\underline{y}(t_{n+1};r) = \underline{y}(t_n;r) + \frac{h}{6}(k_{1,1}\left(t_n,y(t_n;r)\right) + 4\left(\frac{k_{2,1}(t_n,y(t_n;r) + 2\ k_{3,1}(t_n,y(t_n;r)}{2}\right) + k_{4,1}\left(t_n,y(t_n;r)\right)$$
$$\underline{y}(t_{n+1};r) = \underline{y}(t_n;r) + \frac{h}{6}(k_{1,2}\left(t_n,y(t_n;r)\right) + 4\left(\frac{k_{2,2}(t_n,y(t_n;r) + 2\ k_{3,2}(t_n,y(t_n;r)}{2}\right) + k_{4,2}\left(t_n,y(t_n;r)\right)$$
$$\underline{Y}(t;r) = \underline{y}(t;r)e^t$$
$$\underline{Y}(t;r) = \underline{y}(t;r)e^t$$

NUMERICAL RESULTS

Using the Matlab program we get the following results

Exact solutions

Table 1: Table of results

r	$\underline{Y}$	$\overline{Y}$
0	2.310539554	2.990110011
0.1	2.351313782	2.962927193
0.2	2.392088009	2.935744375
0.3	2.432862237	2.908561557
0.4	2.473636464	2.881378738
0.5	2.514410691	2.85419592
0.6	2.555184919	2.827013102
0.7	2.595959146	2.799830283
0.8	2.636733374	2.772647465

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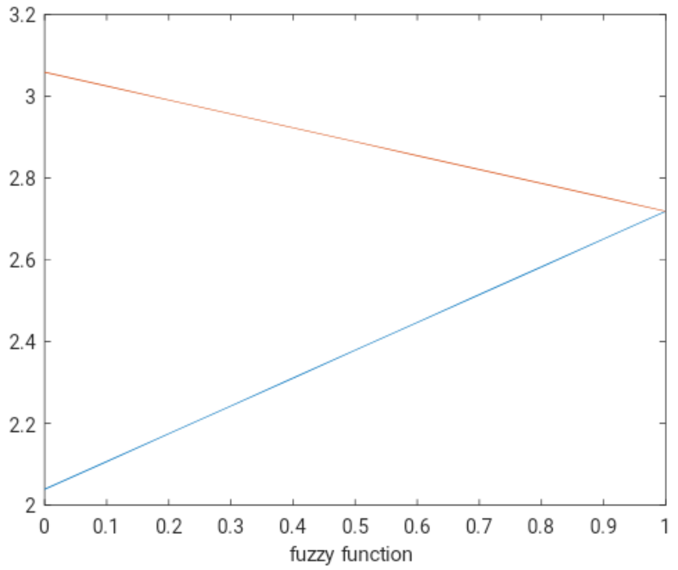
0.9	2.677507601	2.745464647
1	2.718281829	2.718281829

Approximated solution at h =0.01

r	$\underline{Y}$	$\overline{Y}$
0	2.038	3.058
0.1	2.1068	3.0243
0.2	2.1748	2.9904
0.3	2.2428	2.9564
0.4	2.3107	2.9224
0.5	2.3787	2.8884
0.6	2.4466	2.8544
0.7	2.5146	2.8204
0.8	2.5826	2.7865
0.9	2.6506	2.7525
1	2.7185	2.7185



Example (2):

Let the fuzzy initial value problem

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$$y'(t) = t\,y(t), \quad t \in [-1,1]$$
$$y(-1) = \left(\sqrt{e} - 0.5(1-r), \qquad \sqrt{e} + 0.5(1-r)\right), \qquad 0 < r \leq 1$$

Solve:

The exact solution is obtained as:

$$\underline{Y}(t;r) = \underline{y}(t;r)e^{\frac{t^2}{2}}$$
$$\underline{Y}(t;r) = \underline{y}(t;r)e^{\frac{t^2}{2}}$$

And by using the same way that is use in example (1) we have the following

Table 1: Table of results

r	$\underline{Y}$	$\overline{Y}$
0	0.696857202110789	1.309813785190604
0 . 1	0.712224986300327	1.299603428706493
0 . 2	0.730043929114086	1.276666324045802
0 . 3	0.772345602129690	1.269899343438809
0 . 4	0.800032178907357	1.240565002589583
0 . 5	0.816333610893120	1.213245087076441
0 . 6	0.867777732105656	1.202312310779002
0 . 7	0.884238907123690	1.194004658720914
0 . 8	0.923987056500236	1.803496725890742
0 . 9	1.148769032908787	1.692356708368020
1	1.152364689203983	1.152364689203983

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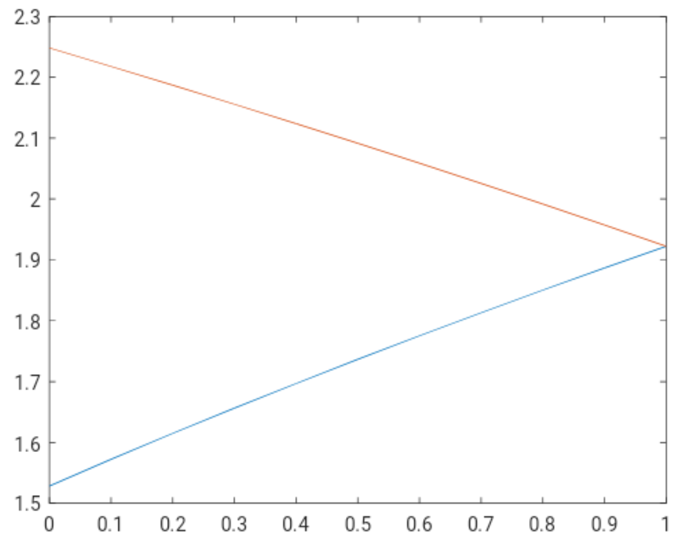
Using the Matlab program we get the following results

Approximated solution at h =0.01

r	$\underline{Y}$	$\overline{Y}$
0	0.926991239529380	1.363649610994994
0.1	0.953584164655085	1.345186626429590
0.2	0.979455338439511	1.326466718221489
0.3	1.004660520232769	1.307478851413578
0.4	1.029248639460408	1.288211177750099
0.5	1.053262912452451	1.268650949199094
0.6	1.076741734894929	1.248784419280649
0.7	1.099719402657452	1.228596730031087
0.8	1.122226699690389	1.208071781961549
0.9	1.144291381779442	1.187192083775436
1	1.165938577855379	1.165938577855379



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