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Using Darsy Theory of Filtration of Multiphase Systems in the Cardiovascular System

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ABSTRACT: This article is considered as a review part for creating a mathematical model and software for the pathology of the circulatory system in the cardiovascular system. At the beginning, the cardiovascular system, organ system, pathology of the cardiovascular system, the international classifier of diseases of heart and vascular diseases are studied, and a mathematical model of blood circulation in the cardiovascular system and Darcy's law is presented..

KEYWORDS*:* mathematical model and software pathology, organ system, international classifier of diseases of heart disease and blood vessels.

I.INTRODUCTION

Authors O.F. Voropaeva, Yu.I. Shokin in his article on the topic "Numerical modeling in medicine: Some statements of problems and results of calculations *" upset the problems of the cardiovascular system, the study of blood cells and various processes in the blood-sucking system at the micro- and nano level. O.F. Voropayeva and Yu.I. Shokin used methods of hemodynamic models on graphs. In this article, classical models of hemodynamics are used based on the Navier-Stokes equations and 2 D and 3 D modeling of blood, large and small blood vessels [10-11,38, 39 pg] is connected and the modeling of the vascular network, cerebral circulation is separately attached [12,38,39,40 pg].

"Mathematical modeling of the circulatory system and its practical applications" in the journal "Avtomat and Telemach" in 2006, the authors A.P. Proshin, Yu. V. Solodyannikov, used the construction of a mathematical model of the circulatory system as a nonlinear oscillatory system and made a computer model of theories of neural networks. They used the neurocomputer analogy of the atrioventricular node of the heart as the conductor of the His-Purkin system [12,176,177,178,179 pg].

Kiselev, I.N., Semisalov. B.V, Biberdorf.A, Sharipov, R.N, Blokhin. A.M, Kalpakov.F.A in the article "Modular modeling of the cardiovascular system of man and agent modeling". The model is made in BioUML and this mathematical model was derived from the Navier-Stokes equations [13,710,712 pg].

Authors from Portugal Alexandra Buyalno de Moura Technical University Alibona and together with Angola in 2012 written article about "Mathematical Models and Simulations of the Human Cardiovascular System". The article proposes a simulation of the organism and blood circulation and uses the Navier-Stokes equations and proposes nonlinear systems of Newton's equations and hydrodynamics. Made 3 D modeling.

For simplicity, we will assume that, $A_0 = const$ and $\beta = const$ (т.е. $R_0 = const$, $h_0 = const$ and modul Yunga E = const). In this formula written in a quasilinear form
$\partial_t U + B(U)\partial_z U = S(U)$ for the vector of unknowns where

$$B = \begin{pmatrix} 0 & 1 \\ \dfrac{\beta}{2\rho A_0}\sqrt{A} - \alpha\dfrac{Q^2}{A^2} & 2\alpha\dfrac{Q}{A} \end{pmatrix},\ S = \begin{pmatrix} 0 \\ -K_r\dfrac{Q}{A} \end{pmatrix}.$$

Under the natural requirement A> 0, system (1.1) is a one-dimensional quasilinear hyperbolic system of equations. The matrix B has two real and different eigenvalues.

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$$\lambda_{1,2} = \alpha \frac{Q}{A} \pm \sqrt{d}\,, \qquad (1.2)$$

where

$$d = \frac{\beta}{2\rho A_0}\sqrt{A} + \alpha(\alpha - 1)\left(\frac{Q}{A}\right)^2$$

($\alpha \geq 1 \Rightarrow d \geq 0$). Moreover, with standard physiological conditions typical values of blood velocity $u = Q/A$ and mechanical characteristics of the vessel walls are such $\sqrt{d} >> \alpha u$ that, i.e.

$$\lambda_1 > 0 \text{ и } \lambda_2 < 0\,. \qquad (1.3)$$

Thus, the hyperbolic system (2.1) on the interval (0, L) has one outgoing characteristic at the boundaries z = 0 and z = L, and therefore it requires one boundary condition at each boundary.

System (2.1) on smooth solutions can also be rewritten in a conservative form, more precisely in the form of a hyperbolic system of balance laws.

$$\partial_t U + \partial_z (F(U)) = S(U)\,, \qquad (1.4)$$

where

$$F = \begin{pmatrix} 0 \\ \alpha \frac{Q^2}{A} + \frac{\beta}{3\rho A_0} A\sqrt{A} \end{pmatrix}$$

We now discuss briefly the question of reducing the system (1.1) to the canonical form. Note that a hyperbolic system of two equations can always be localized to canonical form, i.e. in a sufficiently small neighborhood of an arbitrary point U. However, for the particular case α = 1, there also exist global Riemann invariants. Let (l_1, l_2) and (r_1, r_2) be a pair of left and right eigenvectors of the matrix B. Let us introduce the matrices.

$$L = \begin{pmatrix} l_1^T \\ l_2^T \end{pmatrix},\ R = (r_1 \quad r_2),\ \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

where, without breaking community $LR = I$. Then matrix B can be represented as $B = L\Lambda R$,
and system (1.1) is equivalently rewritten as:

$$L\partial_t U + \Lambda L \partial_z U = LS\,.$$

The Riemann invariants (for our system) are, as is known, the quantities and, satisfying the relations

$$\frac{\partial R_1}{\partial U} = l_1,\ \frac{\partial R_2}{\partial U} = l_2$$

Believing, $R = (R_1, R_2)^T$ bring system (1.1) to a diagonal form. The finding of Riemann invariants is discussed in detail in this article. We here give only the result (for the case α = 1):

$$R_{1,2} = \frac{Q}{A} \pm 4\left(A^{\frac{1}{4}} - A_0^{\frac{1}{4}}\right)\sqrt{\frac{\beta}{2\rho A_0}}$$

Note that knowledge of the Riemann invariants is important, for example, for the correct formulation of the boundary conditions at z = 0 and z = L. So besides the fact that, by (1.3), for each boundary for system (1.1), one boundary condition must be set it is necessary that these boundary conditions be resolved for "leaving" Riemann invariants, that is, for Riemann invariants that correspond to the leaving characteristics. Consider first the case of a single artery. As already noted, for an artery of length L on the left (at z = 0) and on the right border (at z = L) it is necessary to put on one boundary condition. From a mathematical point of view, it is absolutely unimportant what these conditions will be, and what is important is that they can be reduced to the form when the "leaving" Riemann invariants

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are expressed in terms of the “coming” ones. Since in our case, by virtue of (1.3), the “outgoing” Riemann invariant on the left boundary is, and on the right boundary, it is necessary that the boundary conditions be reduced to

$$R_1|_{z=0} = S_1(t) R_2|_{z=0} + g_1(t),\ R_2|_{z=L} = S_2(t) R_1|_{z=L} + g_2(t),$$

Where $S_{1,2}(t)$ and $g_{1,2}(t)$ -are some functions.

On the other hand, the boundary conditions must be reasonable from a physical point of view. On the left border, such a reasonable boundary condition is the pressure setting:

$$p|_{z=0} = q_p(t),$$

where $q_p(t)$ - some input pressure profile, (in the normal case, we considered a piece of sinusoid) simulating the pressure at the outlet of the heart.

Darcy's law is applicable to filtering fluids that obey Newton's viscous friction law (Navier – Stokes law). For filtering non-Newtonian fluids (for example, some oils), the relationship between the pressure gradient and the filtration rate can be non-linear or non-algebraic (for example, differential).

For Newtonian liquids, the scope of Darcy's law is limited to low filtration rates (Reynolds numbers, calculated from the characteristic pore size, are less than or on the order of unity). At high speeds, the dependence between the pressure gradient and the filtration rate is not linear (good agreement with the experimental data is given by the quadratic dependence — Forchheimer's filtration law).

It is planned to write down the Darcy law (the law of filtration of blood in tissues) for the right boundary conditions and introduce it into the model.

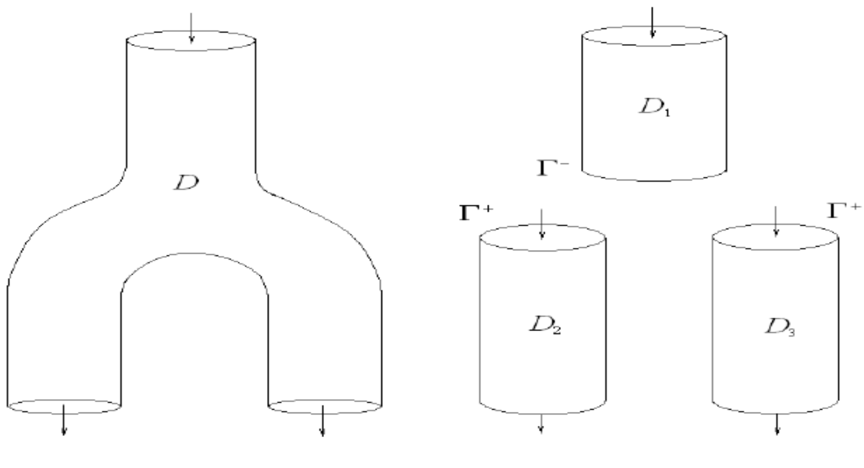

Fig. 1.1. One-dimensional model of artery bifurcation (area partitioning method).

The arterial system is characterized by the presence of branches. The flow of blood in places of artery branching is essentially three-dimensional. However, using the method of partitioning a region, one can describe it within the framework of a one-dimensional model. In Fig. 1.1 shows a diagram of one-dimensional bifurcation. We simplify the real geometric structure of the bifurcation, assuming that branching occurs at one point, and neglecting the effects associated with the branch angles. In order to solve three systems of equations in the domains (the main branch), and accordingly, we again need to find the interface conditions at the point z = Γ (= Γ ±). Since the systems are hyperbolic and have one outgoing characteristic on the boundary {z = Γ,}, it is required to set three interface conditions. For the continuity equation (the first in system (1.1)), we obtain the continuity of the volumetric blood flow at the point z = Γ (the condition for the preservation of blood mass when passing through the bifurcation point), i.e.

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$Q_1 = Q_2 + Q_3$ with $z = \Gamma$. (1.5).

As for the interface condition for the equation of motion (the second in system (1.1)), the continuity condition for pressure p is often used in the literature.

In the works of Formaggia and Quarteroni [8], it was shown that for the case α = 1, the interface condition for the problem in the subdomains ensures the same conditions for “numerical” stability as for the original problem in domain D is the continuity condition on the discontinuity total pressure

$$P = p + \frac{1}{2}\rho\bar{u}^2 = p + \frac{\rho}{2}\left(\frac{Q}{A}\right)^2 \quad (1.6)$$

Thus, the second condition is the continuity of the total pressure at the branch point: as z=Γ.

$$P_1 = P_2 = P_3 \quad z=\Gamma \ (1.7).$$

As a result, we obtain three required interface conditions (1.5) and (1.7).

Similarly, you can get a one-dimensional model of hemodynamics for the entire arterial system of a person, consisting of 55 large arteries of the human body.

Table 1.1. List and characteristics of 55 major arteries of the human body.

#	Artery name	length (sм)	Cross-sectional area (sм^2)	Beta (β) (кg/s^2)	Reflection coefficient *(R_t)*
1	Ascending Aorta	4.0	5.983	388	-
2	Aortic Arch I	2.0	5.147	348	-
3	Brachiocephalic	3.4	1.219	932	-
4	R. Subclavian I	3.4	0.562	1692	-
5	R. Carotid	17.7	0.432	2064	-
6	R. Vertebral	14.8	0.123	10360	0.302
7	R. Subclavian II	42.2	0.510	1864	-
8	R. radial	23.5	0.106	11464	0.273
9	R. Ulnar I	6.7	0.145	8984	-
10	R. Interosseous	7.9	0.031	51576	0.319
11	R. Ulnar II	17.1	0.133	9784	0.298
12	R. internal Carotid	17.6	0.121	10576	0.261
13	R. external Carotid	17.7	0.121	9868	0.26
14	Aortic Arch II	3.9	3.142	520	-
15	L. Carotid	20.8	0.430	2076	-
16	L. internal Carotid	17.6	0.121	10576	0.261
17	L. external Carotid	17.7	0.121	9868	0.264
18	Thoracic Aorta I	5.2	3.142	496	-
19	L. Subclavian I	3.4	0.562	1664	-
20	Vertebral	14.8	0.123	10360	0.302

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21	L. Subclavian II	42.2	0.510	1864	-
22	L. Radial	23.5	0.106	11464	0.274
23	L. Ulnar I	6.7	0.145	8984	-
24	L. Interosseous	7.9	0.031	51576	0.319
25	L. Ulnar II	17.1	0.133	9784	0.298
26	Intercostals	8.0	0.196	3540	0.209
27	Thoracic Aorta II	10.4	3.017	468	-
28	Abdominal I	5.3	1.911	668	-
29	Celiac I	2.0	0.478	1900	-
30	Celiac II	1.0	0.126	7220	-
31	Hepatic	6.6	0.152	4568	0.308
32	Gastric	7.1	0.102	6268	0.307
33	Splenic	6.3	0.238	3224	0.31
34	Superior Mesenteric	5.9	0.430	2276	0.311
35	Abdominal II	1.0	1.247	908	-
36	L. Renal	3.2	0.332	2264	0.287
37	Abdominal III	1.0	1.021	1112	-
38	R. Renal	3.2	0.159	4724	0.287
39	Abdominal IV	10.6	0.697	1524	-
40	Inferior Mesenteric	5.0	0.080	7580	0.306
41	Abdominal V	1.0	0.578	1596	-
42	R. common Iliac	5.9	0.328	2596	-
43	L. common Iliac	5.8	0.328	2596	-
44	L. external iliac	14.4	0.252	5972	-
45	L. internal Iliac	5.0	0.181	12536	0.308
46	L. Femoral	44.3	0.139	10236	-
47	L. deep Femoral	12.6	0.126	10608	0.295
48	L. posterior Tibial	32.1	0.110	23232	0.241
49	L. anterior Tibial	34.3	0.060	36972	0.239
50	R. external Iliac	14.5	0.252	5972	-
51	R. internal Iliac	5.1	0.181	12536	0.308
52	R. Femoral	44.4	0.139	10236	-
53	R. deep Femoral	12.7	0.126	10608	0.296
54	L. posterior Tibial	32.3	0.110	23232	0.241
55	R. anterior Tibial	34.4	0.060	36972	0.239

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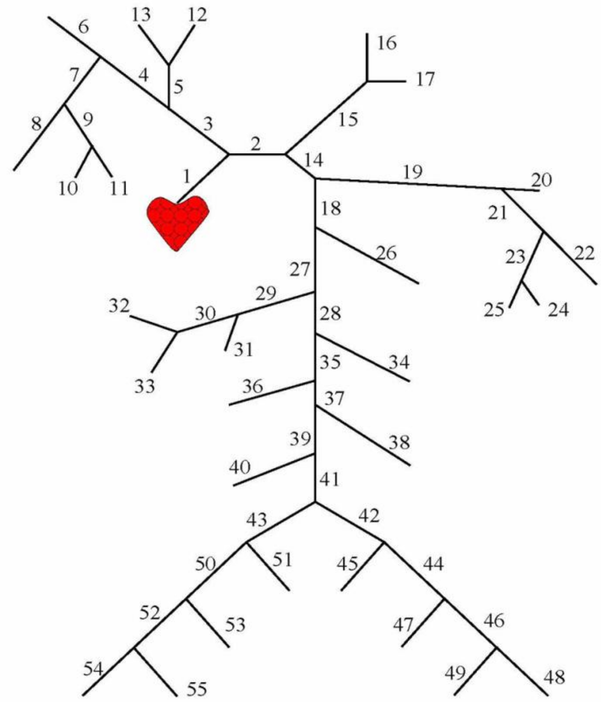

Fig. 1.1. A tree diagram of 55 large human arteries.

To do this, the boundary condition of the form (1.5) is set at the entrance of the first artery (aorta), which simulates the heart, at the exit of the end arteries the boundary conditions of the form (1.6), and at the branch points of the arteries we require the interface conditions (1.5) and (1.7) to be met. In addition, in each individual artery the blood flow is described by the hemodynamic system (1.1). Assuming that the initial radius of the arteries does not change with a change in their length, we set the initial data for the axial section area A for each artery:

$$A_i\big|_{t=0} = \mathrm{A}_{0,i} \text{ for } D_i, i = \overline{1,55} \qquad (1.8)$$

where $A_{0,i}$ - the cross-sectional areas of individual arteries are taken from the table of parameters of the human arterial system (Table 1.1)

As for the initial condition for the volumetric blood flow Q, in practice in the numerical calculations of the arterial system usually use the condition. $\mathrm{Q}\big|_{t=0} = 0$.

The values of such physical parameters as vessel length and coefficient were also taken from the work of Lamponi [1].

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