ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

# Implementation of Various Laws of the Interaction of the Pipeline with the Soil Environment under Static Loading

**T.R. Rashidov, B.M. Mardonov, E.V. An**

Professor, Laboratory Seismodynamics of underground structures and Foundation Engineering, Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, Tashkent, Uzbekistan
Professor, Doctor of Physical and Mathematical Sciences. Faculty of software engineering, Tashkent University Information and Technologies, Tashkent, Uzbekistan.
PhD, Laboratory Seismodynamics of underground structures and Foundation Engineering, Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, Tashkent, Uzbekistan

**ABSTRACT**: The problem of the equilibrium of the pipeline in the ground when on the contact surface two zones: preliminary displacement zone and slip zone is decided in the paper. The algorithm of finding a conjugate point between two interaction zones of pipeline with soil is described. To solve the problem we used the analytical and numerical methods. Good agreement between the results of solving the problem using these methods is obtained. The influence of different parameters on the stress-strain state of the pipeline is determined. It is noted that, before the transition zone the values of longitudinal displacements and stresses of pipeline change according to the linear law and then the exponential law. The maximum value is achieved in the first part - before the transition zone.

**KEYWORDS**: underground pipeline, interaction model of the «pipeline – soil» system, laying depth, axial force, stress-strain state of the pipeline.

## I. INTRODUCTION

The analysis of the effects of strong earthquakes showed that the efficiency of the underground structure depends on the properties of the surrounding soils [1 – 9]. The most unfavorable pipelines laying conditions when they were damaged during earthquakes were revealed: i) pipeline crosses a fault; ii) landslides; iii) soil liquefaction when the pipeline located in the waterlogged zones and water-saturated soils and 4) soils with different properties along the axis of the pipeline when it passes from hard to soft soil. Therefore, in this work, we study the stress-strain state of the underground pipeline when on the contact surface of two interaction zones of pipeline with the soil of different stiffness – the slip zone and the zone of elastic interaction with the soil.

## II. LITERATURE SURVEY

In seismodynamics theory of underground pipelines, the determining moment is the consideration of the interaction of the «pipeline-soil» system.
In the case of a longitudinal movement of the pipeline, a linear model of the interaction of the «pipeline-soil» system was used in [10–15]. In these studies the force of soil impact on the pipeline is directly proportional to the longitudinal movement of the soil.
Various interaction models of the «pipeline-soil» system are given in the paper [16]. The interaction model of the landslide massif with the pipeline and the interaction model of frozen soil with the pipeline were proposed by the authors of this paper.
The stress-strain state of the trunk pipeline using the interaction model of the «underground pipeline - frozen soil» system was studied in the paper [17].
In the case of a transverse movement of the pipeline upwards, the interaction model used in calculating the pipeline for longitudinal stability is applied in the papers [12, 18–20]. Further this interaction model was developed in our investigations [21, 22]. To investigate the problem of stability of the underground pipeline located in water-saturated

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

soils this model was added by members that take into account the damping properties of the interaction of the «underground pipeline – soil» system.
Nowadays, in the seismodynamics theory of underground pipelines, all of the above investigations were provided using one interaction model of the «underground pipeline – soil» system along the axis of the pipe. However, according to the analysis of the effects of strong earthquakes follows that the greatest damages of the underground pipelines are observed in soils with different properties along their axis therefore one of the actual problems is the study of the stress-strain state of the pipeline located in soils with different properties along its axis. Moreover, as the analysis of experimental studies [11–13] shows the contact force of the interaction of the «underground pipeline – soil» system depends on the mechanical properties of the pipe and soil, contact geometry and the magnitude of the acting force. At low values of the force on the contact surface of the pipeline with the soil, relative displacement occurs due to deformation in the contact area. With increasing force, the contact force is increased and determined by Coulomb's law.

## III. METHODOLOGY

Consider the problem of the equilibrium of the underground pipeline when on the contact surface two interaction zones. In the first zone $0<x<x_0$, the pipeline interacts with the soil according to the Coulomb law, in the second zone $x_0<x<l$, the interaction force of the pipe with the soil depends on the displacement of the pipe section according to the linear law. The axial force $P_0$ acts on the end of the pipeline $x$=0. The second end of the pipe is elastic fixed to the motionless structure. The friction force counteracts the section movement in the first zone. The movement occurs in the opposite direction of the axis $Ox$ then the friction force directs in the positive direction of this axis and its value is determined by the lateral pressure from the soil. Given this condition, the equilibrium equation of the pipeline in each zone is as follows

$$EF\frac{d^2u_1}{dx^2}=-f\xi\pi D\gamma_c h(x)\ \text{when}\ 0<x<x_0, \tag{1}$$

$$EF\frac{d^2u_2}{dx^2}=k_x u\ \text{when}\ x_0<x<l\ , \tag{2}$$

where $u_1=u_1(x)$, $u_2=u_2(x)$ – displacement of pipeline sections in zones $0<x<x_0$, $x_0<x<l$, respectively; $f$ – friction coefficient between the pipeline surface and the surrounding soil; $\xi$ – side pressure coefficient; $D$ – pipe external diameter; $\gamma_c=\gamma_c(x)$ – variable density of the soil along the pipeline axis ; $h=h(x)$ – variable depth of the pipeline in the soil; $k_x$ – coefficient of longitudinal shear in the system « underground pipeline – soil».
Equations (1) and (2) are integrated under boundary conditions

$$EF\frac{du}{dx}=P_0\quad \text{when}\ \ x=0\,, \tag{3}$$

$$EF\frac{du}{dx}=-k_0 u\quad \text{when}\ \ x=l \tag{4}$$

and pairing conditions between two zones

$$u_1=u_2\,,\, EF\frac{du_1}{dx}=EF\frac{du_2}{dx}\qquad \text{when}\ x=x_0\,. \tag{5}$$

where $E$ – elasticity coefficient of the pipe material; $F$ –cross-sectional area of the pipe; if $x=x_0$ it is the point when the slip zone of the pipeline begins to form.
The algorithm for finding $x_0$: at first $x_0$=0 and to find a solution of the problem without a slip zone formation, i.e the elastic problem is solved. Further, using the obtained solution the side force value at the points of the pipeline is obtained which compared with the value of dry friction coefficient per unit length. At all points where the side force value exceeds the dry friction coefficient, these areas are dry friction zones. The point $x_0$ is obtained in the first approximation by this way. Then the problem with two zones is solved and the value of $x_0$ is refined. Further the refining process of the value $x_0$ is repeated until the specified accuracy of the calculations is achieved.
The axial stress is determined by integrating equation (1) under condition (3)

$$\sigma_1=E\frac{du_1}{dx}=-q\int_0^x \gamma_c(x)h(x)dx+\sigma_0\,. \tag{6}$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

Here $q=\dfrac{4f\xi D}{(D^2-d^2)}$, $\sigma_0=P_0/F$.

The solution of equation (2) satisfying condition (4) takes the following form

$$u_2=A[sh\alpha(l-x)+\frac{l\alpha}{\bar{k}}ch\alpha(l-x)]\ , \tag{7}$$

where $shz$ и $chz$ – hyperbolic functions; $\alpha=\sqrt{\dfrac{k}{EF}}$, $\bar{k}=\dfrac{k_0 l}{EF}$.

The constant $A$ is determined from conditions (5)

$$A=-\frac{\sigma_{np}}{E\alpha[ch\alpha(l-x_0)+\dfrac{l\alpha}{\bar{k}}sh\alpha(l-x_0)]},$$

where $\sigma_{np}$ –stress in the point $x_0$.

The displacement of the pipeline sections in the slip zone is determined by integrating equation (6) under the condition $u_1(x_0)=u_{20}=u_2(x_0)$

$$u_1=-\frac{q}{E}\left[\int_0^x(x-\xi)\gamma_c(\xi)h(\xi)d\xi-\int_0^{x_0}(x_0-\xi)\gamma_c(\xi)h(\xi)d\xi\right]+\frac{\sigma_0}{E}(x-x_0)+u_{20}. \tag{8}$$

Consider special cases

1. $h(x)=const=h_0$, $\gamma_c=\gamma_{c0}$, $k_x(x)$=const= $k_x$.

Equation (8) is written in the form

$$u_1=-q\frac{(x^3-x_0^3)}{6E}+\sigma_0(x-x_0)/E+u_{20}. \tag{9}$$

The analysis of solution (7) shows that on the distribution of the pipeline movement along its axis is significantly affected by the lateral $q$ and elastic resistance $k_0$ coefficients and the parameter $\alpha$ which characterizes the surrounding soil resistance.

## IV. RESULTS

The distribution curves of the sections pipeline displacement and stresses along the length of the pipe for various values of parameters $k$, $k_0$, $P_0$ when $P_{np}$=0.95$P_0$, $E$=2·10$^{10}$ Pa, $l$=10 m, $D$=0.3 m, $d$=0.29 m, $f$=0.3 m, $k$=0.4 m, $\rho_c$=1600 kg/m$^3$ are showed in the figures 1 and 2.

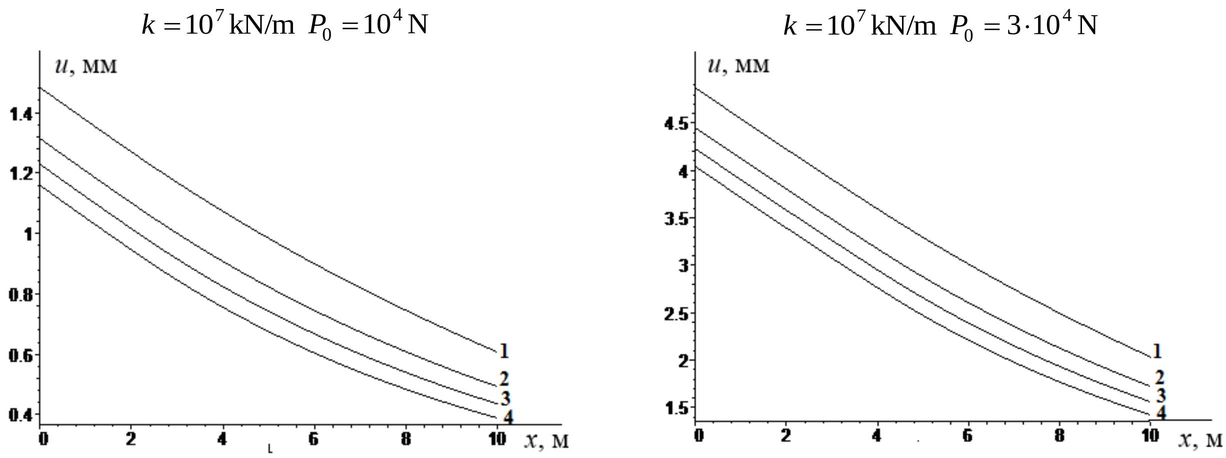



Figure 1. Distribution of displacements in sections along the length of the pipe for various values of parameters $k$, $P_0$, $k_0$: *1* – $k_0$=5 MN/m; *2* – $k_0$=0.8 MN/m; *3* – $k_0$=1 MN/m; *4* – $k_0$=1.2 MN/m

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

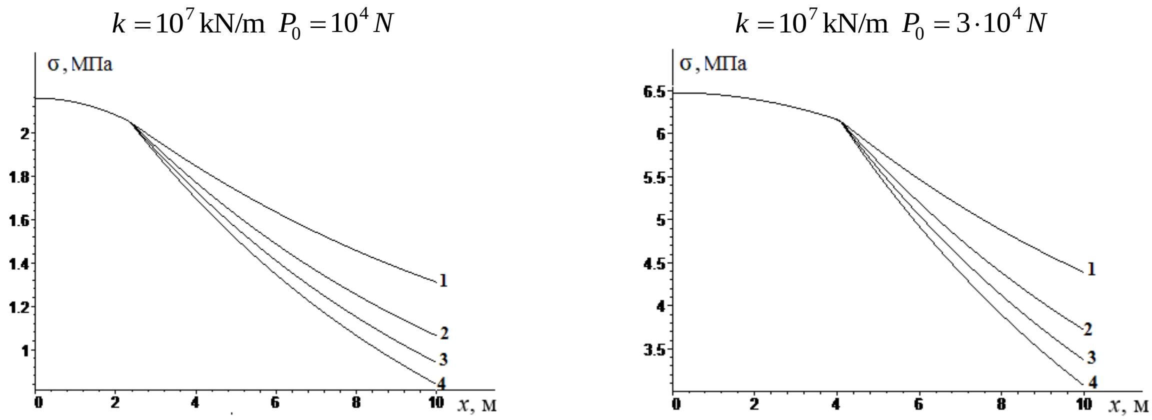



Figure 2. Distribution of stresses in sections along the length of the pipe for various values of parameters $k$, $P_0$, $k_0$:
*1* – $k_0$=5 MN/m; *2* – $k_0$=0.8 MN/m; *3* – $k_0$=1 MN/m; *4* – $k_0$=1.2 MN/m

2. $h = h_0 + \beta \dfrac{x(l-x)}{l}$, $\gamma_c = \gamma_{c0}$, $k_x(x)$=const.

The graphs of changes of depth of the pipeline in the soil are showed in the figure 3. The graph of change of the conjugate point $x_0$ between two interactions models by depending on the depth of the pipeline which varies along its length is showed in the figure 4. The curves of stress distribution of the pipe along its length for different values of parameter $\overline{\alpha} = \alpha l$ and constant value of the interaction coefficient ($k_x$) along the axis of the pipeline is showed in the figure 5.

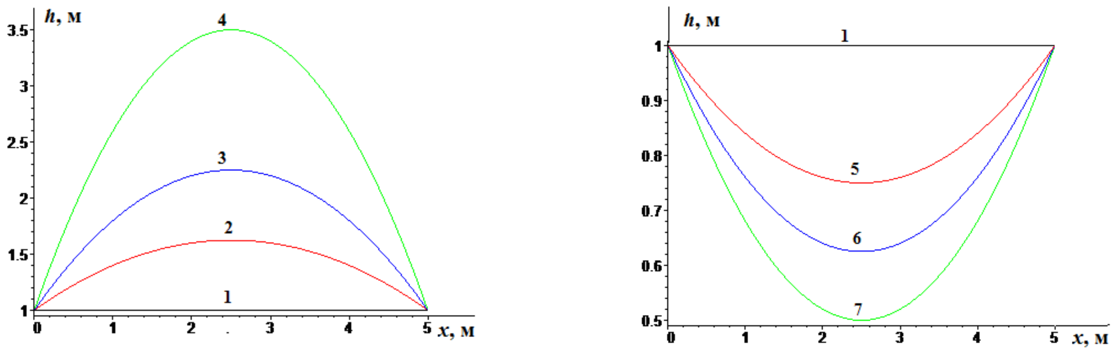



Figure 3. The graphs of changes of depth of the pipeline in the soil for different values of parameter $\beta$:
*1* – $\beta$=0; 2 – $\beta$=0.5; *3* – $\beta$=1; *4* – $\beta$=2; *5* – $\beta$= – 0.2; *6* – $\beta$= – 0.3; *7* – $\beta$= – 0.4

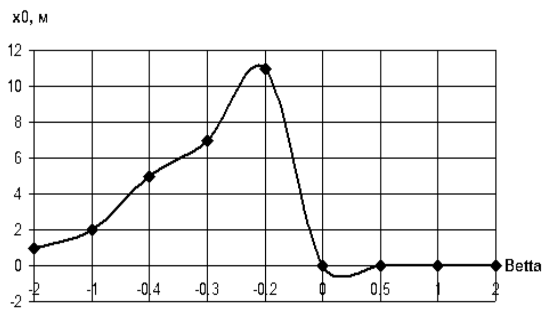



Figure 4. The graph of change of the conjugate point $x_0$ by depending on the depth of the pipeline $\beta$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

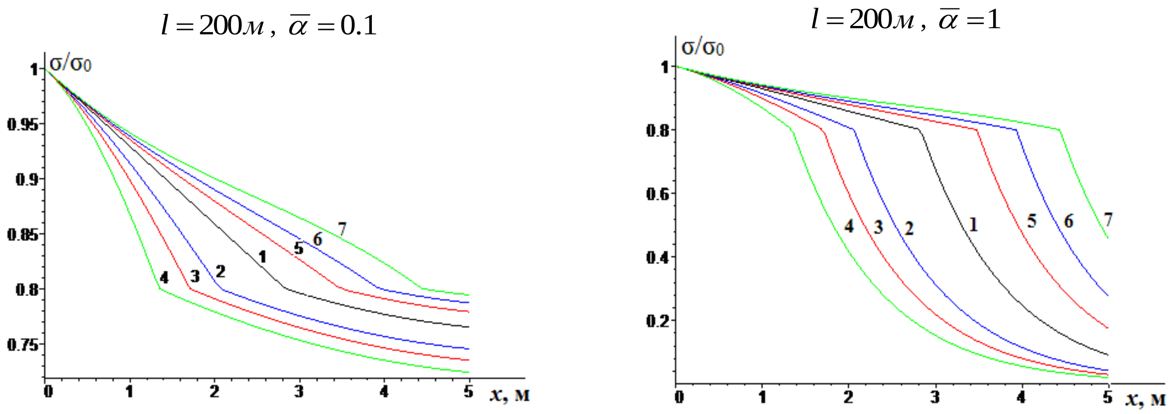



Figure 5. The curves of stress distribution of the pipe along its length:
*1* – $\beta$=0; 2 – $\beta$=0.5; *3* – $\beta$=1; *4* – $\beta$=2; *5* – $\beta$= – 0.2; *6* – $\beta$= – 0.3; *7* – $\beta$= – 0.4

3. $h=h_0+\beta\dfrac{x(l-x)}{l}$, $\gamma_c=\gamma_{c0}$, $k_x(x)\neq$const.

The formula from the paper [12] is used to determine $k_x(x)$ which depends on the depth of the pipe.
The finite difference method is used to solve the problem in this case.
Dimensionless variables are declared: $\bar{u}_1=u_1R;\quad \bar{u}_2=u_2R;\quad \bar{x}=xl$ .
Then equations (1) and (2) take the following form:

$$\frac{EFR}{l^2}\frac{\partial^2 u_1}{\partial x^2}=-f\xi\pi D\gamma_c(x)h(x)\qquad \text{when}\qquad 0<x<x_0, \tag{8}$$

$$\frac{EFR}{l^2}\frac{\partial^2 u_2}{\partial x^2}=2\pi R^2k_x(x)u_2\qquad \text{when}\qquad x_0<x<1. \tag{9}$$

Boundary conditions:

$$\frac{EFR}{l}\frac{du_1}{dx}=P_0 \text{ when } x=0, \tag{10}$$

$$\frac{EFR}{l}\frac{du_2}{dx}=-Rk_0u_2 \text{ when } x=1. \tag{11}$$

Pairing conditions between two zones:

$$u_1=u_2,\qquad \frac{EFR}{l}\frac{du_1}{dx}=\frac{EFR}{l}\frac{du_2}{dx}\quad \text{when } x=x_0 \tag{12}$$

Central difference schemes are used to approximate equations (8) - (9), boundary conditions (10) - (11) and conjugation conditions (12). Then the boundary conditions (10) - (11) and the conjugation conditions (12) are transformed to the following form:

$$\frac{EFR}{l}\frac{du_1}{dx}\bigg|_{x=0}=P_0\Rightarrow\frac{EFR}{2hl}\left(-3u_{1\,0}+4u_{1\,1}-u_{1\,2}\right)=P_0\Rightarrow u_{1\,0}=\left(\frac{-2hlP_0}{EFR}+4u_{1\,1}-u_{1\,2}\right)\Big/3$$

$$\frac{EFR}{l}\frac{du_2}{dx}\bigg|_{x=n}=-Rk_0u_2\big|_{x=l}\Rightarrow\frac{EFR}{2hl}\left(3u_{2\,n}-4u_{2\,n-1}+u_{2\,n-2}\right)=-Rk_0u_{2\,n}\Rightarrow$$

$$u_{2\,n}=\left(4u_{2\,n-1}-u_{2\,n-2}\right)\Big/\left(\frac{2hl\,k_0}{EF}+3\right).$$

And equations (8) – (9) after approximation are solved to decide on $u_{1\,i+1}, u_{2\,i+1}$:

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

$$u_{1\,i+1} = \frac{-2f\xi\pi\gamma_c(x)h(x)l^2h^2}{EF} + 2u_{1\,i} - u_{1\,i-1}, \tag{13}$$

$$u_{2\,i+1} = \left(\frac{2\pi Rk_x(x)l^2h^2}{EF} + 2\right)u_{2\,i} - u_{2\,i-1}. \tag{14}$$

Equations (13) – (14) are solved by the sweep method.

The reverse run method is used to solve the following systems of algebraic equations:

$$u_{1\,1} = \alpha_{1\,2}u_{1\,2} + \beta_{1\,2},\; u_{1\,i} = \alpha_{1\;i+1}u_{1\,i+1} + \beta_{1\,i+1},$$

$$u_{1\;N-2} = \alpha_{1\,N-1}u_{1\,N-1} + \beta_{1\,N-1},\; u_{1\,N-1} = \beta_{1\,N},$$

$$u_{2\,1} = \alpha_{2\,2}u_{2\,2} + \beta_{2\,2},\; u_{2\,i} = \alpha_{2\,i+1}u_{2\,i+1} + \beta_{2\,i+1},$$

$$u_{2\,N-2} = \alpha_{2\,N-1}u_{2\,N-1} + \beta_{2\,N-1},\; u_{2\,N-1} = \beta_{2\,N},$$

where $\alpha_{1i}$, $\beta_{1i}$, $\alpha_{2i}$, $\beta_{2i}$ – coefficients.

Normal stresses are determined by the following formulas:

$$\sigma_1 = E\frac{\partial \bar{u}_1}{\partial \bar{x}},\; \sigma_2 = E\frac{\partial \bar{u}_2}{\partial \bar{x}}.$$

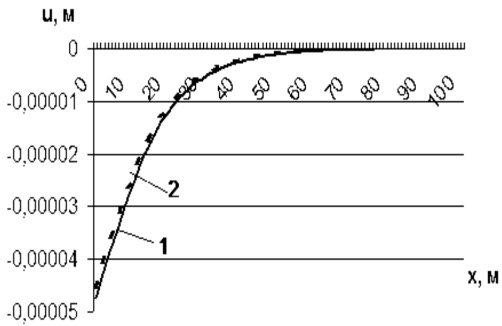
Dimensionless variables are declared: $\bar{u}_1 = u_1R;\quad \bar{u}_2 = u_2R;\quad \bar{x} = xl$.

The finite difference method is used to decide on $\sigma_{1\,i}$, $\sigma_{2\,i}$

$$\sigma_{1\;i} = \frac{E}{2h}\frac{R}{l}\left(u_{1\,i+1} - u_{1\,i-1}\right), \qquad \sigma_{2\,i} = \frac{E}{2h}\frac{R}{l}\left(u_{2\,i+1} - u_{2\,i-1}\right).$$

The following numerical values of the parameters which characterize the pipeline and its interaction with the surrounding soil are taken in the calculations: $E$=2·10$^5$ MPa/m$^2$, $l$=100 m, $D$=0.3 m, $d$=0.25 m, $P_0$=10 kN, $P_{np}$=8 kN, $\gamma_c$=2000 kg/m$^3$, $f$=0.3, ξ=0.6, $h_0$=1 м, $k_0$=10 kN/m.

Graphs of comparing of analytical and numerical solutions are shown in figure 6. Distribution curves of the longitudinal interaction coefficient of the pipeline with surrounding soils are shown in figure 7. Distribution curves of displacements and stresses of the pipeline along its length for various values of parameters $\beta$и$P_0$ are shown in figures 8 and 9.



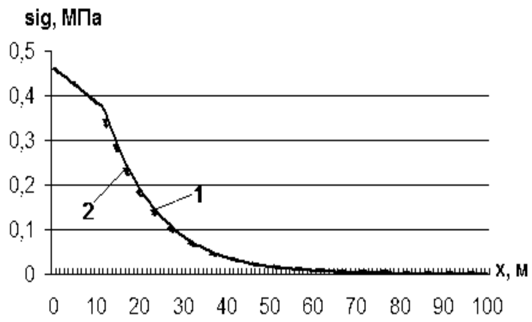




Figure 6. The graphs of changes of displacements and stresses of the pipeline along its length when $\beta$=0 и$\gamma_c$=1 kg/m$^3$: *1* – analytical solution; *2* – numerical solution

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

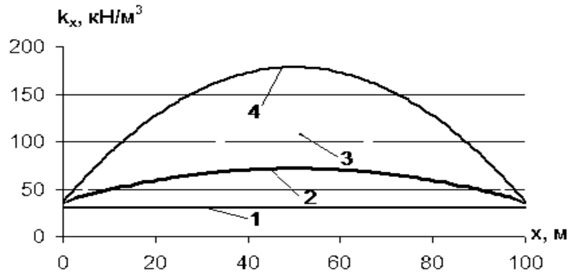



Figure 7. The graphs of changes of the longitudinal interaction coefficient of the pipeline with surrounding soils along its length: *1 – β*=0; 2 – *β*=0.5; *3 – β*=1; *4 – β*=2

*а* *б*

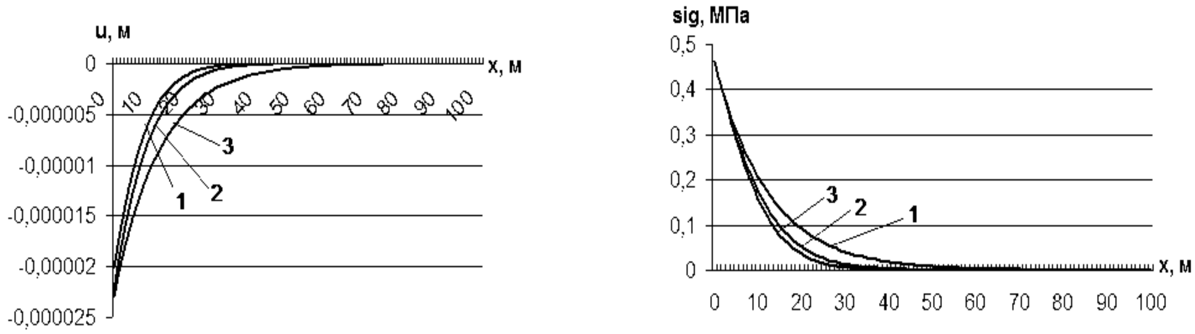



Figure 8. The graphs of changes ofdisplacements (*а*) иstresses (*б*) by depending on the coordinate: *1 – β*=0; *2 – β*=1; *3 – β*=2

*а* *б*

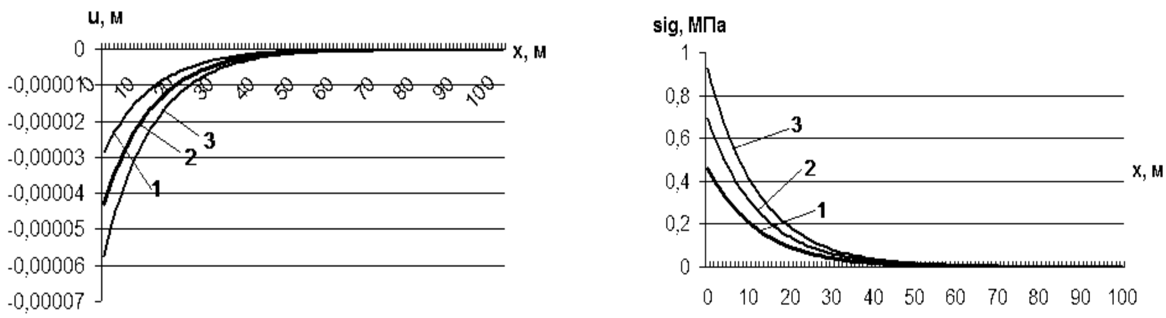



Figure 9. The graphs of changes of displacements (*а*) иstresses (*б*) by depending on the coordinate when *β*=0: *1 – $P_0$*=10; 2 – $P_0$=15; *3 – $P_0$*=20

## V. CONCLUSION

From the analysis of the curves are presented in the figure 1 follows that if the longitudinal shear coefficient $k_0$ is increased then displacements of the pipeline sections are decreased and when $k_0$>1.5 MN/m they independent of this coefficient. This is observed in the pipeline section *x=l* when the coefficient of elastic resistance of the soil is increased. From the analysis of the stress curves are presented in the figure 2 follows that the coefficient $k_0$ affects the stress state only in the area where the pipeline surface interacts with the surrounding soil according to Winkler's law. It is noted that if the value of this coefficient increases then the value of the stress is increased too.

Comparative analysis is allowed obtaining good agreement between the results of solving the problem using analytical and numerical methods (figure 6).

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 9 , September 2019

It was found that before the transition zone $x_0$ the values of longitudinal displacements and stresses of pipeline change according to the linear law and then the exponential law.

It was revealed that the maximum value is achieved in the first part – before the transition zone$x_0$ (figures 5, 6, 8, 9). The influence of the variable coefficient of the longitudinal interaction of the underground pipeline with the surrounding soil along its length affects the displacements and stresses of the pipeline in the case of a hill; it is presented in the figure 8. It was found that if the initial force increases then the values of displacements and stresses of the pipeline are increase (figure 9).

## REFERENCES

[1] Selcuk Toprak, Filiz Taskin. Estimation of Earthquake Damage to Buried Pipelines Caused by Ground Shaking // Natural Hazards. January 2007. Vol. 40. Issue 1. P. 1 – 24.
[2] Abdoun T.H., Ha D., O'Rourke M.J., Symans M.D., O'Rourke T.D., Palmer M.C., Stewart H.E. Factors Influencing the Behavior of Buried Pipelines Subjected to Earthquake Faulting // Soil Dynamics and Earthquake Engineering. Vol. 29. 2009. No. 3. Pp. 415 – 427.
[3] Ai Xiaoqiu, Li Jie. Research on Seismic Response of Underground Pipelines in Solid-Liquid Media // Earthquake Engineering and Engineering Vibration. Vol. 25. 2005. Issue 2. Pp. 136 – 140.
[4] Azadi M., Mir Mohammad Hosseini S.M. Analysis of the Effect of Seismic Behavior of Shallow Tunnels in Liquefiable Grounds // Tunneling and underground space technology. Vol. 25. 2010. No. 5. Pp. 543 – 552.
[5] Cubrinovski M., Hughes M., O'Rourke T.D. Impacts of Liquefaction on the Potable Water System of Christchurch in the 2010-2011 Canterbury (NZ) Earthquakes // Journal of Water Supply: Research and Technology – AQUA. Vol. 63. 2014. Pp. 95 – 105
[6] Hazarika H., Boominathan A. Liquefaction and Ground Failures During the 2001 Buhl Earthquake, India // Proceeding of International Conference on Performance-Based Design in Earthquake Geotechnical Engineering: Earthquake Geotechnical Case Histories for Performance-Based Design. Tokyo, 2009. Pp. 201 – 226.
[7] Lee D.H., Kimb B.H., Lee H., Kong J.S. Seismic Behavior of a Buried Gas Pipeline Under Earthquake Excitations // Journal of Structural Engineering. Vol. 31. 2009. Pp. 1011 – 1023.
[8] Lee W.F., Chu B.L., Lin C.C., Chen C.H. Liquefaction Induced Ground Failures at Wu Fang Caused by Strong Ground Motion During 1999 Chi-Chi earthquake // Proceeding of International conference on performance-based design in earthquake geotechnical engineering: Earthquake Geotechnical Case Histories for Performance-Based Design. Tokyo, 2009. Pp. 289 – 310.
[9] O'Rourke M., Deyoe E. Seismic Damage to Segmented Buried Pipe // Earthquake Spectra. Vol. 20. 2004. Pp. 1167 – 1183.
[10] Florin V.A. Fundamentals of soil mechanics [in Russian]. Leningrad: Stroyizdat, 1959. Vol. 2. – 541 p.
[11] Rashidov T. Dynamic Theory of Seismic Stability of Complex Systems of Buried Structures [in Russian]. Tashkent: Fan, 1973. – 180 p.
[12] Rashidov T.R., Khozhmetov G.Kh. Seismic Resistance of Buried Pipelines [in Russian]. Tashkent: Fan, 1985. – 152 p.
[13] Rashidov T., Khozhmetov G., Mardonov B. Oscillations of structures interacting with soil [in Russian]. Tashkent: Fan, 1975. – 176 p.
[14] Rashidov T.R., Nishonov N.A. Seismic behavior of underground polymer piping with variable interaction coefficients // Soil mechanics and foundation engineering. Vol. 53. 2016. Pp. 196 – 201.
[15] Rashidov T.R., Yuldashev T., Bekmirzaev D.A. Seismodynamics of Underground Pipelines with Arbitrary Direction of Seismic Loading. J. Soil Mechanics and Foundation Engineering. New York. September 2018, Volume 55, Issue 4, Pp. 243–248.
[16] Bashirzade S.R., Ovchinnikov I.G. Predicting the behavior of pipeline structures in complex soil and geological conditions. Part 2. Soil – pipeline interaction models // Internet-journal «Naukovedenie» [in Russian]. Vol. 9. No. 1. 2017. Pp. 1 – 18.
[17] Nikolaeva M.I., Atlasov R.A., Berdiev S.S., Ivanov A.G. Interaction model of the "underground pipeline - frozen soil" system // Journal «Arktika. XXI century». Technical sciences [in Russian]. 2015. No. 1 (3). Pp. 44 – 50.
[18] Aynbinder A.B. Calculation of trunk and field pipelines for strength and stability [in Russian]. Moscow: Nedra, 1991. – 287 p.
[19] Yasin E.M. The Stability of Buried pipelines [in Russian]. Moscow: Nedra, 1967. – 120 p.
[20] Borodavkin P.P. Soil mechanics [in Russian]. Moscow: Nedra, 2003. – 349 p.
[21] Rashidov T.R., An E.V. Seismodynamics of underground pipelines interacting with water-saturated fine-grained soil // Mechanics of solids. Vol. 50. 2015. Issue 3. Pp. 305 – 317.
[22] Rashidov T.R., Mardonov B.M., An E.V. Transverse vibrations of buried pipelines under axial loading within geometrically nonlinear theory // International Applied Mechanics. 2019. Vol. 55. No. 2. Pp. 229 – 238.