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Stable Estimation of Covariance Noise Matrix Based on Iterative Algorithms

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ABSTRACT: The algorithms for the stable estimation of the covariance noise matrix of a controlled object based on the iterative computational algorithms are presented in the article. The possibilities of the secant method using regular methods are analyzed to determine the covariance noise matrix of an object. Lavrentiev and Tikhonov methods for regularization parameters selection based on the methods of quasi-optimality and relations are used for regularization of desired hyperplanes construction. The presented algorithms make possible to perform a stable estimation of the covariance noise matrix of an object and thereby increase the accuracy of the adaptive estimation.

KEY WORDS: an operated object, object noise, the covariance matrix, steady estimation, an iterative algorithm, the secant method, regularization methods, regularization parameters.

I.INTRODUCTION

Let's consider a linear continuous stochastic dynamic system, which can be described by the equations in discrete time:

$$x_{i+1} = A_i x_i + B_i u_i + \Gamma_i w_i \,, \qquad (1)$$

$$z_i = H x_i + v_i \,, \qquad (2)$$

where x_i – vector of a dimension condition system *n*, u_i – dimension control vector *l*; z_i – dimension observation vector *m*, w_i and v_i – object noise vectors and observation noise of *q* and *p* dimensions, respectively, which are a sequence of the form of Gaussian white noise with characteristics $E[w_i]=0$, $E\left[w_i w_k^T\right] = Q\delta_{ik}$, $E[v_i]=0$, $E\left[v_i v_k^T\right] = R\delta_{ik}$, $E\left[w_i v_k^T\right] = 0$; A_i, B_i, Γ_i and H_i – matrixes of corresponding dimensions. These sequences also do not depend on the random initial state of the system x_0 with the mathematical expectation $\bar{x}_0$ and the covariance P_0.

To estimate the state vector x_i of the dynamic system (1), (2), traditional Kalman filter equations [1-9] are usually used:

$$\hat{x}_{i+1|i} = A\hat{x}_{i|i} + Bu_i \,,$$

$$\hat{z}_i = H\hat{x}_{i|i-1} \,,$$

$$y_i = z_i - \hat{z}_i = z_i - H\hat{x}_{i|i-1} \,,$$

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + K_i(z_i - \hat{z}_i) = \hat{x}_{i|i-1} + K_i z_i \,,$$

Apriori information on the mathematical model of an object, on the input and measurement noise statistics is required to implement the Kalman filter. Inaccuracy in a priori data can cause divergence [2,4,7].

II. FORMULATION OF THE PROBLEM

One of the possible ways to develop adaptive filtering algorithms is to use the correlation properties of the updated sequence in order to design estimates of the covariance matrices of the input *Q* and *R* noise measurement [2,4,8]. This algorithm is suitable for stationary objects, stationary input and measuring noise. Before using the adaptive algorithm, the state vector is estimated based on the classical Kalman algorithm, in which the matrices *Q* and *R* are given. In this case, it is necessary to determine the degree of difference between the used matrices $\tilde{Q}$ and $\tilde{R}$ from the real

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covariance matrices of the input noise Q and measuring noise R. If $\tilde{Q}$ and $\tilde{R}$ are slightly different from the real values of Q, R, the Kalman filter actually works in the optimal mode and we can assume that $\tilde{Q} = Q$, $\tilde{R} = R$. Thus, testing the optimal properties of the Kalman filter with selected $\tilde{Q}$ and $\tilde{R}$, it is possible to solve the problem of the correctness of the initial choice of $\tilde{Q}$ and $\tilde{R}$. Testing can be based on the statistical processing of the updated sequence [2,4,7,8]. In case where v_i is white random noise, the filter with the initially selected $\tilde{Q}$ and $\tilde{R}$ works practically optimally, and we can assume that $\tilde{Q} = Q$, $\tilde{R} = R$. If v_i is not white random noise, then the filter operates in a suboptimal mode and it is necessary to evaluate the real values of Q and R, which are different from the initial matrices $\tilde{Q}$ and $\tilde{R}$.

A number of methods that enable to evaluate or identify the elements of these covariance matrices [2,3,7,10] are known. The significant part of the identification algorithms for the covariance noise matrices Q_i and R_i can be based on the analysis methods for sequence updating or measurements residual $v_i = z_i - H_i \hat{x}_{i|i-1}$ in Kalman filter.
For a suboptimal filter, the updating process represents a non-white Gaussian process with the following correlation properties [2,4]:

$$C_0 = M\left(v_i v_i^T\right) = HP'H^T + R, \tag{3}$$

$$C_k = M\left(v_i v_{i-k}^T\right) = H\left[A(I - KH)\right]^{k-1} A\left[P'H^T - KC_0\right],$$

where $P' = A(I - KH)P'(I - KH)^T A^T + AKRK^T A^T + \Gamma Q \Gamma^T$ - a priori covariance matrix of error estimation of Kalman suboptimal filter.
The estimation of the covariance matrices quantity C_k can be obtained using the ergodic properties of a stationary updating sequence [2]:

$$\hat{C}_k = \frac{1}{N}\sum_{i=k}^{N} v_i v_{i-k}^T .$$

The estimation of the measuring noise matrix is based on the equation (3), i.e.

$$\hat{R} = \hat{C}_0 - H\left(P'H^T\right).$$

Let's restrict to the case when the number of unknown elements of the matrix Q is less than or equal to $n \times m$. Then we can write [2]:

$$\begin{gathered}\sum_{j=0}^{k-1} HA^j \Gamma Q \Gamma^T \left(A^{j-k}\right)^T H^T = \left(P'H^T\right)^T \left(A^{-k}\right)^T H^T - \\ - HA^k \left(P'H^T\right)^T - \sum_{j=0}^{k-1} HA^j \hat{V} \left(A^{j-k}\right)^T H^T, \quad k = 1,2,\ldots,n,\end{gathered} \tag{4}$$

where $\hat{V} = A\left[-K\left(P'H^T\right)^T - \left(P'H^T\right)K^T + K\hat{C}_0 K^T\right] A^T$.
Let's rewrite the system of equations (4) in the following form

$$f(q) = 0, \tag{5}$$

where $q = \{q_1, q_2, \ldots, q_{n+m}\}$ – vector composed of elements of the covariance matrix Q.

III. SOLUTION OF THE TASK

To solve equation (5), we will use the secant method [11,12] in $p = n + m$-dimensional space.
According to the method of secant equation, $p+1$- points in p-dimensional space and p hyperplanes can be written as:

$$q^{i_\gamma} = (q_1^{i_\gamma}, q_2^{i_\gamma}, \ldots, q_p^{i_\gamma})^T, \quad \gamma = 1,2,\ldots, p,$$

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$$L^k(q) \equiv \sum_{j=1}^{p} a_j^k q_j + a_{p+1}^k = 0, \;\; k = 1,2,...,p\,. \tag{6}$$

In this case, hyperplanes must satisfy a condition of the form:

$$L^k(q^{i_\gamma}) = f_k(Q_\gamma^i), \;\; \gamma = 0,1,2,...,p, \;\; k = 1,2,...,p\,,$$

or $L^k(q^{i_\gamma}) - L^k(q^{i_0}) \equiv \sum_{j=1}^{p} a_j^k (q_j^{i_\gamma} - q_j^{i_0}) = f_k(q^{i_\gamma}) - f_k(q^{i_0}), \;\; \gamma = 1,2,...,p, \;\; k = 1,2,...,p\,.$

To develop these p hyperplanes, it is necessary to find the elements of matrix A based on an equation of the form:

$$(\Delta Q)\mathrm{A} = \Delta F\,, \tag{7}$$

where

$$\mathrm{A} = \begin{pmatrix} a_1^1 & a_1^2 & \cdots & a_1^p \\ a_2^1 & a_2^2 & \cdots & a_2^p \\ \cdots & \cdots & \cdots & \cdots \\ a_p^1 & a_p^2 & \cdots & a_p^p \end{pmatrix}, \; \Delta Q = \begin{pmatrix} q_1^{i_1} - q_1^{i_0} & q_2^{i_1} - q_2^{i_0} & \cdots & q_p^{i_1} - q_p^{i_0} \\ q_1^{i_2} - q_1^{i_0} & q_2^{i_2} - q_2^{i_0} & \cdots & q_p^{i_2} - q_p^{i_0} \\ \cdots & \cdots & \cdots & \cdots \\ q_1^{i_p} - q_1^{i_0} & q_2^{i_p} - q_2^{i_0} & \cdots & q_p^{i_p} - q_p^{i_0} \end{pmatrix},$$

$$\Delta F = \begin{pmatrix} f_1(q_1^{i_1}) - f_1(q_1^{i_0}) & f_2(q_2^{i_1}) - f_2(q_2^{i_0}) & \cdots & f_p(q_p^{i_1}) - f_p(q_p^{i_0}) \\ f_1(q_1^{i_2}) - f_1(q_1^{i_0}) & f_2(q_2^{i_2}) - f_2(q_2^{i_0}) & \cdots & f_p(q_p^{i_2}) - f_p(q_p^{i_0}) \\ \cdots & \cdots & \cdots & \cdots \\ f_1(q_1^{i_p}) - f_1(q_1^{i_0}) & f_2(q_2^{i_p}) - f_2(q_2^{i_0}) & \cdots & f_p(q_p^{i_p}) - f_p(q_p^{i_0}) \end{pmatrix}.$$

When solving (7) in order to give greater numerical stability, it is advisable to calculate the matrix A on the basis of the expression:

$$\mathrm{A} = g_\alpha(\Delta Q)\Delta F\,,$$

where – $g_\alpha(\Delta Q) = (\Delta Q + \alpha I)^{-1}$ – generating function system for the regularization method; α –regularization parameter; I – identity matrix. Here, it is necessary to determine the regularization parameters based on the methods of quasi-optimality, relativity or cross-significance [13-16].

As a new point, by means of one iteration using the secant method, we take the intersection point of the developed hyperplanes, i.e. the system solution (6) or $L^k(q) - L^k(q^{i_0}) \equiv \sum_{j=1}^{p} a_j^k (q_j - q_j^{i_0}) = -f_k(q^{i_0}), \;\; k = 1,2,...,p$, and in matrix form $\mathrm{A}^T(q - q^{i_0}) = -f(q^{i_0})$. Consequently,

$$q = q^{i_0} - (\mathrm{A}^T)^{-1} f(q^{i_0}) = q^{i_0} - ((g_\alpha(\Delta Q)\Delta F)^T + \beta I)^{-1} f(q^{i_0})\,, \tag{8}$$

where β - regularization parameter, which is determined on the basis of the model sample method [13].

Taking $i_0 = i$ and considering

$$q^{i_\gamma} = (q_1^i, q_2^i, ..., q_{\gamma-1}^i, \varphi_\gamma(q^i), q_{\gamma+1}^i, ..., q_p^i)^T, \;\; \gamma = 1,2,...,p\,, \tag{9}$$

$$\varphi_\gamma(q^i) = q_\gamma^i - f_\gamma(q^i)\,,$$

We can write

$$\Delta Q' = \begin{pmatrix} \varphi_1(q^i) - q_1^i & 0 & \cdots & 0 \\ 0 & \varphi_2(q^i) - q_2^i & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \varphi_p(q^i) - q_p^i \end{pmatrix}.$$

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$$((\Delta Q')^{-1}\Delta F)^T = \mathrm{f}(q^i,\varphi(q^i)) = (u_{k\gamma}),$$

where $u_{k\gamma} = (f_k(q_1^i,q_2^i,...,q_{\gamma-1}^i,\varphi_\gamma(q^i),q_{\gamma+1}^i,...,q_p^i) - f_k(q^i)) \big/ (\varphi_\gamma(q^i) - q_\gamma^i)$.

Then, taking into account (8) for the $(i+1)$ -st approximation, we can write the following expression for the iterative process

$$q^{i+1} = q^i - (\mathrm{f}(q^i,\varphi(q^i)))^{-1} f(q^i). \tag{10}$$

In the practical use of the iterative algorithm (10), computational difficulties arise due to the fact that such iterative procedures have significant instability - with an increase in the number of iterations, a significant accumulation of errors can occur [17-19]. Let us regularize algorithm (10) based on A.N.Tikhonov iterative method [17,20]. Thus, the estimation algorithm can be formed:

$$q^{i+1} = q^i - (\mathrm{f}(q^i,\varphi(q^i)) + \alpha_i B_i)^{-1} f(q^i). \tag{11}$$

Based on the research results [19,20], it can be shown that for obtaining a minimizing sequence, less sensitive to the choice of the initial approximation and converged to the set of the minimum points, parameter α_i in (11) can be chosen in the form of $\alpha_i = \mathrm{B}(i+1)^{-1}, \quad \mathrm{B} >> 0$.

Based on (9), we can develop an expression of the following form: $\mathrm{f}(q^i,\varphi(q^i)) = I - \Phi(q^i,\varphi(q^i))$, where

$$\Phi(q^i,\varphi(q^i)) = (v_{k\gamma}) = \left(\frac{\varphi_k(q_1^i,q_2^i,...,q_{\gamma-1}^i,\varphi_\gamma(q^i),q_{\gamma+1}^i,...,q_p^i) - \varphi_k(q^i)}{\varphi_\gamma(q^i) - q_\gamma^i} \right).$$

$$\mathrm{f}(q^i,\varphi(q^i)) = (u_{k\gamma}) = \left(\frac{f_k(q^i) - f_k(q_1^i,q_2^i,...,q_{\gamma-1}^i,q_\gamma^i - f_\gamma(q^i),q_{\gamma+1}^i,...,q_p^i)}{f_\gamma(q^i)} \right), \quad k,\gamma = 1,2,...,p.$$

It can be noted that in one iteration of the iterative process (11), only one system of linear algebraic equations is solved as in the Newton method: $\mathrm{f}(q^i,\varphi(q^i))(q - q^i) = -f(q^i)$.

It can be marked that the iterative algorithm (11) does not require the calculation or approximation of partial derivatives, which makes it distinct from the iterative algorithms of the first and second orders.
Based on the research results [17-20], it can be shown that the iterative process (11) converges from any initial approximation $q^0 \in N_q$, i.e. $\lim_{i\to\infty} \left\| q^\bullet - q^i \right\| = 0$, where $N_q = \{ \left\| q - q^\bullet \right\| \le \rho \}$, $\rho > 0$ – is a fairly small number; $q^\bullet = (q_1^\bullet, q_2^\bullet, ..., q_p^\bullet)$ – is the only solution to system $f(q) = 0, \quad q = (q_1, q_2, ..., q_p), \quad f = (f_1, f_2, ..., f_p)$.

VI.CONCLUSION

Thus, the presented algorithm makes possible to obtain a regularized solution of the equation (11) for the stable determination of the elements of the covariance noise matrices of an object and measurement residuals and their subsequent use at calculating intensification coefficient of Kalman filter, and thereby adapt the filter to the variable values of the covariance matrices of disturbing actions.

REFERENCES

[1] Krasovsky A.A. The directory of the automatic control theory. - Moscow: Science, 1987. – 712 p.
[2] Leondes K.T. Filtration and stochastic management in dynamic systems. - Moscow: World, 1980. – 407 p.
[3] Peltsverger S.B. Algorithmic support of estimation processes in dynamic systems under conditions of uncertainty. - Moscow: Science, 2004. - 116 p.
[4] Ogarkov M.A. Methods of statistical estimation of random process quality. - Moscow: Energoatomizdat, 1990. – 208 p.
[5] Sinitsyn I.N. Filter of Kalman and Pugachev. - Publishing house: Lagos, 2006. – 640 p.
[6] Kolos M.V., Kolos I.V. Linear optimal filtering methods. - Moscow: Science, 2000.- 158p.
[7] Pervachev S.V., Perov A.I. Adaptive message filtering. - Moscow: Radio and communications, 1991. -160 p.
[8] Kuzovkov N.T., Salychev O.S. Inertial and optimal filtering. - Moscow: Mechanical Engineering, 1982. - 216 p.
[9] Perov A.I. Adaptation of linear filtration systems // Radio engineering and electronics, 1987. - T.33, No. 8. - p. 1617-1625.

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[10] Igamberdiyev H.Z., Yusupbekov A.N., Zaripov O.O., Sevinov J.U. Algorithms of adaptive identification of uncertain operated objects in dynamical models // Procedia Computer Science, Volume 120, 2017, Pages 854-861. 9th International Conference on Theory and Application of Soft Computing, Computing with Words and Perception, ICSCCW 2017; Budapest; Hungary; 22-23 August 2017.
[11] Kahaner D., Mowler K., Nash S. Numerical methods and software: Per. from English - Ed. the second is a stereotype. - Moscow: Mir, 2001. - 575 p.
[12] Aeyels D., Lamnabhi-Lagarrigue F., Van der Schaft A. (Eds.) Stability and Stabilization of Nonlinear Systems. London: Springer, 2003. - 387 pp.
[13] Verlan A.F., Sizikov V.S. Integral equations: methods, algorithms, programs. Kiev: Naukova Dumka, 1986.- 542 p.
[14] Tikhonov A.N., Arsenin V.Y. Incorrect problems decision methods. - Moscow: Science, 1979. – 285 p.
[15] Tikhonov A.N., Goncharsky A.V., Stepanov V.V., Yagola A.G. Numerical methods for solving ill-posed problems, Moscow: Science, 1990.
[16] Voskoboynikov Y.E. Sustained methods and algorithms of parametric identification. - Novosibirsk: NGASU (Sibstrin), 2006. – 180 p.
[17] Bakushinsky A.B., Goncharsky A.V. Iterative methods for solving ill-posed problems. - Moscow: Science, 1989.-128 p.
[18] Bakushinsky A.B., Kokurin M.Yu. Iterative methods for solving irregular equations. - Moscow: Lenand, 2006.- 214 p.
[19] Vasiliev F.P. Methods for solving extreme problems. - Moscow: Science, 1981. - 400 p.
[20] Vainikko G.M., Veretennikov A.Yu. Iterative procedures in incorrect tasks. -Moscow: Science, 1986.