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Propagation of Non-Stationary Waves Of Transverse Displacement from a Spherical Cavity in an Elastic Half-Space

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ABSTRACT. The task about propagation of axisymmetric non-stationary waves of transverse displacement from a spherical cavity in an elastic half-space is studied. Integrated transformation of Laplace on dimensionless time was applied to the solution of a task. In space of images the task is reduced to the infinite system of the linear algebraic equations which solution is looked for in the form of an infinite series on exponents that allows to receive the solution of infinite system without use of a method of a reduction. Formulas for a component of a vector of movement and a tension of tensor are received. Transition to originals is carried out by means of the the theory of residues. The example of calculation is given.

KEY WORDS: Elastic half-space, spherical cavity, Laplace's transformation, infinite system, axisymmetric non-stationary wave, elastic medium, disturbance.

I. INTRODUCTION

Progress of various areas of the equipment and creation of the new constructions working at non-stationary dynamic influences, modern tasks of aircraft construction, shipbuilding, geophysics and seismology, and also some problems of other tendencies of scientific and technical character promote increase in relevance of dynamics of deformable bodies. Such problems as these questions belong to propagation and diffraction of shock waves on various types of heterogeneousness (inclusions, cavities, etc.).

The problem of propagation and diffraction of non-stationary waves in elastic medium with different obstacles is one of fundamental and applied problems in dynamics of a deformable solid body.

This article is devoted to studying of a task about propagation of axisymmetric non-stationary waves of transverse displacement from a spherical cavity in an elastic half-space.

The purpose of the work - elaboration of mathematical model of process of propagation of axisymmetric non-stationary waves of transverse displacement from a spherical cavity in an elastic half-space and definition a component of the intense deformed condition of the medium.

Results of a research of propagation of non-stationary waves of transverse displacement from a spherical cavity in an elastic half-space are used in applied problems of geophysics, seismography, geology, astronautics, and also in actions for carrying out deep excavations of various configuration and deep underground tanks – depository of oil and gas.

II. SIGNIFICANCE OF THE PROBLEM

This work contains mathematical model and an algorithm of the solution of a task on propagation of non-stationary waves of transverse displacement from a spherical cavity in an elastic half-space. The research of the literary review is presented in the section III, the Methodology is explained in the section IV, the section V covers the numerical results of an experiment of the study, and the section VI discusses the future research and Conclusion.

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III. LITERATURE SURVEY

Studying of propagation and diffraction of stationary and non-stationary waves of various type in continuous mediums was a subject of numerical scientific - research works.

At present time there is a significant number of publications in periodic literature [1 - 9] and also the monograph [10, 11]. They are dedicated to a research of propagation and diffraction of the steady-state and unsteady-state waves in elastic and acoustic mediums.

In the first part of work [1] propagation in an elastic spherical layer of waves of transverse displacement generated by rotary influence is studied. The exact solution of a task will be transformed to the sum of interferential waves, each of which is represented in integral like Fourier. In the second part [2] is studied the non-stationary field of shifts resulting from an interference of waves of SH reflected from borders of an elastic spherical layer. Conditions under which propagation of an interferential wave in a spherical layer happens the same as in a layer plane-parallel borders become clear.

In work [3] the task about diffraction on the motionless sphere of a flat shift wave of torsion (SH wave) in an infinite elastic medium is considered. The exact solution of a task is found in the form of convolution integral. By means of Watson's transformation physical interpretation of components of the diffraction field is given.

Diffraction of a stationary flat elastic wave of shift on cylindrical cavities in an isotropic half-space is considered in [4]. The regional task comes down to the solution of infinite system of the linear algebraic equations of rather amplitude coefficients of scattered waves. The task about diffraction of shift waves on cavities and rigid inclusions in a half-space with free border from forces and fixed, is investigated in article [5]. Calculations were carried out for cylindrical cavities and rigid inclusions of elliptic section.

The task about diffraction of a wave on the noncircular cylindrical cavity located in the circular cylinder is studied in the publication [6]. The regional problem was solved with use of the combined finite element method. The dynamic tension and moving to vicinities of a noncircular cavity are found. In [7] the task about propagation of shift indignations from a spherical cavity to an infinite elastic medium is considered. At the same time Fourier's transformation on time was used. Formulas for the shift and tension are received.

Article [8] is devoted to construction of an algorithm of the solution of a task about propagation of non-stationary waves from a thin spherical shell at an elastic half-space. The motion of a shell is described by the system of the equations connected with a name of S.P. Timoshenko. Formulas for a component of the intense deformed condition of the medium and kinematics parameters of a shell, and also for contact pressure are received.

The axisymmetric task about diffraction of non-stationary waves is studied in the publication [9]. As an obstacle absolutely rigid motionless sphere in an elastic half-space is chosen. The infinite system of the linear algebraic equations in space of images of transformation of Laplace on time is received. The intense deformed state of the medium in the neighborhood of a sphere is investigated.

Results of work [10] are devoted to a systematic research of propagation and diffraction of non-stationary waves in an elastic medium with a body of spherical shape and also the broad parametrical research is conducted. Exact solutions with use of the theory of the generalized spherical waves and methods of integrated transformations are received.

In the monograph [11] problems of diffraction of the steady-state and unsteady-state elastic waves are studied and dynamic tension near concentrators of tension of various form is defined. The large number of specific tasks of diffraction of elastic waves is given.

IV. METHODOLOGY

Formulation of the problem. Let in an elastic homogeneous isotropic half-space of $z \geq 0$ at h depth from the $z=0$ plane on $O_2 z$ axis (the point of O_2 lies on half-space border) the center O of a spherical cavity of radius R is situated ($R < h$) (Fig. 1). Let's consider two systems of coordinates: spherical r, θ, ϑ with the center in a point O and cylindrical ρ, ϑ, z from the beginning in O_2 point.

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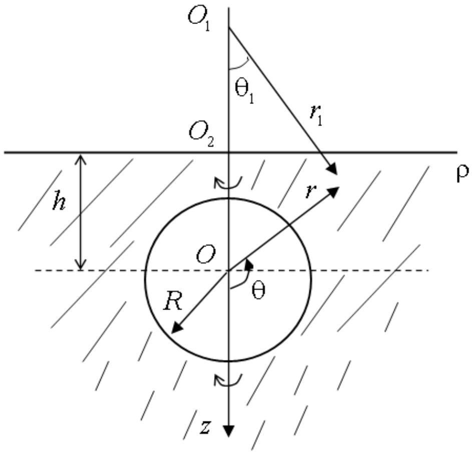

Fig. 1

Let us assume that on border of a cavity there are following conditions:

$$\sigma_{rr}\big|_{r=1} = 0, \tag{1}$$

$$\sigma_{r\theta}\big|_{r=1} = 0, \tag{2}$$

and also

$$\sigma_{r\vartheta}\big|_{r=1} = q(\tau,\theta). \tag{3}$$

Using connection of a vector of movement with scalar and vector potentials and also Hooke's law, it is easy to show that for performance of equalities (1) and (2) and also axisymmetric character of a condition (3) it is enough to put $\varphi = \psi_r = \psi_\vartheta \equiv 0$ and $\psi_\theta = \psi(r,\theta,\tau)$. At the same time for components of a vector of movement and a tensor of tension in the spherical system of coordinates we will receive the following expressions:

$$w = \frac{\partial\psi}{\partial r} + \frac{\psi}{r}, \; u = v \equiv 0, \tag{4}$$

$$\sigma_{r\vartheta} = \frac{1}{\eta^2}\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right), \; \sigma_{\theta\vartheta} = \frac{1}{\eta^2 r}\left(\frac{\partial w}{\partial \theta} - w\,\mathrm{ctg}\,\theta\right), \tag{5}$$

$$\sigma_{rr} = \sigma_{\theta\theta} = \sigma_{\vartheta\vartheta} = \sigma_{r\theta} \equiv 0,$$

and the motion of the medium is described by one wave equation concerning ψ nonzero components of vector potential

$$\eta^2\ddot{\psi} = \Delta\psi - \frac{\psi}{r^2\sin^2\theta}\,, \tag{6}$$

where Δ - Laplace's operator in the spherical system of coordinates; η - quantity, the inverse of speeds of propagation of waves *SH*.

For a formulation of the boundary conditions on the $z = 0$ plane providing existence only of axisymmetric transverse waves (the intense deformed state (4), (5) at independence its component from ϑ angle), we use connection of movements and coordinates of a vector of tension on this surface in the spherical and cylindrical systems of coordinates:

$$u_\rho = u\sin\theta + v\cos\theta, \; u_\vartheta = w, \; u_z = u\cos\theta - v\sin\theta, \tag{7}$$

$$\sigma_{\rho z} = \frac{\sigma_{rr} - \sigma_{\theta\theta}}{2}\sin 2\theta + \sigma_{r\theta}\cos 2\theta, \; \sigma_{\vartheta z} = \sigma_{r\vartheta}\cos\theta - \sigma_{\theta\vartheta}\sin\theta, \tag{8}$$

$$\sigma_{zz} = \frac{\sigma_{rr} + \sigma_{\theta\theta}}{2} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{2}\cos 2\theta - \sigma_{r\theta}\sin 2\theta,$$

where u_ρ, u_ϑ, u_z, $\sigma_{\rho z}$, $\sigma_{\vartheta z}$, σ_{zz} - components of a vector of movement and a tensor of tension in the cylindrical system of coordinates.

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For axisymmetric transverse waves, according to expressions (4) and (5), formulas (7) and (8) take the following form:

$$u_\rho = u_z = 0,\ u_\vartheta = w, \tag{9}$$

$$\sigma_{\rho z} = \sigma_{zz} = 0,\ \sigma_{\vartheta z} = \sigma_{r\vartheta}\cos\theta - \sigma_{\theta\vartheta}\sin\theta. \tag{10}$$

Consequently, for specified of the intense deformed state on the $z=0$ plane boundary conditions need to be put in the following form:

$$u_\rho\big|_{z=0} = 0 \text{ or } \sigma_{\rho z}\big|_{z=0} = 0, \tag{11}$$

$$u_z\big|_{z=0} = 0 \text{ or } \sigma_{zz}\big|_{z=0} = 0, \tag{12}$$

and also

$$w\big|_{z=0} = V_0(\tau,\rho) = V_0(\tau, -h\,\mathrm{tg}\theta) \tag{13}$$

or

$$\sigma_{\vartheta z}\big|_{z=0} = q_0(\tau,\rho) = q_0(\tau,-h\,\mathrm{tg}\theta). \tag{14}$$

Here it is considered that on the $z=0$ plane equality $\rho = -h\,\mathrm{tg}\theta$ is fair as $\rho = r\sin\theta$ and the equation of the $z=0$ plane in the spherical system of coordinates has $r\cos\theta = -h$ appearance where $\pi/2 \le \theta \le \pi$.

Private case of conditions (13) and (14) are conditions of "a rigid wall"

$$w\big|_{z=0} = 0 \tag{15}$$

and free surface

$$\sigma_{\vartheta z}\big|_{z=0} = 0. \tag{16}$$

Thus, the equation (6), non-trivial equalities in (4) and (5), and also boundary conditions (3), (13) or (14) are components of an initial-boundary task which becomes closed with the corresponding initial conditions:

$$\psi\big|_{\tau=0} = \dot\psi\big|_{\tau=0} = 0 \tag{17}$$

and with requirement of limitation of the solution on infinity

$$\lim_{r\to\infty}\psi = 0\cdot \tag{18}$$

Further, we will be limited to a task about propagation of axisymmetric transverse waves from a spherical cavity in the presence of a rigid wall (condition (15)) or of free border of a half-space (condition (16)).

In the formulation of the initial-boundary problem (3) - (6), (15) - (18) and further the following dimensionless quantities are used (dimension parameters are denoted by a prime):

$$r = \frac{r'}{R},\ \tau = \frac{c_1 t}{R},\ \eta = \frac{c_1}{c_2},\ h = \frac{h'}{R},\ \sigma_{\alpha\beta} = \frac{\sigma'_{\alpha\beta}}{\lambda + 2\mu},\ w = \frac{w'}{R},\ \psi = \frac{\psi'}{R^2},$$

where λ, μ are elastic constants to Lama; c_1 and c_2 are speeds of propagation of waves of stretching - compression and shift; τ is dimensionless time.

Method of solution. To the initial-boundary problem (3) - (6), (15) - (18) we apply Laplace's transformation on τ time. In space of images we write down the solution of the equation (6) taking into account a condition of limitation (18) in the form [10]:

$$\psi^L = -\frac{\sin\theta}{\sqrt{r}}\sum_{n=1}^{\infty} A_n^L(s) K_{n+1/2}(r\eta s) C_{n-1}^{3/2}(\cos\theta) - \frac{\sin\theta_1}{\sqrt{r_1}}\sum_{n=1}^{\infty} B_n^L(s) K_{n+1/2}(r_1\eta s) C_{n-1}^{3/2}(\cos\theta_1), \tag{19}$$

where $K_{n+1/2}(x)$ are modified Bessel's functions of the second kind; $C_{n-1}^{3/2}(x)$ are Gegenbauer's polynomials; $A_n^L(s)$ and $B_n^L(s)$ are unknown functions; r_1, θ_1, ϑ_1 are the additional spherical system of coordinates received by transfer along an axis of $O_2 z$ of the center O of initial spherical system in O_1 point symmetric to O point concerning the $z=0$ plane (see fig. 1); s is the transformation parameter and the superscript L indicates a transformant.

Now taking into account (4), (5), substituting Eqs (19) into boundary conditions (15), (16) and also taking into account the relations

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$$r\big|_{z=0} = r_1\big|_{z=0}, \qquad \theta\big|_{z=0} + \theta_1\big|_{z=0} = \pi, \tag{20}$$

and the equality of Gegenbauer's polynomials [14]

$$C_{n-1}^{3/2}(-x) = (-1)^n C_{n-1}^{3/2}(x), \tag{21}$$

We obtain that these boundary conditions are satisfied at $z=0$, if we require that the arbitrary function are related to one another as follows ($n \geq 1$)

$$B_n^L(s) = \mp(-1)^n A_n^L(s). \tag{22}$$

The upper sign corresponds to the free boundary while the lower sign corresponds to the absolutely rigid boundary.

Further, substituting the relations (22) into expression (19) and using the addition theorem [13] for the $K_{n+1/2}(x)$ functions, and also taking into account the relations of these functions through elementary functions [12, 14], we represent a series (19) and the coefficients so

$$\psi^L = -\sin\theta \sum_{n=1}^{\infty} \psi_n^L(r,s) C_{n-1}^{3/2}(\cos\theta), \tag{23}$$

$$\psi_n^L(r,s) = \frac{1}{r^{n+1}(\eta s)^n}\left[R_{n0}(r\eta s) A_n(s) e^{-r\eta s} + G_{n0}(r\eta s) \times , \times \sum_{p=1}^{\infty} S_{np}(s) A_p(s) e^{-2\eta h s} \right], \tag{24}$$

$$S_{np}(s) = \frac{\mp(-1)^p(2n+3)}{2(n+1)(n+2)} \sum_{\sigma=|p-n|}^{p+n} b_\sigma^{(n1p1)} \frac{R_{\sigma 0}(2\eta h s)}{(2\eta h s)^{\sigma+1}}, \quad R_{n0}(s) = \sum_{k=0}^{n} A_{nk} s^{n-k}, \quad A_{nk} = \frac{(n+k)!}{(n-k)!k!2^k} \quad (0 \leq k \leq n),$$

$$A_{nk} = 0 \quad (k<0, k>n), \quad G_{ni}(s) = R_{ni}(-s)e^s - R_{ni}(s)e^{-s}, \quad i = \overline{0,4}$$

where $b_\sigma^{(n1p1)}$ are Klebsch-Gordon's coefficients [13, 14].

Analogously (23), in space of the image we represent movements w^L and tension $\sigma_{r\vartheta}^L$ in the form of a series on Gegenbauer's polynomials:

$$w^L = -\sin\theta \sum_{n=1}^{\infty} w_n^L(r,s) C_{n-1}^{3/2}(\cos\theta), \quad \sigma_{r\vartheta}^L = -\sin\theta \sum_{n=1}^{\infty} \sigma_{r\vartheta n}^L(r,s) C_{n-1}^{3/2}(\cos\theta). \tag{25}$$

Now, according to formulas (4) and (5), and also taking expression (24) into account we will receive the following expressions for coefficients of series (25):

$$w_n^L(r,s) = -\frac{1}{r^{n+2}(\eta s)^n}\left[R_{n3}(r\eta s) A_n(s) e^{-\eta r s} + G_{n3}(r\eta s) T_n(s) \right],$$

$$\sigma_{r\vartheta n}^L(r,s) = \frac{1}{\eta^{n+2} s^n r^{n+3}}\left[R_{n4}(r\eta s) A_n(s) e^{-r\eta s} + G_{n4}(r\eta s) T_n(s) \right], \tag{26}$$

$$T_n(s) = \sum_{p=1}^{\infty} S_{np}(s) A_p(s) e^{-2\eta h s}, \quad R_{n3}(s) = R_{n1}(s) - 2R_{n0}(s),$$

$$R_{n4}(s) = R_{n2}(s) - R_{n0}(s), \quad R_{n1}(s) = \sum_{k=0}^{n+1} B_{nk} s^{n+1-k}, \quad R_{n2}(s) = \sum_{k=0}^{n+2} C_{nk} s^{n+2-k},$$

$$B_{nk} = A_{nk} + kA_{n,k-1} \; (k \neq 0), \quad B_{n0} = A_{n0},$$

$$C_{nk} = B_{nk} + kB_{n,k-1} \; (k \neq 0), \quad C_{n0} = B_{n0},$$

where the polynomial $R_{n0}(s)$ is connected with $R_{n1}(s)$ and $R_{n2}(s)$ with relations [10]:

$$\left[R_{n0}(s) s^{-n-1} e^{-s} \right]' = -R_{n1}(s) s^{-n-2} e^{-s}, \quad R_{n1}(s) = R_{n+1,0}(s) - nR_{n0}(s),$$

$$\left[R_{n1}(s) s^{-n-2} e^{-s} \right]' = -R_{n2}(s) s^{-n-3} e^{-s}.$$

Further, in space of images, we represent the right part of a boundary condition (3) in form of a series in Gegenbauer's polynomials

$$q^L(s,\theta) = -\sin\theta \sum_{n=1}^{\infty} q_n^L(s) C_{n-1}^{3/2}(\cos\theta), \tag{27}$$

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where

$$q_n^L(s)=\frac{2n+3}{2(n+1)(n+2)}\int_0^{\pi} q^L(s,\theta)\sin^2\theta C_{n-1}^{3/2}(\cos\theta)d\theta .$$

Then from (3) follows that coefficients of a series (27) have to meet a boundary condition of

$$\sigma_{r\vartheta n}^L\Big|_{r=1}=q_n^L(s). \qquad (28)$$

Substituting expression (26) in a boundary condition (28), we obtain an infinite system of the linear algebraic equations in the unknown functions $A_n^L(s)$ which we will write down in the form of the matrix equation:

$$\mathbf{M}(s)\mathbf{A}(s)y^2+\mathbf{F}^{(1)}(s)\mathbf{A}(s)x-\mathbf{F}^{(2)}(s)\mathbf{A}(s)xy^2=\mathbf{p}(s)y \qquad (29)$$

$$\mathbf{A}(s)=\left\|A_1^L(s),A_2^L(s),...\right\|^T,\ \mathbf{p}(s)=\left\|p_1^L(s),p_2^L(s),...\right\|^T$$

$$x=e^{-2h\eta s}, \qquad y=e^{-\eta s}.$$

where $\mathbf{M}(s)$ is an infinite diagonal matrix with the $M_n(s)$ elements; $\mathbf{F}^{(l)}(s)$ are infinite matrixes of the $F_{np}^{(l)}(s)$ ($l=1,2$) elements; $\mathbf{p}(s)$ - an infinite vector-columns with the $p_n^L(s)$ elements; $\mathbf{A}(s)$ - an infinite unknown vector-columns with the $A_n^L(s)$ components, and the $M_n(s)$, $F_{np}^{(l)}(s)$ and $p_n^L(s)$ functions have the following appearance:

$$M_n(s)=R_{n4}(\eta s),\ p_n^L(s)=\eta^{n+2}s^n q_n^L(s),$$

$$F_{np}^{(1)}(s)=M_n(-s)S_{np}(s),\ F_{np}^{(2)}(s)=M_n(s)S_{np}(s).$$

We search for the solution of system of the equations (29) in the form of an infinite series in exponential functions

$$\mathbf{A}(s)=\sum_{i,j=0}^{\infty}\mathbf{a}_{ij}(s)x^i y^{-j-1}, \qquad \mathbf{a}_{ij}(s)=\left\|a_{ij}^{(1)}(s),a_{ij}^{(2)}(s),...\right\|^T. \qquad (30)$$

Substituting an infinite series (30) in the matrix equation (29) and equating coefficients of the left and right parts at equal powers of variables x and y (the right part of a series contains only one nonzero member), we obtain the system of recurrence relations relative to the functions $a_{ij}^{(n)}(s)$ and the corresponding initial conditions to them:

$$a_{00}^{(n)}(s)=\frac{p_n^L(s)}{M_n(s)},\ \mathbf{a}_{0j}(s)=0 \qquad j\ge 1, \qquad (31)$$

$$\mathbf{a}_{i1}(s)=0,\ i\ge 0, \qquad (32)$$

$$\mathbf{a}_{i0}(s)=\mathbf{S}(s)\mathbf{a}_{i-1,0}(s),\ i\ge 1, \qquad (33)$$

$$\mathbf{a}_{ij}(s)=\mathbf{S}(s)\mathbf{a}_{i-1,j}(s)-\mathbf{N}(s)\mathbf{S}(s)\mathbf{a}_{i-1,j-2}(s),\ i\ge 1, j\ge 2,$$

(34)

$$\mathbf{N}(s)=\left\|\frac{M_n(-s)}{M_n(s)}\delta_{nm}\right\|,\ \mathbf{S}(s)=\left\|S_{np}(s)\right\|, \qquad n\ge 1, p\ge 1$$

where δ_{nm} is the Kronecker delta, $n, m=1,2,...$.

Follows from initial (31), (32) and recurrence relations (34) that at odd values of the lower second indexes the following equalities are right

$$\mathbf{a}_{i,2j+1}(s)=0 \text{ when } i\ge 1,\ j\ge 1.$$

Analogously, for even values of low second indexes from initial (31) and recurrence relations (34) that follows

$$\mathbf{a}_{i,2j}(s)=0 \text{ when } j>i,\ i\ge 1,\ j\ge 2.$$

Hence it follows that $2hi-j>i-j>0$, and a certain right half-plane $\mathrm{Re}\, s>\alpha$ exists in which

$$\left|x^i y^{-j}\right|=\left|e^{-(2hi-j)\eta s}\right|<1$$

and series (30) converges n this half-plane.

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Recurrence relations (31)-(34) enable us to determine all the elements of the corresponding columns $\mathbf{a}_{ij}(s)$ without using reduction of the infinite system of equations. An analysis of these relations shows that the elements of the required columns are rational functions of the Laplace transformation parameter, which enables us to calculate their originals, and, consequently, also the originals of the coefficients of the series for the potential, the movement and the tension of the medium using the theory of residues.

Final formulas for images of coefficients of series (25) of movements w^L and tension $\sigma^L_{r\vartheta}$ on Gegenbauer's polynomials follows from the expressions (26) and solution of system of the equations (30):

$$w_n^L(r,s) = -\frac{1}{r^{n+2}(\eta s)^n}\sum_{i,j=0}^{\infty}\left\{R_{n3}(\eta rs)a_{ij}^{(n)}(s)y^r + +G_{n3}(s)\sum_{p=1}^{\infty}S_{np}(s)a_{ij}^{(p)}(s)x\right\}x^i y^{-j-1},$$

$$\sigma_{r\vartheta n}^L(r,s) = \frac{1}{r^{n+3}\eta^{n+2}s^n}\sum_{i,j=0}^{\infty}\left\{R_{n4}(\eta rs)a_{ij}^{(n)}(s)y^r + G_{n4}(s)\sum_{p=1}^{\infty}S_{np}(s)a_{ij}^{(p)}(s)x\right\}x^i y^{-j-1}. \quad (35)$$

Now we will show that formulas for movement and tension in the analogous problem for an elastic homogeneous isotropic space follow from the solution obtained. For this purpose limit transition at $h\to\infty$ ($x\to 0$) from the received results for a half-space of an elastic medium it is possible to find solutions of an analogous task for space.

In this case $S_{np}(s)=0$ and at $i\geq 1, j\geq 0$ all coefficients of $a_{ij}^{(n)}(s)$ are equal to zero, except $a_{00}^{(n)}(s)$.

Then, formulas for movement and tension (35) for space of an elastic medium take the following forms:

$$w_n^L(r,s) = -\frac{\eta^2}{r^{n+2}}\frac{R_{n3}(\eta rs)q_n^L(s)}{R_{n4}(\eta s)}y^{r-1},\ \sigma_{r\vartheta n}^L(r,s) = \frac{1}{r^{n+3}}\frac{R_{n4}(\eta rs)q_n^L(s)}{R_{n4}(\eta s)}y^{r-1}.$$

V. NUMERICAL RESULTS

Fig. 2 corresponds to propagation of axisymmetric non-stationary waves of transverse displacement from a spherical cavity. Here the dependences of $\sigma_{r\vartheta}$ tension time received taking into account four members of series on Gegenbauer's polynomials for a half-space from aluminum ($\eta=1.9853$) at the free boundary plane and a depth of inclusion of $h=1.5$ are given. The graphics correspond to a cavity with a uniform pressure in the form of the single Heaviside function of $q(\tau,\theta)=H(\tau)$ and the following points: $r=1.2$ and $\theta=3\pi/4$ (curve 1), $r=1.2$ and $\theta=\pi/2$ (curve 2). From the analysis of graphs follows that on a period $0.4\leq\tau\leq 3$ in a point of $r=1.2$, $\theta=3\pi/4$ (a curve 1) is absent influence of the wave reflected from flat border. Further, at $\tau>3$ flat border exert impact on distribution of tension of $\sigma_{r\vartheta}$.

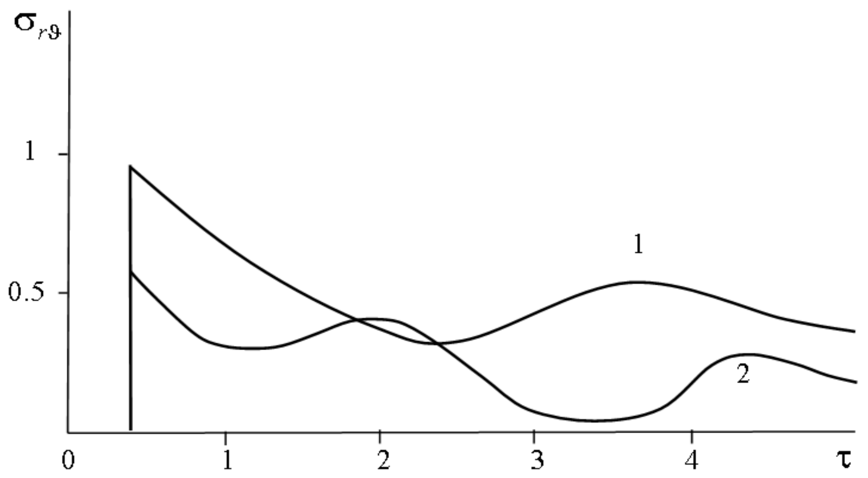

Fig. 2.

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VI. CONCLUSION AND FUTURE WORK

The algorithm of modeling of process of propagation of axisymmetric non-stationary waves of transverse displacement from a spherical cavity in an elastic half-space is developed. In space of images of Laplace the infinite system of the linear algebraic equations which solution is constructed in the form of an infinite series on exponents that allowed to receive the solution of infinite system without use of a method of a reduction is received. Schedules of dependence on time of tension of $\sigma_{r\vartheta}$ show that the waves reflected from flat border exert impact on the intense deformed condition of the medium.

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