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Digital processing of geophysical signals measured at unequal intervals by cubic splines

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ABSTRACT: This article is about building local and natural spline models for signals measured at unequal intervals.As an example, local spline and natural spline interpolation models were constructed for digital processing of geophysical signals measured at unequal intervals.The article also presents the results of interpolation and estimated errors.

KEY WORDS: geophysical signal, spline function, interpolation cubic spline, interpolation, unequal interval, classical polynomial, cubic spline, coefficient.

I.INTRODUCTION

In recent years, digital functions have been used extensively to address and analyze digital signals.An analysis of the existing literature shows that interpolation and grinding splines are used by the approximation method, and polynomial and basissplines by type of imaging.Interpolation discs are such splines that satisfy the set of boundary conditions and conditions at the points of the function definition, while grinding splines are related to the optimization of various functions [1].

Cubic splines have a great mathematical advantage. They are the only function that has the minimum plane of all the functions of the second derivative, interpolating the given points and integrating the square.Cubic splines are characterized by high accuracy in digital processing of signals and simplicity of construction compared to classical polynomials.

We are familiar with many ways in mathematics to approximate functions with polynomials. But these methods have a number of drawbacks, the most important of which is that the state of such functions around a point is inextricably linked to their complete state. Moreover, as a disadvantage of polynomials, they do not always approach the interpolated function.

Recently, methods have been developed in computational mathematics that are free of these shortcomings. One of them is the Spline function, introduced to science by Schönberg in 1946. Spline features developed extensively after the 50s.

Well-known scientists on the development of the theory of spline functions, their construction and application I.J. Schoenberg, S. de Boor, JLHolladay, Dj.Alberg, E.Nilson, Dj.Walsh, S.B.Stekin, LLSchumaker, B.D.Bojanov, Yu.S.Zavyalov, B.I.Isroilov, H.M.Shadimetov, A.R.Hayotov, H.N.Zaynidinov, S.A.Bakhromov and others. .

II. DISCUSSING THE PROBLEM

It is known that the $S(x)$ function, which corresponds to the $f(x)$function and passing through the given $\{x\}_{i=0}^{N}$ nodes, is considered to be interpolated cubic spline and meets the following conditions.

1) In each $[x_i, x_{i+1}]$ $(i = \overline{0,n})$ range, the $S(x)$ function is a tertiary multiplier;
2) The first and second order derivatives of the $S(x)$function must be continuous in [a,b];
3) $S(x_i) = f(x_i), i = \overline{0,n}$.

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The last condition is called the interpolation condition and the function satisfying all three conditions is called the interpolation cubic spline [2,6,9].
It is considered building a offline model based on the specified conditions.The main advantage of the splines from the classical polynomials is the locality of the splines, that is, the construction of two spots.
It can be see in Figure 1.

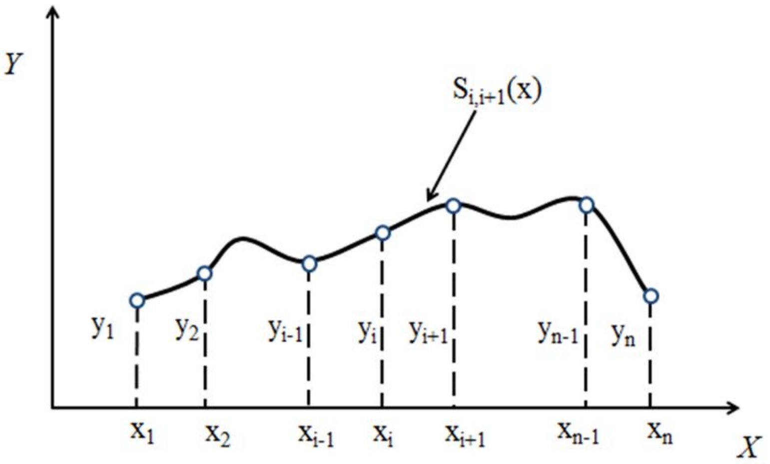

Figure 1. View of unequal intervals in the construction of $S_3(x)$cubic spline.

So, taking a look at the construction of the $S_{i,i+1}(x)$cubic spline function in Figure1.

III. BUIL A LOCAL CUBIC SPLINE

The local cubicle spline considered below also has the maximum convergence order $O(h^3)$, followed by the interpolation process.Consider building a local cubic spline
The considered node points are given in the a,b in the range $\Delta: x_i = a + h_i$, $i = 0,1,\dots,N, h_i = x_i - x_{i-1}$
Δ'nets are filled with two $x_{-1}$and x_{N+1}node points:

$$\Delta' = \Delta \cup \{x_{-1}\} \cup \{x_{N+1}\},$$

Let the net $f(x)$function be given in this grid:

$$f_{-1}, f_0, f_1, \dots, f_{N-1}, f_N, f_{N+1} \quad (1)$$

Based on this table, it is constructed a local cubic spline with $S_3(x) = S_3(f;x)$to interpolate the $f(x)$function in the Δnet.
There is built two parabola for this

$$y_i(x) = a_i x^2 + b_i x + c_i \text{ва} y_{i+1}(x) = a_{i+1} x^2 + b_{i+1} x + c_{i+1},$$

Accordingly, the parabola passes through the following nutrients

$$(x_{i-1}, f_{i-1}), \quad (x_i, f_i), \quad (x_{i+1}, f_{i+1})$$

and

$$(x_i, f_i), \quad (x_{i+1}, f_{i+1}), \quad (x_{i+2}, f_{i+2})$$

For convenience, there is performed$t = (x - x_i)/h_i$replacement
Here$h_i = x_i - x_{i-1}$.

$$y_i(x) = y_i(t) = -0{,}5t(1-t)f_{i-1} + (1-t^2)f_i + 0{,}5t(1+t)f_{i+1}, \quad (2)$$

$$y_{i+1}(x) = y_{i+1}(t) = 0{,}5(1-t)(2-t)f_i + t(2-t)f_{i+1} - 0{,}5t(2-t)f_{i+2}. \quad (3)$$

It is gained the following view through the combination of the above parabolas

$$S_i(x) = S_i(t) = (\alpha_1 + \alpha_2 t)y_i(x) + (\alpha_3 + \alpha_4 t)y_{i+1}(x).$$

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From the condition of interpolation

$$S_i(x_i) = f_i, \quad S_i(x_{i+1}) = f_{i+1}$$

There is constructed equations (4) to determine the $\alpha_1, \alpha_2, \alpha_3$ and α_4

$$\alpha_1 + \alpha_2 = 1, \ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1, (4)$$

there is found α_3 and α_4, $\alpha_3 = 1 - \alpha_1, \ \alpha_4 = -\alpha_2$.
The system of equations (5) and (6) is formed after some simplification of the interpolation conditions of the first $S_i'(x)$, $S_{i+1}'(x)$ and second $S_i''(x)$, $S_{i+1}''(x)$ sequences of the x_{i+1} node.

$$\alpha_1(\Delta^2 f_{i+1} - \Delta^2 f_{i-1}) + \alpha_2 \Delta^3 f_{i-1} = \Delta^3 f_i, \quad (5)$$

$$\alpha_1(\Delta^4 f_{i-1}) - \alpha_2(\Delta^2 f_{i-1} + \Delta f_i - \Delta f_{i+2}) = \Delta^3 f_i \quad (6)$$

Δ- the operator difference.
If it is said that the system of equations (4) - (6) is solved by $\alpha_1^*, \alpha_2^*, \alpha_3^*$ and α_4^*

$$S_3(x) = S_3(t) = (\alpha_1^* + \alpha_2^* t) y_i(x) + (\alpha_3^* + \alpha_4^* t) y_{i+1}(x).$$

the spline will have 1 defect.But $f_{i-1}, f_i, f_{i+1}, f_{i+2}, f_{i+3}$ are complex rational functions given the coefficients.Therefore, such splines are unfavorable to the computer.If it is said $\alpha_1 = 1, \ \alpha_2 = -1$, then $\alpha_3 = 0$ and $\alpha_4 = 1$.These values satisfy equations (4) - (5) but do not satisfy equation (6).
Therefore, the deflection of the interpolation spline in $[x_i, x_{i+1}]$ interval is 2 defecte.

$$S_3(f;x) = S_3(t) = (1-t) y_i(t) + t y_{i+1}(t) (7)$$

Entering a markup

$$\varphi_1(t) = -0{,}5t(1-t)^2, \quad \varphi_2 = 0{,}5(1-t)(2+2t-3t^2),$$
$$\varphi_3(t) = 0{,}5t(1+4t-3t^2), \quad \varphi_4 = -0{,}5(1-t)t^2$$

If it is put the (2) and (3) expressions in place of the $y_i(t)$ and $y_{i+1}(t)$ given in (7), it will have a (8) view for the $[x_i, x_{i+1}]$ interval.

$$S_3(f;x) = \sum_{j=0}^{3} \phi_{j+1}(t) f(x_{i+j-1}) (8)$$

Based on spline control [3], the (8) function in this $[x_i, x_{i+1}]$ range can be called a spline function.

IV. NATURAL CUBIC SPLAYING CONSTRUCTION

The second spline is a natural cubic spline, and it be built as follows.
There is searched for the $S(x) = S_i(x)$ function in each $[x_i, x_{i+1}]$ $(i = \overline{1, n-1})$ interval in the form of a tertiary multiplex.

$$S_i(x) = a_i + b_i(x - x_i) + \frac{c_i}{2}(x - x_i)^2 + \frac{d_i}{6}(x - x_i)^3, \quad (9)$$
$$x_{i-1} \le x \le x_i, i = 1,2,..., N,$$

Here are the coefficients that a_i, b_i, c_i, d_i needs to determine.
These coefficients are calculated as follows.

$$S_i'(x) = b_i + c_i(x - x_i) + \frac{d_i}{2}(x - x_i)^2,$$
$$S_i''(x) = c_i + d_i(x - x_i), \quad S_i'''(x) = d_i,$$

the coefficients should be written based on the following

$$a_i = S_i(x_i), \ b_i = S_i'(x_i), \ c_i = S_i''(x_i), \ d_i = S_i'''(x).$$

the coefficients should be defined $a_i = f(x_i), \quad i = 1,2,..., N$, under the conditions of interpolation.In addition, the continuity of the function should be recorded as follows.

$$S_i(x_i) = S_{i+1}(x_i), \quad i = 1,2, \ldots, N-1.$$

Given the expressions for the $S(x)$ cubic spline, express the $i = 1,2,..., N-1$ equations as follows

$$a_i = a_{i+1} + b_{i+1}(x_i - x_{i+1}) + \frac{c_{i+1}}{2}(x_i - x_{i+1})^2 + \frac{d_{i+1}}{6}(x_i - x_{i+1})^3.$$

If we define $h_i = x_i - x_{i-1}$, in general terms the equations come to (10).

$$h_i b_i - \frac{h_i^2}{2}c_i + \frac{h_i^3}{6}d_i = f_i - f_{i-1},\ i = 1,2,...,N \quad (10)$$

Write the condition of continuity for the first order yield

$$S_i'(x_i) = S'_{i+1}(x_i),\ \ i = 1,2,...,N-1$$

After the above substitution, the statements appear (11)

$$h_i c_i - \frac{d_i^2}{2}h_i = b_i - b_{i-1},\ \ i = 2,3,...,N. \quad (11)$$

From the continuum condition of the second order derivative, the equations (12)

$$h_i d_i = c_i - c_{i-1},\ \ i = 2,3,...,N. \quad (12)$$

By combining (10) - (12), a system will be gained of 3N - 2 equations for an unknown
$b_i, c_i, d_i,\ \ i = 1,2,\ldots,N$ of 3N.

The two missing equations can be obtained by setting a boundary condition for cubic spline.For instance, it is said that the $f(x)$ function satisfies the $f''(a) = f''(b) = 0$ condition.From this it will be gained the following $S_1''(x_0) = 0, S_N''(x_N) = 0$, it's, $c_1 - d_1 h_1 = 0,\ c_N = 0$.

Note that the condition $c_1 - d_1 h_1 = 0,$ coincides with equation (12) for $i = 1$, if it put $c_0 = 0$. Thus, a closed system is come of equations for determining the coefficients of the cubic spline:

$$h_i d_i = c_i - c_{i-1},\ \ i = 1,2,\ldots,N,\ \ c_0 = c_N = 0, \quad (13)$$

$$h_i c_i - \frac{h_i^2}{2}d_i = b_i - b_{i-1},\ \ i = 1,2,\ldots,N, \quad (14)$$

$$h_i b_i - \frac{h_i^2}{2}c_i + \frac{h_i^3}{6}d_i = f_i - f_{i-1},\ i = 1,2,\ldots,N. \quad (15)$$

to make sure that this system has aunique solution[6,9]. It is eliminated the variables $b_i, d_i,\ \ i = 1,2,...,N-1$ from (13) - (15), and it will be obtain a system containing only $c_i,\ \ i = 1,2,...,N-1$. To do this, consider two neighboring equations (15):

$$b_i = \frac{h_i}{2}c_i - \frac{h_i^2}{6}d_i + \frac{f_i - f_{i-1}}{h_i},$$

$$b_{i-1} = \frac{h_{i-1}}{2}c_{i-1} - \frac{h_{i-1}^2}{6}d_{i-1} + \frac{f_{i-1} - f_{i-2}}{h_{i-1}}$$

and subtract the second equation from the first. Then we get

$$b_i - b_{i-1} = \frac{1}{2}(h_i c_i - h_{i-1}c_{i-1}) - \frac{1}{6}(h_i^2 d_i - h_{i-1}^2 d_{i-1}) + \frac{f_i - f_{i-1}}{h_i} - \frac{f_{i-1} - f_{i-2}}{h_{i-1}}$$

Substituting the expression found for $b_i - b_{i-1}$ the right side of equation (14), we obtain

$$h_i c_i + h_{i-1}c_{i-1} - \frac{h_{i-1}^2}{3}d_{i-1} - \frac{2h_i^2}{3}d_i = 2\left(\frac{f_i - f_{i-1}}{h_i} - \frac{f_{i-1} - f_{i-2}}{h_{i-1}}\right). \quad (16)$$

Next, from equation (13) it is obtain

$$h_i^2 d_i = h_i(c_i - c_{i-1}),\ \ h_{i-1}^2 d_{i-1} = h_{i-1}(c_{i-1} - c_{i-2})$$

and substituting these expressions in (15), it is arrived at the equation

$$h_{i-1}c_{i-2} + 2(h_{i-1} + h_i)c_{i-1} + h_i c_i = 6\left(\frac{f_i - f_{i-1}}{h_i} - \frac{f_{i-1} - f_{i-2}}{h_{i-1}}\right).$$

Finally, to determine the coefficients c_i, the system is obtained of equations

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$$h_i c_{i-1} + 2(h_i + h_{i+1})c_i + h_{i+1}c_{i+1} = 6\left(\frac{f_{i+1} - f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}\right). \quad (17)$$

$$i = 1,2,..., N,\ c_0 = c_N = 0.$$

Due to the diagonal prevalence, system (17) has a unique solution.Since the matrix of the system is three diagonal, the solution is obtained by the progonka method.From the found coefficientsc_i, (i=1,…,n) the coefficients b_i and d_i are determined using explicit formulas

$$d_i = \frac{c_i - c_{i-1}}{h_i}, \quad b_i = \frac{h_i}{2}c_i - \frac{h_i^2}{6}d_i + \frac{f_i - f_{i-1}}{h_i}, \quad (18)$$

$$i = 1,2,..., N.$$

Thus, it is proved that there exists a unique cubic spline defined by conditions 1) - 2) and the boundary conditions $S''(a) = S''(b) = 0$[4,5].

V. THE RESULTS OF THE NUMERICAL OPERATION OF GEOPHYSICAL SIGNALS WITH THE HELP OF INTERPOLATION CUBIC SPLINES

The construction of the interpolation cubic spline model was performed using the geophysical signal preliminary data presented in Table 1[7,8].

Table 1.Values of geophysical signals measured at unequal intervals

№	Distance, km	agnetic field, nTs	№	Distance, km	Magnetic field, nTs
1.	0.2	0.6	26.	10.2	17.1
2.	0.7	0.9	27.	10.3	19.2
3.	1	1.1	28.	10.7	22.29
4.	1.3	1.3	29.	10.9	24.79
5.	2.1	1.6	30.	11.3	25.7
6.	2.8	2	31.	11.5	25.6
7.	3.3	2.5	32.	11.8	24.9
8.	3.8	3	33.	12	23.1
9.	4.3	3.7	34.	12.3	20.2
10.	5	4.9	35.	12.40	17
11.	5.1	5.5	36.	12.6	15.4
12.	5.4	6.1	37.	12.7	14.6
13.	5.7	7	38.	12.9	13.4
14.	5.9	7.8	39.	13.2	13.1
15.	6.2	8.9	40.	13.5	12.7
16.	6.5	10	41.	13.8	12.4
17.	6.7	10.6	42.	14.2	12.1
18.	6.9	12	43.	14.5	12.4
19.	7.2	13.7	44.	14.7	12.6
20.	7.7	15	45.	14.9	2.7
21.	7.9	15.5	46.	15.1	1.9
22.	8	15.4	47.	15.3	1.7
23.	9.4	15.1	48.	15.7	1.1
24.	9.6	15.4	49.	15.9	0.7
25.	9.8	16	50.	16.6	0.2

Table 2.The error results in the interpolation process of the geophysical signal.

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№	Types of Spline	Absolute error	Relative error
1.	Natural spline	1.0504	8 %
2.	Local spline	0.6375	5 %

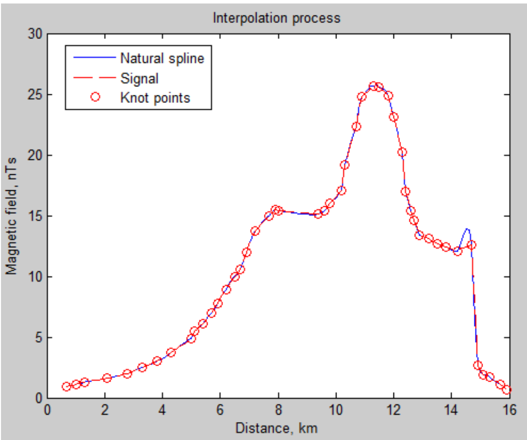

Figure 2. Results of geophysical signal interpolation using natural spline.

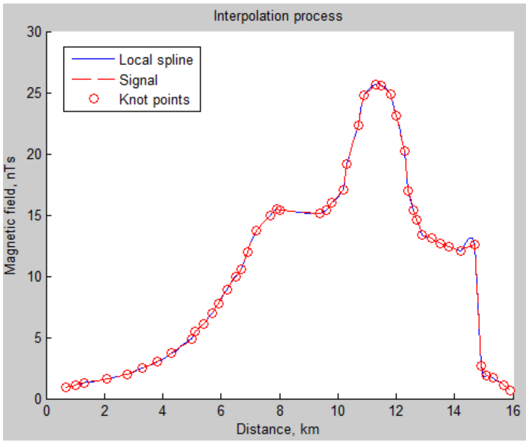

Figure 3. Results of geophysical signal interpolation using local spline.

VI. CONCLUSION

In this paper, geophysical signal interpolation is performed using natural spline and local interpolation cubic splines. In the reviewed process, it was found that the local interpolation cubic spline is high.
This is illustrated in Figures 1 and 2, and the errors are estimated and summarized in Table 2.Comparative analysis of natural spline and local interpolation cubic splines is as follows:in interpolation of geophysical signals, the local interpolation cubic spline approaches the object better than the natural spline (Figure 3).It is found that the local interpolation cubic splay accuracy is 1.6476 times higher than that of natural spline (Table 2).Therefore, the use of local interpolation cubic spline models for the digital processing of biomedicine signals[10].This results in a higher practical validity of the diagnostic analysis performed on patients.

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