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Optimal control of unsteady water movement in the main canals

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ABSTRACT: Mathematical models of sections of the main canal, based on non-linear differential equations of Saint-Venant, describing the transient motion of the water flow in them, have been developed. The restrictions for the implementation of the optimal control problem in the canal, hydraulic structures and lateral-water intakes are determined. Criteria for optimal control have been developed for canals, hydraulic structures and lateral-water intakes. The necessary optimality conditions for the system of Saint-Venant equations are obtained, when controlling within the region of restrictions. The developed mathematical models, criteria for optimal control and the necessary conditions for optimality will provide water consumers with a minimum of water resources losses.

KEY WORDS: mathematical model, unsteady flow of water, main canals, optimal control problems, fundamental solution, differential equations, hydraulic structures.

I.INTRODUCTION

Currently, there is no single systematic approach to modeling the dynamics of waterworks facilities, only there is a wide class of mathematical models of individual objects of varying degrees of complexity, so the choice of mathematical models that will describe the complex processes of water supply and water distribution in waterworks facilities with the required degree of accuracy is a very difficult task problematic.

Dynamic models with distributed parameters are mainly described by the Saint-Venant equation and take into account the distribution of the flow motion parameters in sections of the main canal along the length and time.

The condition of the main canal section is characterized by an unsteady flow of water and is described by the system of differential equations of Saint-Venant in the form of energy conservation laws [1,2,3]

$$B\frac{\partial z}{\partial t}+\frac{\partial Q}{\partial x}=q,$$

$$\frac{1}{g\omega}\left(\frac{\partial Q}{\partial t}+2v\frac{\partial Q}{\partial x}\right)+\left[1-\left(\frac{v}{c}\right)^{2}\right]\frac{\partial z}{\partial x}=\left[i+\frac{1}{B}\left(\frac{\partial \omega}{\partial x}\right)_{h=const}\right]\left(\frac{v}{c}\right)^{2}-\frac{Q|Q|}{K^{2}}, \tag{1}$$

where $v=\frac{Q}{\omega}$; $c=\sqrt{\frac{g\omega}{B}}$; $Q=Q(x,t)$ – water flow rate; $z=z(x,t)$ – the ordinate of the free surface; g – gravitational constant; i – bottom slope; $B=B(z)$ – flow width over a living section surface; $\omega=\omega(z)$ – living cross-sectional area of the stream; $c=c(z)$ – propagation speed of small waves; $K=K(z)$ – flow rate module.

Partial differential equations of the hyperbolic type in the system (1) are the equations of conservation of mass and momentum of the flow and are a mathematical model of the unsteady movement of water in the open channel section.

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The flow rate and the ordinate of the free surface are selected here as functions determining the flow $z(x,t)$. Independent variables are the longitudinal coordinate x and time t.

The riverbed is set by the ordinate of the bottom $z_0(x)$ and cross-sectional width $B(x,t)$ on distance $z(x,t)$ (vertically) from the bottom of the channel, then the depth of the stream: $h(z,t)=z(x,t)-z_0(x,t)$; flow cross-sectional area: $\omega(x,t)=\int_0^h B(x,z)dz$; average speed of flow: $\upsilon=\frac{Q}{\omega}$; small wave propagation speed: $c=\sqrt{g\omega/B}$; bottom slope $i=-dz_0/dx$.

The characteristic form of equations (1) has the form [3,4]

$$\frac{\partial Q}{\partial t}+(v\pm c)\frac{\partial Q}{\partial x}-B(v\mp c)\left[\frac{\partial z}{dt}+(v\pm c)\frac{\partial z}{\partial x}\right]=$$
$$=\left(\phi-\frac{Q|Q|}{K^2}\right)g\omega-(v\mp c)q. \quad (2)$$

where $\phi=\left[i+\frac{1}{B}\left(\frac{\partial\omega}{\partial x}\right)_{h=const}\right]\left(\frac{v}{c}\right)^2$.

The initial conditions are specified as

$$z(x,0)=z_0(x), \qquad Q(x,0)=Q_0(x), \quad (3)$$

where $Q_0(x)$, $z_0(x)$ - knows functions.

Boundary conditions at points $x=0$ and $x=l$ written as follows

$$Q_1(0,t)=g_1\left(z_1(0,t),\ u_1(t)\right), \quad (4)$$
$$Q_2(l,t)=g_2\left(z(l,t),\ u_2(t)\right),$$

where

$$g_1=\mu_1 b_1 u_1(t)\sqrt{2g\left(z_1(x,t)-z_1(0,t)\right)},$$
$$g_2=\mu_2 b_2 u_2(t)\sqrt{2g\left(z_2(0,t)-\varepsilon u_2(t)\right)},$$

$u_i=u_i(t),\ i=1,2-$ control functions applied at the boundary points (the height of the openings of the water gates), $b_i,\ i=1,2$ – the width of the openings of the water gates, $z_1(x,l)$ – the ordinate of the free surface of the water stream of the upstream of the first water gate.
The water flow rate at the points of intake of the canal site, the right side of equation (1) has the form

$$q(x,t)=-\sum_{i=1}^{N} q_i\delta(x-a_i)l(t-T). \quad (5)$$

II. METHODS AND RESULTS

In these models, the analytical solution of equations (1), (2) and (3) under the indicated boundary conditions is absent, since the hydraulic parameters of the water flow is a nonlinear function depending on the shape of the cross-section of the canal site.

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From the expression of the lateral water intakes (5), it is seen that consumers are provided with uniform water supply in time in the form of a step function. With step functions, to solve the problem of optimal control of water distribution, it is necessary to formulate criteria for the quality of water distribution in the main canals of irrigation systems.

The processes occurring in the main canals relate to systems with distributed parameters, and the optimal distribution of water between consumers and the main canals relates to the tasks of optimal control of systems with distributed parameters. For them, we formulate the criteria for the quality of water distribution as criteria for the quality of control of a system with distributed parameters.

Criteria for the quality of control of a site of the main canal as a system with distributed parameters are generally written in the form of a sum of integral functional [3,4]

$$\begin{aligned} I = &\int_0^T \int_0^l F_1\left(x,t,Q(x,t),u(x,t)\right)dxdt + \\ &+\int_0^T F_2\left(t,Q(0,t),u_1(t)\right)dt + \\ &+\int_0^T F_3\left(t,Q(L,t),u_1(t)\right)dt + \\ &+\int_0^l F_4\left(x,Q(x,T)\right)dx, \end{aligned} \tag{6}$$

where F_i, $i=1,...,4$– given continuous functions of their arguments, the first component representing quality criteria for distributed control actions, the second and third for boundary controls, and the fourth for the final states of the controlled process.

Canal site. The quality criterion for hydraulic processes of water distribution in the canal site can be written as follows

$$I_1 = \int_0^T \int_0^l \left[z\left(x,t\right) - z^*\right]^2 dxdt, \tag{7}$$

where $z\left(x,t\right)$ – the actual change in water level in the canal site; z^* - given value of water level.

Functional (7) shows the quality of the change in water level in the canal site during the entire process of water distribution. The solution to the problem of minimizing functional (7) is to reduce the excessive fluctuation of water levels in the canal site.

The next quality criterion is the change in the water level in the canal site is the integral deviation of the water level at the end of the process T from the given distribution of water level $z*(x)$ [5]

$$I_2 = \int_0^l \left[z\left(x,T\right) - z^*(x)\right]^2 dx, \tag{8}$$

Similarly for water flow rate in the canal site

$$I_3 = \int_0^T \int_0^l \left[Q\left(x,t\right) - Q^*\right]^2 dxdt, \tag{9}$$

$$I_4 = \int_0^l \left[Q\left(x,T\right) - Q^*(x)\right]^2 dx, \tag{10}$$

where $Q\left(x,t\right)$ – the actual change in water flow rate in the canal site; Q^* - the given value of water flow rate.

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Functionals (9) and (10) show the quality of the change in water flow rate in the canal site at the beginning of the process and the end of the water distribution process. The solution to the problem of minimizing functionals (9) and (10) is to reduce excessive fluctuations in water flow rate in the canal site.
The main restrictions on hydraulic structures are

$$a_i^{\min}(t) \le a_i(t) \le a_i^{\max}(t) \tag{11}$$

where $a_i^{\min}(t)$, $a_i^{\max}(t)$ minimum and maximum permissible openings of the water gates of hydraulic structures.

Let us consider the formulation of the problem of optimal control of the distribution of water in the canals of irrigation systems under conditions of uniform water supply to consumers, using the mathematical models developed above.

Lateral-water intakes. For lateral water intakes, the quality standard of the water distribution process can be used to choose the root-mean-square integral deviation of the actual water flow rate from the planned (limited) values for the period $[0,T]$ water distribution [6]

$$I_1 = \sum_{j=1}^{N}\int_0^T \left(q_j(t) - q_j^*\right)^2 dt, \tag{12}$$

where $q_j(t)-$ the actual value of water flow rate $j-$th lateral water intake; $q_j^*(t)-$ the planned value of water flow rates $j-$th water intake.

An analog of criterion (12) is the expression

$$I_2 = \left|q_j(t) - q_j^*\right|. \tag{13}$$

Technological restrictions on the operating modes of the canal sites are of the form [7]

$$\begin{aligned} z_i^{\min} &\le z_i\left(x_i, t\right) \le z_i^{\max}, \\ Q_i^{\min} &\le Q_i\left(x_i, t\right) \le Q_i^{\max}. \end{aligned} \tag{14}$$

where $Q_i^{\min}$, $Q_i^{\max}$ – minimum and maximum allowable water flow rate for $i-$th canal site; $z_i^{\min}$, $z_i^{\max}$ - minimum and maximum permissible ordinates of the free surface of the water on $i-$thcanal site.

The main objective of the task, in this case, is to minimize fluctuations in the flow rate of the lateral-water intakes and the water level in the canal site while providing a discrete water supply to the lateral-water intakes by controlling the water flow rate at the beginning of the canal site [6,7]:

$$\begin{gathered} I = \int_0^T\int_0^l \left[Q(x,t) - Q^*\right]^2 dxdt + \sum_{j=1}^{N}\int_0^T \left(q_j(t) - q_j^*\right)^2 dt \to \min, \\ \Omega = \left\{R^N \mid Q_{\min} \le Q(x,t) \le Q_{\min}\right\}. \end{gathered} \tag{15}$$

on conditions that

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$$\begin{cases} B(x,t)\dfrac{\partial z(x,t)}{\partial t}+\dfrac{\partial Q(x,t)}{\partial x}=q(x,t), \\ \dfrac{1}{g\omega(x,t)}\left(\dfrac{\partial Q(x,t)}{\partial t}+2\upsilon(x,t)\dfrac{\partial Q(x,t)}{\partial x}\right)+\left[1-\left(\dfrac{\upsilon(x,t)}{c(x,t)}\right)^2\right]\dfrac{\partial z(x,t)}{\partial x}= \\ =\left[i+\dfrac{1}{B(x,t)}\left(\dfrac{\partial \omega(x,t)}{\partial x}\right)_{h=const}\right]\left(\dfrac{\upsilon(x,t)}{c(x,t)}\right)^2-\dfrac{Q(x,t)\left|Q(x,t)\right|}{K(x,t)^2}, \\ q(x,t)=-\sum\limits_{i=1}^{5} q_i\delta(x-a_i)l(t-T), \\ \upsilon(x,t)=\dfrac{Q(x,t)}{\omega(x,t)},\quad c=\sqrt{\dfrac{g\omega}{B}},\quad K(x,t)=\omega(x,t)C(x,t)\sqrt{R(x,t)i}, \\ C(x,t)=\dfrac{1}{n}R(x,t)^y,\quad y=2,5\sqrt{n}-0,13-0,75\sqrt{R}\left(\sqrt{n}-0,1\right), \\ Q(x,0)=Q_0(x),\quad \omega(x,0)=\omega_0(x), \\ x\geq 0,\quad t\geq 0,\quad v>0 \end{cases} \qquad (16)$$

control actions:

$$Q(0,t)=u_1(t),\qquad Q(l,t)=u_2(t), \qquad (17)$$

The restrictions on the operation modes of the canal sites are of the form

$$\begin{aligned} z_i^{\min} &\leq z_i\left(x_i,t\right)\leq z_i^{\max}, \\ Q_i^{\min} &\leq Q_i\left(x_i,t\right)\leq Q_i^{\max}. \end{aligned} \qquad (18)$$

The minimization problem (16) on condition (17) with the help of the control action (18) and with constraints (18) is the problem of controlling quasilinear systems with distributed parameters with state restrictions.

Expanding (1) in a Taylor power series in a neighbourhood of the points of the optimal trajectory and discarding the terms of the expansion above the first order of smallness, we obtain the equation in variations

$$\frac{\partial \delta z_i}{\partial t}=\left(\frac{\partial f_i}{\partial z_i}\right)_0\delta z_i+\left(\frac{\partial f_i^1}{\partial Q_i^{'}}\right)_0\frac{\partial \delta Q_i}{\partial x_i}+\left(\frac{\partial f_i^1}{\partial q_i^1}\right)_0\delta q_i, \qquad (19)$$

$$\begin{aligned} \frac{\partial \delta Q_i}{\partial t}&=\left(\frac{\partial f_i^2}{\partial z_i}\right)_0\delta z_i+\left(\frac{\partial f_i^2}{\partial Q_i^{'}}\right)_0\delta Q_i+ \\ &+\left(\frac{\partial f_i^2}{\partial z_i'}\right)_0\frac{\partial \delta z_i}{\partial x_i}+\left(\frac{\partial f_i^2}{\partial Q_i'}\right)_0\frac{\partial \delta Q_i}{\partial x_i} \end{aligned}, \qquad (20)$$

where is the symbol $(\ )_0$ means that the corresponding value is calculated along the optimal path $Q^{'}_i=\dfrac{\partial Q_i}{\partial x_i}$.

$$\frac{\partial f_i^1}{\partial z_i}=\frac{B_i'}{B_i^2}\cdot\frac{\partial Q_i}{\partial x_i},\ \frac{\partial f_i^1}{\partial Q_i'}=-\frac{1}{B_i},\ \frac{\partial f_i^1}{\partial q_i}=\frac{1}{B_i},$$

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$$\frac{\partial f_i^2}{\partial z_i} = -\frac{g i_{i0} Q_i^2 \left(2\varpi_i c_i' + \varpi_i' c'\right)}{\varpi_i^2 c_i^3} - \\ -\frac{g Q_i \left|Q_i\right| \left(2\varpi_i K_i - \varpi_i' K_i\right)}{K_i^3} + \frac{2 g \varpi_i' Q_i'}{\varpi_i^2} - \\ -g\left[\varpi_i'\left(1 - \frac{Q_i^2}{\varpi_i^2 c_i^2}\right) + \frac{2Q_i^2\left(\varpi_i c_i' + \varpi_i' c'\right)}{\varpi_i^2 c_i^3}\right] z_i', \\ \frac{\partial f_i^2}{\partial Q_i} = \frac{2 g i_{i0} Q_i}{\varpi_i c_i^2} - \frac{2 g \varpi_i \left|Q_i\right|}{K_i^2} - \frac{2}{\varpi_i} Q_i' + \frac{2 g Q_i}{\varpi_i c_i^2} z_i, \\ \frac{\partial f_i^2}{\partial z_i'} = g\varpi_i\left(1 - \frac{Q_i^2}{\varpi_i^2 c_i^2}\right), \; \frac{\partial f_i^2}{\partial Q_i'} = \frac{2Q_i}{\varpi_i}, \tag{21}$$

here $Q_i' = \frac{\partial Q_i}{\partial x_i}$, $z_i' = \frac{\partial z_i}{\partial x_i}$, $B_i' = \frac{\partial B_i}{\partial z_i}$, $c_i' = \frac{\partial c_i}{\partial z_i}$, $K_i' = \frac{\partial K_i}{\partial z_i}$.

In exactly the same way for variation of the optimality criterion (6) we obtain

$$\delta I = \int_0^T \left\{ \left(\frac{\partial G_c}{\partial z_1(l_1,t)}\right)_0 \delta z_1(l_1,t) + \left(\frac{\partial G_c^1}{\partial u_1^c(t)}\right)_0 \delta u_1^c(t) \right\} - \\ -\int_0^T \left\{ \left(\frac{\partial G_2^2}{\partial z_2(l_2,t)}\right)_0 \delta z_2(l_2,t) + \left(\frac{\partial G_2^2}{\partial u_2(t)}\right)_0 \delta u_2(t) \right\} dt. \tag{22}$$

We introduce conjugate variables or Lagrange multipliers using expressions

$$\int_0^T \int_0^{l_1} \left[\frac{\partial \delta z_1}{\partial t} - \left(\frac{\partial f_1^1}{\partial z_1}\right) \delta z_1 - \left(\frac{\partial f_1^1}{\partial Q_1'}\right)_0 \frac{\partial \delta Q_1}{\partial x_1} - \left(\frac{\partial f_1^1}{\partial q_1^1}\right) \delta q_1 \right] \lambda_1^1 dx_1 dt = 0, \tag{23}$$

$$\int_0^T \int_0^{l_1} \left[\frac{\partial \delta Q_1}{\partial t} - \left(\frac{\partial f_1^2}{\partial z_1}\right)_0 \delta z_1 - \left(\frac{\partial f_1^2}{\partial Q_1'}\right)_0 \delta Q_1 - \left(\frac{\partial f_1^2}{\partial q_1^1}\right)_0 \frac{\partial \delta z_1}{\partial x_1} - \left(\frac{\partial f_1^2}{\partial Q_1'}\right)_0 \frac{\partial Q z_1}{\partial x_1} \right] \lambda_1^2 dx_1 dt = 0, \tag{24}$$

$$\int_0^T \int_0^{l_1} \left[\frac{\partial \delta z_2}{\partial t} - \left(\frac{\partial f_2^1}{\partial z_2}\right) \delta z_2 - \left(\frac{\partial f_2^1}{\partial Q_2'}\right)_0 \frac{\partial \delta Q_2}{\partial x_2} - \left(\frac{\partial f_2^1}{\partial q_2}\right) \delta q_2 \right] \lambda_2^1 dx_2 dt = 0, \tag{24}$$

$$\int_0^T \int_0^{l_2} \left[\frac{\partial \delta Q_2}{\partial t} - \left(\frac{\partial f_2^2}{\partial z_2}\right)_0 \delta z_2 - \left(\frac{\partial f_2^2}{\partial Q_2'}\right)_0 \delta Q_2 - \right. \\ \left. -\left(\frac{\partial f_2^2}{\partial z_2'}\right)_0 \frac{\partial \delta z_2}{\partial x_2} - \left(\frac{\partial f_2^2}{\partial Q_2'}\right)_0 \frac{\partial \delta Q_2}{\partial x_2} \lambda_2^2 \right] dx_2 dt = 0 \tag{25}$$

Subtracting (23) - (25) from (22), we obtain a new record for the variation of the functional

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$$\delta I=\int_0^T\left\{\left(\frac{\partial G_c}{\partial z_1(l_1,t)}\right)_0\delta z_1(l_1,t)+\left(\frac{\partial G_1^c}{\partial u_1^c(t)}\right)_0\delta u_1^c(t)\right\}dt-$$
$$-\int_0^T\left\{\left(\frac{\partial G_2^2}{\partial z_2(l_2,t)}\right)_0\delta z_2(l_2,t)+\left(\frac{\partial G_2^2}{\partial u_2(t)}\right)_0\delta u_2^c(t)\right\}dt+$$
$$+\int_0^T\int_0^{l_1}\left[\left(\frac{\partial H_1}{\partial z_1}\right)_0\delta z_1+\left(\frac{\partial H_1}{\partial Q_1}\right)_0\delta Q_1+\left(\frac{\partial H_1}{\partial z_1}\right)_0\frac{\partial\delta z_1}{\partial x_1'}+\right.$$
$$\left.+\left(\frac{\partial H_1}{\partial Q_1'}\right)_0\frac{\partial\delta Q_1}{\partial x_1}-\lambda_1^1\frac{\partial\delta z_1}{\partial t}-\lambda_1^2\frac{\partial\delta Q_1}{\partial t}\right]dx_1dt+\tag{26}$$
$$+\int_0^T\int_{l_1}^{l_2}\left[\left(\frac{\partial H_2}{\partial z_2}\right)_0\delta z_2+\left(\frac{\partial H_2}{\partial Q_2}\right)_0\delta Q_2+\left(\frac{\partial H_2}{\partial z_2}\right)_0\frac{\partial\delta z_2}{\partial x_2}+\right.$$
$$\left.+\left(\frac{\partial H_2}{\partial Q_2'}\right)_0\frac{\partial\delta Q_2}{\partial x_2'}-\lambda_2^1\frac{\partial\delta z_2}{\partial t}-\lambda_2^2\frac{\partial\delta Q_2}{\partial t}\right]dx_2dt,$$

where H_1 и H_2 – Hamiltonians are defined as follows

$$H_i=\lambda_i^1 f_i^1+\lambda_i^2 f_i^2,\ i=1,2\tag{27}$$

Under differentiation, the Hamiltonians H_i it must be taken into account that f_i^1 independent of Q_i and z_i, f_i^2 independent of q_i, which follows from the main equation.

We integrate in parts the following expressions

$$\int_0^T\int_0^{l_1}\left[\left(\frac{\partial H_1}{\partial z_1'}\right)_0\frac{\partial\delta z_1}{\partial x'_1}\right]dx_1dt=\int_0^T\left\{\left(\frac{\partial H_1}{\partial z_1'}\right)_o\delta z_1\Big|_0^{l_1}-\int_0^{l_1}\frac{\partial}{\partial x_1}\left(\frac{\partial H_1}{\partial z_1'}\right)\delta z_1dx_1\right\}dt\tag{28}$$

$$\int_{l_1}^T\int^{l_2}\left[\left(\frac{\partial H_1}{\partial Q_1'}\right)_0\frac{\partial\delta Q_1}{\partial x_1}\right]dx_1dt=\int_0^T\left\{\left[\left(\frac{\partial H_1}{\partial Q_1'}\right)_0\delta Q_1\Big|_0^{l_1}-\int_0^{l_1}\frac{\partial}{\partial x_1}\left(\frac{\partial H_1}{\partial Q_1'}\right)_0\delta Q_1\right]dx_1\right\}dt,\tag{29}$$

$$\int_0^T\int_{l_1}^{l_2}\left[\left(\frac{\partial H_2}{\partial z_2'}\right)_0\frac{\partial\delta z_2}{\partial x_2'}\right]dx_2dt=\int_0^T\left\{\left[\left(\frac{\partial H_2}{\partial z_2'}\right)_0\delta z_2\Big|_{l_1}^{l_2}-\int_{l_1}^{l_2}\left[\frac{\partial}{\partial x_2}\left(\frac{\partial H_2}{\partial z_2'}\right)_0\delta z_2\right]dx_2\right\}dt,\tag{30}$$

$$\int_0^T\int_{l_1}^{l_2}\left[\left(\frac{\partial H_2}{\partial Q_2'}\right)_0\frac{\partial\delta Q_2}{\partial x_2'}\right]dx_2dt=\int_0^T\left\{\left[\left(\frac{\partial H_2}{\partial Q_2'}\right)_0\delta Q_2\right]_{l_1}^{l_2}-\int_{l_1}^{l_2}\left[\frac{\partial}{\partial x_2}\left(\frac{\partial H_2}{\partial Q_2'}\right)_0\delta Q_2\right]dx_2\right\}dt,\tag{31}$$

$$\int_0^T\int_0^{l_2}\left[\lambda_1^1\frac{\partial\delta z_1}{\partial x_1'}\right]dx_1dt=\int_{l_1}^{l_2}\left\{\left[\lambda_1^1\delta z_1\right]_0^T-\int_0^T\left[\frac{\partial\lambda_1^1}{\partial t}\delta z_1\right]dt\right\}dx_1,\tag{32}$$

$$\int_0^T\int_0^{l_2}\left[\lambda_1^1\frac{\partial\delta Q_1}{\partial x_1'}\right]dx_1dt=\int_{l_1}^{l_2}\left\{\left[\lambda_1^1\delta Q_1\right]_0^T-\int_0^T\left[\frac{\partial\lambda_1^1}{\partial t}\delta Q_1\right]dt\right\}dx_1,\tag{33}$$

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$$\int_0^T \int_{l_1}^{l_2} \left[\lambda_2^1 \frac{\partial \delta z_2}{\partial x_2'} \right] dx_2 dt = \int_{l_1}^{l_2} \left\{ \left[\lambda_2^1 \delta z_2 \right]_0^T - \int_0^T \left[\frac{\partial \lambda_2^1}{\partial t} \delta z_2 \right] dt \right\} dx_2, \tag{34}$$

$$\int_0^T \int_{l_1}^{l_2} \left[\lambda_2^1 \frac{\partial \delta Q_2}{\partial x_2'} \right] dx_2 dt = \int_{l_1}^{l_2} \left\{ \left[\lambda_2^1 \delta Q_2 \right]_0^T - \int_0^T \left[\frac{\partial \lambda_2^1}{\partial t} \delta Q_2 \right] dt \right\} dx_2, \tag{35}$$

Substituting (28) - (35) in (26), we obtain

$$\begin{aligned}
\delta I = & \int_0^T \int_0^{l_1} \left\{ \left[\left(\frac{\partial H_1}{\partial z_1} \right)_0 - \frac{\partial}{\partial x_1'} \left(\frac{\partial H_1}{\partial z_1'} \right)_0 + \frac{\partial \lambda_1^1}{\partial x_1'} \right] \delta z_1 + \left[\left(\frac{\partial H_1}{\partial Q_1} \right)_0 - \frac{\partial}{\partial x_1'} \left(\frac{\partial H_1}{\partial Q_1'} \right)_0 + \frac{\partial \lambda_1^2}{\partial x_1'} \right] \delta Q_1 + \left(\frac{\partial H_1}{\partial q_1} \right)_0 \delta q_1 \right\} dx_1 dt + \\
& + \int_0^T \int_{l_1}^{l_2} \left\{ \left[\left(\frac{\partial H_2}{\partial Q_2} \right)_0 - \frac{\partial}{\partial x_2'} \left(\frac{\partial H_2}{\partial z_2'} \right)_0 + \frac{\partial \lambda_2^1}{\partial x_2'} \right] \delta z_2 + \left[\left(\frac{\partial H_2}{\partial Q_2} \right)_0 - \frac{\partial}{\partial x_2} \left(\frac{\partial H_2}{\partial Q_2'} \right)_0 + \frac{\partial \lambda_2^2}{\partial x_2'} \right] \delta Q_2 \left(\frac{\partial H_2}{\partial q_2} \right)_0 \delta q_2 \right\} dx_2 dt + \\
& + \int_0^T \left\{ \left[\left(\frac{\partial G_c}{\partial z_1(l_1,t)} \right)_0 + \left(\frac{\partial H_1}{\partial z_1'} \right)_0 (l_1,t) \right] \delta z_1(l_1,t) + \left[\left(\frac{\partial G_2^2}{\partial z_2(l_2,t)} \right)_0 + \left(\frac{\partial H_2}{\partial z_2'} \right)_0 (l_2,t) \right] \delta z_2(l_2,t) + \right. \\
& + \left(\frac{\partial G_2^2}{\partial u_2} \right)_0 \delta u_2(t) + \left(\frac{\partial G_c}{\partial u_c} \right)_0 \delta u_c(t) + \left(\frac{\partial H_2(l_2,t)}{\partial Q_2'} \right)_0 \delta Q_2(l_2,t) + \\
& + \int_0^T \left\{ \left[\left(\frac{\partial H_2}{\partial Q_2} \right)_0 - \frac{\partial}{\partial x_2} \left(\frac{\partial H_2}{\partial Q_2''} \right)_0 \right]_{x_2=l_2} \delta Q_2(l_2,t) \right\} dt + \\
& + \left(\frac{\partial H_1}{\partial Q_1'}(l_1,t) \right)_0 \delta Q_1(l_1,t) - \left(\frac{\partial H_1}{\partial Q_1'}(0,t) \right)_0 \delta Q_1(0,t) - \left(\frac{\partial H_1}{\partial Q_1'}(0,t) \right)_0 \delta z_1(l_1,t) - \\
& \left. - \left(\frac{\partial H_2}{\partial Q_2'}(l_1,t) \right)_0 \delta Q_2(l_1,t) - \left(\frac{\partial H_2}{\partial z_2'}(l_1,t) \right)_0 \delta z_2(l_1,t) \right\} dt - \\
& - \int_0^{l_1} \left[\lambda_1^1(x_1,T) \delta z_1(x_1,T) + \lambda_1^2(x_1,T) \delta Q_1(x_1,T) \right] dx_1 + \int_{l_1}^{l_2} \left[\lambda_2^1(x_2,T) \delta z_2(x_2,T) + \lambda_2^2(x_2,T) \delta Q_2(x_2,T) \right] dx_2 .
\end{aligned} \tag{36}$$

Now in (36) the influence of variations δu_1, δu_2, δQ_1, δQ_2 on variations δI expressed explicitly. Since it was assumed that these variations are arbitrary, the remaining necessary conditions for optimality of controls u_1, u_2, Q_1, Q_2 are obtained by equating to zero the coefficients of these variations.

From expression (36) it can be seen that the explicit dependence of the variation of the functional δI from $\delta z_1(x_1,t)$, $\delta z_2(x_2,t)$, $\delta Q_1(x_1,t)$, $\delta Q_2(x_2,t)$ disappear, if $\lambda_i^j(x_2,t)$ satisfy the conditions

$$\begin{aligned}
\frac{\partial \lambda_i^1}{\partial x_i} &= \left(\frac{\partial H_i}{\partial z_i} \right)_0 - \frac{\partial}{\partial x_i} \left(\frac{\partial H_i}{\partial z_i} \right)_0 , \\
\frac{\partial \lambda_i^2}{\partial x_i} &= \left(\frac{\partial H_i}{\partial z_i} \right)_0 - \frac{\partial}{\partial x_i} \left(\frac{\partial H_i}{\partial z_i} \right)_0 , \quad i = 1, 2,
\end{aligned} \tag{37}$$

since in this case, the expressions in square brackets of double integrals are equal to zero. Now, if at the end of the process, the conjugate variables satisfy the conditions

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$$\lambda_i^1(x_i,T)=0,\ \lambda_i^2(x_i,T)=0,\ i=1,2, \tag{38}$$

then the last two integrals in (36) are always zero.

For variation conditions (1) - (3), we obtain the expressions

$$\delta Q_1(l_1,t)=\left(\frac{\partial g_1^c}{\partial z_1(l_1,t)}\right)_0\delta z_1(l_1,t)+\left(\frac{\partial g_1^c}{\partial z_2(l_1,t)}\right)_0\delta z_2(l_1,t)+ +\left(\frac{\partial g_1^c}{\partial u_1^c(t)}\right)_0\delta u_1^c(t)+\left(\frac{\partial g_1^c}{\partial u_2^c(t)}\right)_0\delta u_2^c(t), \tag{39}$$

$$\delta Q_2(l_1,t)=\left(\frac{\partial g_2^c}{\partial z_1(l_1,t)}\right)_0\delta z_1(l_1,t)+ +\left(\frac{\partial g_2^c}{\partial z_2(l_1,t)}\right)_0\delta z_2(l_1,t)++\left(\frac{\partial g_2^c}{\partial u_2^c(t)}\right)_0\delta u_2^c(t). \tag{40}$$

Similarly, for variation of the boundary conditions

$$\delta Q_1(0,t)=\left(\frac{\partial g_1}{\partial z_1(0,t)}\right)_0\delta z_1(0,t)+\left(\frac{\partial g_1}{\partial u_1(l_1,t)}\right)_0\delta u_1(t), \tag{41}$$

$$\delta Q_2(0,t)=\left(\frac{\partial g_2}{\partial z_2(0,t)}\right)_0\delta z_2(0,t)+\left(\frac{\partial g_2}{\partial u_2(t)}\right)_0\delta u_2(t). \tag{42}$$

It follows from (36) for the variation of the functions that the conjugate variables satisfy the following conjugation conditions

$$\left(\frac{\partial G_c}{\partial z_2(l_1,t)}\right)_0+\left(\frac{\partial H_1}{\partial z_1'}(l_1,t)\right)_0+\left(\frac{\partial H_1}{\partial Q_1}(l_1,t)\right)_0\left(\frac{\partial g_1^c}{\partial z_1(l_1,t)}\right)_0- -\left(\frac{\partial H_2}{\partial Q_2'}(l_1,t)\right)_0\left(\frac{\partial g_2^c}{\partial z_2(l_1,t)}\right)_0=0, \tag{43}$$

$$\left(\frac{\partial H_2}{\partial z_2^1}(l_1,t)\right)_0+\left(\frac{\partial H_2}{\partial Q_2}(l_1,t)\right)_0\left(\frac{\partial g_2^c}{\partial z_2(l_1,t)}\right)_0- -\left(\frac{\partial H_1}{\partial Q_1'}(l_1,t)\right)_0\left(\frac{\partial g_2^c}{\partial z_2(l_1,t)}\right)_0=0. \tag{44}$$

Boundary conditions

$$\left(\frac{\partial H_1}{\partial Q_1'}(0,t)\right)_0\left(\frac{\partial g_1}{\partial z_1(0_1,t)}\right)_0+\left(\frac{\partial H_1}{\partial z_1'}(0,t)\right)_0=0$$
$$\left(\frac{\partial G_2^2}{\partial z_2(l_2,t)}\right)_0+\left(\frac{\partial H_2}{\partial z_2'}(l_2,t)\right)_0+\left(\frac{\partial H_2}{\partial Q_2'}(l_2,t)\right)_0\left(\frac{\partial g_2}{\partial z_2(l_2,t)}\right)_0=0. \tag{45}$$

Given certain equations for conjugate variables, conjugation conditions, and boundary conditions, we obtain an expression for the variation of the functional

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$$\delta I=\int_0^T\left\{\left[\left(\frac{\partial H_2}{\partial Q_2'}(l_2,t)\right)_0\left(\frac{\partial g_2}{\partial u_2(t)}\right)_0+\left(\frac{\partial G_2^2}{\partial u_2(t)}\right)_0\right]\delta u_2(t)-\right.$$
$$-\left(\frac{\partial H_1}{\partial Q_1'}(0,t)\right)_0\left(\frac{\partial g_1}{\partial u_1(t)}\right)_0\delta u_1(t)+\left(\frac{\partial G_c}{\partial u_1^c(t)}\right)_0\delta u_1^c(t)+ \tag{46}$$
$$\left.+\left(\frac{\partial H_1}{\partial Q_1'}(l_1,t)\right)_0\left[\left(\frac{\partial g_1^c}{\partial u_1^c(t)}\right)_0\delta u_1^c(t)+\left(\frac{\partial g_1^c}{\partial u_2^c(t)}\right)_0\delta u_2^c(t)\right]\right\}dt.$$

We determined explicit expressions of the variation of the functional from the variation of controls $\delta u_1(t), \delta u_2(t), \delta u_1^c(t), \delta u_2^c(t)$. Since it was assumed that these variations are arbitrary, the necessary condition for optimality is the equality to zero of the expressions for these variations.

Thus, the maximum principle for the optimal control problem under consideration is formulated as follows: in order to control $u_1(t)$, $u_2(t)$, $u_1^c(t)$, $u_2^c(t)$ was optimal, taking into account restrictions, it is necessary to fulfil the following conditions [7.8]

$$-\left(\frac{\partial H_1}{\partial Q_1'}(0,t)\right)_0\left(\frac{\partial g_1}{\partial u_1(t)}\right)_0=0 \tag{47}$$

$$\left(\frac{\partial H_2}{\partial Q_2'}(l_2,t)\right)_0\left(\frac{\partial g_2}{\partial u_2(t)}\right)_0+\left(\frac{\partial G_2^2}{\partial u_2(t)}\right)_0=0 \tag{48}$$

$$\left(\frac{\partial G_c}{\partial u_1^c(t)}\right)_0+\left(\frac{\partial H_1}{\partial Q_1'}(l_1,t)\right)_0\left(\frac{\partial g_1^c}{\partial u_1^c(t)}\right)_0=0 \tag{49}$$

$$\left(\frac{\partial H_2}{\partial Q_2'}(l_1,t)\right)_0\left(\frac{\partial g_2}{\partial u_2^c(t)}\right)_0+\left(\frac{\partial G_c}{\partial u_2^c(t)}\right)_0=0 \tag{50}$$

When controlling within the region of constraints, the corresponding values in the left-hand sides of equalities (47) - (50) must be non-positive if the upper bounds are reached [8].

When controlling inside the constraint region for the considered problem (16) - (18), the variation of the optimality criterion has the form [9]

$$\delta I=\int_0^l\left[z(x,T)-z^*\right]\delta z(x,T)dx+\sum_{j=1}^{N}\int_0^T 2\left(q_j(t)-q_j^*\right)\delta q_j(t)dt-$$
$$-\int_0^T\left\{\left[\left(\frac{\partial H(l,t)}{\partial Q'}\right)_0\left(\frac{\partial g_2}{\partial u_2(t)}\right)_0\right]\delta u_2(t)+\left(\frac{\partial H(0,t)}{\partial Q'}\right)_0\left(\frac{\partial g_1}{\partial u_1(t)}\right)_0\delta u_1(t),\right. \tag{51}$$

where $H=\lambda^1 f_1+\lambda^2 f_2$.

Computing the derivative of the Hamiltonian H by Q', we get

$$\frac{\partial H}{\partial Q'}=-\frac{1}{B}\lambda^1-\frac{2Q}{\omega}\lambda^2 \tag{52}$$

Now we have defined explicit expressions of the variation of the functional from the variation of controls $\delta u_1(t)$, $\delta u_2(t)$. Since it was assumed that these variations are arbitrary, a necessary condition for optimality is the equality to zero of the expressions for these variations.

Thus, the final necessary optimality conditions for the system of Saint-Venant equations, when controlled within the region of constraints, have the form

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$$\left[\left(\frac{1}{B}\right)_0 \lambda^1 - \left(\frac{2Q}{\omega}\right)_0 \lambda^2\right]_{x=0} \left(\frac{\partial g_1}{\partial u_1(t)}\right)_0 = 0, \tag{53}$$

$$\left[\left(\frac{1}{B}\right)_0 \lambda^1 - \left(\frac{2Q}{\omega}\right)_0 \lambda^2\right]_{x=l} \left(\frac{\partial g_1}{\partial u_1(t)}\right)_0 = 0. \tag{54}$$

In the case of achieving control of its upper (lower) boundaries, the corresponding quantities in the left-hand sides of expressions (53), (54) must be non-positive (non-negative).

III. CONCLUSION

The mathematical models of the main canals of irrigation systems are analysed, the essence of which is a stepwise change in flow rate through the hydraulic structures of the canal.

The criteria for the quality of water distribution in the main canal site and lateral-water intakes, as well as the main restrictions on the operating modes of the sites and hydraulic structures of the main canal, are determined.
For the model of unsteady water movement in the main canals of irrigation systems, when controlled within the area of restrictions, the necessary conditions for the optimality of water distribution are determined.

REFERENCES

[1]M.S. Grushevsky, Unsteady water movement in rivers and canals, L.: Gidrometeoizdat, p.288, 1982.
[2]DenisovYu.M. Mathematical modeling of the flow process in mountain rivers // Collection of scientific papers, Tashkent: SANIIRI, No. 39, pp.30–36, 1968.
[3]Kuchmend L.S., Singh V.P. Hydrological system modeling, EOLSS Publications, Oxford, United Kingdom, vol.1, p.209, 2009,.
[4]G.P Kalinin, P.P. Milyukov, An approximate calculation of the unsteady movement of water masses, Proceedings of the TsIP, No. 66, p. 72, 1958.
[5][12] KunzhZh.A., Holly F.M., Vervey A., Numerical methods in problems of river hydraulics, M.: Energoatomizdat, 1985, 253 pp.
[6]Dooge, J.C.I. Linear theory of hydrologic systems EGU Reprint Series, European Geosciences Union. pp.1-327 2003.
[7]A.A. Atavin, O.F. Vasiliev., A.F. Voevodin, S.M. Shugrin, Computational methods for solving one-dimensional hydraulic problems, Water Resources, No. 4, pp. 38-47, 1983.
[8]A.F. Voevodin, S.M. Shugrin, Numerical methods for calculating one-dimensional systems, Novosibirsk: Nauka, pp. 208, 1981.
[9]S.A.Khristianovich, Some questions of the mechanics of a continuous medium, M.: Publishing House of the Russian Academy of Sciences, p. 407, 1938.
[10]N.A. Kartvelishvili, Regulation of river flow, L.:Gidrometeoizdat, p. 218, 1970.
[11]N.A.Kartvelishvili, Unsteady open streams, L.: Gidrometeoizdat, p.125, 1968.
[12]E.E. Makovsky, Automation of hydraulic structures in cascade control systems for water consumption, Frunze: Ilim, p.302, 1972.
[13]V.I. Kurotchenko, Principles for the construction of means for centralized management of the water distribution process, Frunze: Ilim, p.272, 1979.
[14]V.I. Kurotchenko, Centralized Automatic Water Distribution Management, Frunze: Ilim, p.288, 1983.
[15]Mathematical modeling in hydrology. In book Overview information. Ser. Hydrology of land, vol. 1, Obninsk, p.467, 1979.
[16]V.I Koren., Investigation of the stability of some explicit schemes for the integration of the Saint-Venant equations, Metrology and Hydrology, No. 1, pp..42–48, 1967.
[17]V.I.Koren, L.S. Kuchment, On the formulation of boundary conditions for the numerical integration of the Saint-Venant equations, Metrology and Hydrology, No. 6, pp. 105–107,1967.
[18]V.I. Koren, L.S. Kuchment, Numerical integration of Saint-Venant equations according to explicit schemes for calculating the unsteady movement of water in rivers, Collection of scientific. Tr. hydrometeorologicalcenter of the Russian Federation, No. 8, pp..42–61, 1967.
[19]L.S. Kuchment, Mathematical modeling of river flow, L.:Gidrometeoizdat, p. 192, 1972.
[20]S.K. Godunov, V.S. Ryabensky, Difference schemes, Moscow: Nauka, p. 499, 1977.
[21]V.A. Bolshakov, T.P. Kleshevnikova, Application of the finite element method in solving problems of hydraulics of open channels and structures, Hydrotechnical construction, –No. 11, pp..48–50, 1980.
[22]V.K. Kantorovich, L.S. Kuchment, Application of finite element methods to calculations of unsteady water motion according to the equations of Saint-Venant, Water Resources, No. 6, pp. 45–53, 1981.
[23]B.L. Historian, Calculation of unsteady water movement in open channels on electronic machines, Collection of scientific. trHydroproject, No. 12, pp..127–138, 1964.
[24]A.A. Atavin, Calculation of the unsteady flow of water in the branched systems of river channels and channels, Dynamics of a continuous medium. Vol. - 22, Novosibirsk, pp..25-39,1975.
[25]A.F. Voevodin, On the implicit difference scheme for integrating systems of hyperbolic equations, Comput. mate. and mate. physics. - Novosibirsk: SB RAS, No. 2, pp..367–376, 1977.
[26]A.G. Butkovsky, Characteristics of systems with distributed parameters, M.: Nauka, p.225, 1979.
[27]Sh.Kh. Rakhimov, Management of machine water-lifting systems, Tashkent Fan, pp. 137, 1986.

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