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On the geodetic iteration number and geodetic number of a fuzzy graph based on sum distance

SameehaRehmani, M.S.Sunitha

Assistant Professor, Department of Mathematics Sullamussalam Science College,Areacode-673639 Kerala ,India.
Sr.Assistant Professor, Department of Mathematics National Institute of Technology Calicut,Kozhikode-673601 Kerala, India

ABSTRACT: In this paper, the concepts of s-geodetic iteration number and s-geodetic number of a fuzzy graph based on sum distance in fuzzy graphs are introduced. Some properties related to these concepts are established. The s-geodetic iteration number of fuzzy trees, fuzzy cycles and complete bipartite fuzzy graphs subject to certain conditions are identified. A necessary and sufficient condition for a connected fuzzy graph $G : (V, \sigma, \mu)$ to have its s-geodetic number as $|V|$ is established. An upper and lower bound for the s-geodetic number of a fuzzy graph is discussed along with suitable examples. The s-geodetic number of complete bipartite fuzzy graphs and of fuzzy cycles is examined. The concept of extreme s-geodesic fuzzy graphs is introduced and some of its properties are examined. Finally, an attempt is made to define the concept of a minimum s-geodetic fuzzy subgraph along with some of its properties.

I. INTRODUCTION

Zadeh in 1965 [30] brought the concept of fuzzy sets into existence which gave a platform for describing the uncertainties prevailing in day-to-day life situations. Later on, the theory of fuzzy graphs was developed by Rosenfeld in the year 1975 [22]along with Yeh and Bang [29]. Rosenfeld also obtained the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness along with some of their properties [22] and the concept of fuzzy trees [19],automorphism of fuzzy graphs [2], fuzzy interval graphs [16],cycles and cocycles of fuzzy graphs [17] etc. has been established by several authors during the course of time. Fuzzy groups and the notion of a metric in fuzzy graphs were introduced by Bhattacharya [1]. The concept of strong arcs [5] and geodesic distance in fuzzy graphs [4] were introduced by Bhutani and Rosenfeld in the year 2003. The definition of fuzzy end nodes and some of their properties were established by the same authors in [3]. Several other important works on fuzzy graphs can be found in [21, 14, 26]. Studies in fuzzy graphs using μ-distance was carried out by Rosenfeld [23] in 1975 and was further studied by Sunitha and Vijayakumar in [26]. In crisp graph, the concept of geodetic iteration number was first introduced by Harary and Nieminen in 1981 [12]. This concept along with that of geodetic numbers in graphs was again discussed by several authors in [8],[10] and [9]. Later on, these concepts were extended to fuzzy graphs using geodesic distance by Suvarna and Sunitha in [28] and the same based on μ-distance was introduced by Linda and Sunitha in [13].The concept of sum distance and some of its metric aspects was introduced by Mini Tom and Sunitha in [15].

In this paper, s-geodetic iteration number and s-geodetic number of a fuzzy graph based on sum distance are introduced and certain properties satisfied by them are identified. The concepts of Extreme s-geodesic fuzzy graph and Minimum s-geodetic fuzzy subgraph are also explained.

II. PRELIMINARIES

A **fuzzy graph** [18] is a triplet $G : (V, \sigma, \mu)$ where V is vertex set, σ a fuzzy subset of V and μ a fuzzy relation on σ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, $\forall u, v \in V$.

We assume that V is finite and non-empty, μ is reflexive (i.e., $\mu(x, x) = \sigma(x), \forall x$) and symmetric (i.e., $\mu(x, y) = \mu(y, x), \forall (x, y)$). Also we denote the underlying crisp graph [11] by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V: \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V: \mu(u, v) > 0\}$. Here we assume $\sigma^* = V$.

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A fuzzy graph H : (V, τ, ν) is called a **partial fuzzy subgraph** [18] of G : (V, σ, μ) if τ (u) ≤ σ(u) for every u ∈ τ * and ν(u, v) ≤ μ(u, v) ∀(u, v) ∈ ν * . In particular, we call H : (V, τ, ν) a **fuzzy subgraph** of G : (V, σ, μ) if τ (u) = σ(u), ∀u ∈ τ * and ν(u, v) = μ(u, v), ∀(u, v) ∈ ν * and if in addition τ * = σ * , then H is called a **spanning fuzzy subgraph** of G. A fuzzy graph H : (P, τ, ν) is called a **fuzzy subgraph** of G : (V, σ, μ) **induced by P** if P ⊆ V , τ (u) = σ(u) for all u in P and ν(u, v) = μ(u, v) for all u, v in P.

A fuzzy graph G : (V, σ, μ) is called **trivial** if |σ * | = 1. Otherwise it is called **non-trivial**.

A fuzzy graph G : (V, σ, μ) is a **complete fuzzy graph** [18] if μ(u, v) = σ(u)∧σ(v) ∀u, v ∈ σ *.

A **weakest arc** of G : (V, σ, μ) is an arc with least non zero membership value. A **path** P of length n is a sequence of distinct nodes u_0, u_1, ..., un such that $\mu(u_{i-1}, u_i) > 0$, i = 1, 2, 3, ..., n and the degree of membership of a weakest arc in the path is defined as its **strength**.

The path becomes a **cycle** if $u_0 = u_n$, n ≥ 3 and a cycle is called a **fuzzy cycle** [19] if it contains more than one weakest arc. The **strength of connectedness** between two nodes u and v is defined as the maximum of the strengths of all paths between u and v, and is denoted by $CONN_G(u, v)$. A u − v path P is called a **strongest u − v path** if its strength equals $CONN_G(u, v)$. A fuzzy graph G : (V, σ, μ) is **connected** if for every u, v in σ * , $CONN_G(u, v) > 0$.

An arc (u, v) of a fuzzy graph is called **strong** if its weight is at least as great as the strength of connectedness of its end nodes u, v when the arc (u, v) is deleted and a u − v path P is called a **strong path** if P contains only strong arcs [5]. Depending on the $CONN_G(u, v)$ of an arc (u, v) in a fuzzy graph G, strong arcs are further classified as α-strong and β-strong and the remaining arcs are termed as δ-arcs [14] as follows. Note that G − (u, v) denotes the fuzzy subgraph of G obtained by deleting the arc (u, v) from G. An arc (u, v) in G is called **α-strong** if $\mu(u, v) > CONN_{G-(u,v)}(u, v)$.

An arc (u, v) in G is called **β-strong** if $\mu(u, v) = CONN_{G-(u,v)}(u, v)$. An arc (u, v) in G is called a **δ-arc** if $\mu(u, v) < CONN_{G-(u,v)}(u, v)$.A δ-arc (u, v) is called a **δ * -arc** if μ(u, v) > μ(x, y) where (x, y) is a weakest arc of G.

A node is a **fuzzy cut node** of G : (V, σ, μ) if removal of it reduces the strength of connectedness between some other pair of nodes [22]. Two nodes u and v in a fuzzy graph G : (V, σ, μ) are **neighbors** if μ(u, v) > 0 and v is called a **strong neighbor** of u if the arc (u, v) is strong. Also N(u) denotes the set of neighbors of u other than u and **degree** of u is deg(u) = |N(u)|. A node u with deg(u) = 1 is an **end node** and a node u with deg(u) > 1 is an **internal node**. A node v is called a **fuzzy end node** of G if it has exactly one strong neighbor in G [3]. A connected fuzzy graph G : (V, σ, μ) is called a **fuzzy tree** [22] if it has a spanning fuzzy subgraph F : (V, σ, ν) which is a tree such that for all arcs (u, v) not in F,$CONN_F (u, v) > \mu(u, v)$. A **maximum spanning tree** (MST) [24] of a connected fuzzy graph G : (V, σ, μ) is a fuzzy spanning subgraph T : (V, σ, ν), such that T* is a tree, and for which $\Sigma_{u \neq v} \nu(u, v)$ is maximum. A fuzzy graph G is said to be **bipartite** [25] if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Further if μ(u, v) = σ(u) V σ(v) ∀u ∈ V_1 and v ∈ V_2, then G is called a complete bipartite fuzzy graph and is denoted by $K_{\sigma 1,\sigma 2}$, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2.

For any path P : $u_0 - u_1 - u_2 - ... - u_n$, **length** of P, L(P), is defined as the sum of the weights of the arcs in P. That is, $L(P) = \sum_{i=1}^{n} \mu(u_{i-1}, u_i)$. If n = 0, define L(P) = 0 and for n ≥ 1, L(P) > 0.

For any two nodes u, v in G : (V, σ, μ), if P = {P_i : P_i is a u-v path, i = 1, 2, 3, ...},then the **sum distance** between u and v is defined as $d_s(u, v) = \min\{L(P_i) : P_i \in P, i = 1, 2, 3, ...\}$. The **eccentricity**$e_s(u)$ of a node u in the connected fuzzy graph G : (V, σ, μ) is the sum distance to a node farthest from u. i.e, $e_s(u) = \max\{d_s(u, v) : v \in V\}$. The **radius**$r_s(G)$ is the minimum eccentricity of the nodes, whereas the **diameter** $d_s(G)$ is the maximum eccentricity. A node u is an **s-peripheral node** if $e_s(u) = d_s(G)$. A **diametral path** of a fuzzy graph is a shortest path whose length is equal to the diameter of the fuzzy graph.

Throughout this paper we consider only connected fuzzy graphs.

III. s-GEODETICITERATION NUMBER OF A FUZZY GRAPH [s-gin(G)]

In crisp graph, the concept of a geodesic and of geodesic iteration number is discussed in [6] and [11]. Later on, these ideas were extended to fuzzy graphs using g-distance by Suvarna in [28] and using μ-distance by Linda in [13]. Here we are extending these ideas to fuzzy graphs using sum distance $d_s(u, v)$. Depending on sum distance, we define s-geodesic, s-geodetic closure and s-geodetic iteration number as follows.

Definition 3.1. Any path P from x to y whose length is $d_s(x, y)$ is called **s-geodesic** from x to y.

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Definition 3.2. Let $S \subseteq V$ be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Then the **s-geodetic closure** of S, with respect to sum distance, is the set of all nodes of S as well as all nodes that lie on s-geodesics between nodes of S and is denoted by (S).

Example 3.3. Consider the fuzzy graph given in Fig.1.

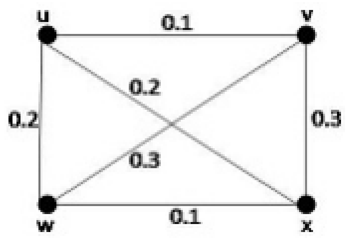

Fig.1

Here, $d_s(u, v) = \min\{0.1, 0.5, 0.6\} = 0.1$. Similarly $d_s(v, x) = 0.3$, $d_s(u, x) = 0.2$, $d_s(u, w) = 0.2$, $d_s(v, w) = 0.3$ and $d_s(w, x) = 0.1$. Now if $S = \{v, x\}$, then since $d_s(v, x) = 0.3$, both (v, x) and $v - u - x$ are s-geodesics from v to x and so (S) = $\{u, v, x\}$. Similarly if $S = \{u, v, x\}$, then also (S) = $\{u, v, x\}$.

Definition 3.4. Let $S \subseteq V$ be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Let S^1 = (S), $S^2 = (S^1)$ = ((S)) etc where S^1, S^2, ..., are s-geodetic closures.
Since we consider only finite fuzzy graphs, the process of taking closures must terminate with some smallest n such that $S^n = S^{n+1}$. The smallest value of n for which $S^n = S^{n+1}$ is called **s-geodetic iteration number of S**, denoted by s-gin(S). The maximum value of s-gin(S) for all $S \subseteq V(G)$ is called **s-geodetic iteration number of G**, denoted by s-gin(G).

Example 3.5. Consider the fuzzy graph given in Fig.1. Taking $S = \{v, x\}$, S^1 = (S) = $\{u, v, x\}$ $S^2 = S^1$. Therefore s-gin(S) = 1. It can be verified that maximum value of s-gin(S) is 1 for all $S \subseteq V(G)$. Therefore s-gin(G) = 1.

Remark 3.6. For a trivial fuzzy graph G, s-gin(G) = 0.

Proposition 3.7. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph on n nodes in which each pair of nodes in G is joined by an arc which is the unique s-geodesic between them. Then the s-geodetic iteration number, s-gin(G) = 0.

Proof. Let $S \subseteq V(G)$. Then since every pair of nodes in S is connected by an arc which is the unique s-geodesic between them, any s-geodesic between a pair of nodes u, v of S is the arc (u, v) and so S^1 = (S) = S. Since this is true for any $S \subseteq V(G)$, we get s-gin(G) = 0.

Proposition 3.8. The s-geodetic iteration number of a fuzzy tree $G : (V, \sigma, \mu)$ such that G^* is a star, is 1.

Proof. Since G^* is a star graph, it is a tree and hence there is always a unique path between any two nodes of G [11]. Let us consider the following cases.

Case(1): $S \subseteq V(G)$ contains the node x of G where $\deg(x) > 1$.
Then since x lies on the s-geodesic between any two nodes of S and since $x \in S$, we get S^1 = (S) = S. Hence s-gin(S) = 0.

Case(2): $S \subseteq V(G)$ does not contain the node x of G where $\deg(x) > 1$.
In this case, the node x that lies on the s-geodesic joining any two nodes of S does not belong to S and so $x \in$ (S) = S^1. Therefore $S \neq S^1$.
But by case(1), since S^1 contains the node x, we get $S^2 = (S^1) = S^1$. Hence in this case, s-gin(S) = 1. From the above two cases, we get s-gin(G) = max{0, 1} = 1.

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Remark 3.9. In general, the s-geodetic iteration number of a fuzzy tree G : (V, σ, μ), on $n \geq 3$ nodes, such that G* is a tree is 1.

Remark 3.10. In a fuzzy cycle C, for any $S \subseteq V(C)$, s-gin(S) is either 0 or 1 and so the s-geodetic iteration number of a fuzzy cycle C, s-gin(C) = max{0, 1} = 1.

Example 3.11. Consider the fuzzy cycle C in Fig.2.

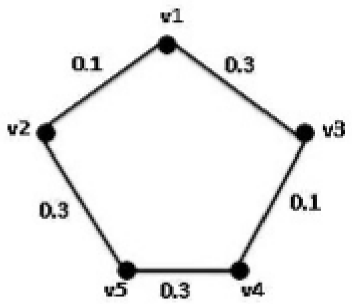

Fig.2

Here if $S = \{v_1, v_4\}$, then $S^1 = (S) = \{v_1, v_3, v_4\}$ and $S^2 = (S^1) = S^1$. Therefore s-gin(S) = 1. Also if $S = \{v_1, v_2\}$, then $S^1 = (S) = S$ and so s-gin(S) = 0. It can be seen that for any $S \subseteq V(G)$, s-gin(S) is either 0 or 1 and so s-gin(C) = max{0, 1} = 1.

Proposition3.12. Let $K_{\sigma 1,\sigma 2} = (V_1 \cup V_2, \sigma, \mu)$ be a complete bipartite fuzzy graph such that $|V_1| = 2 = |V_2|$, then the s-geodetic iteration number of $K_{\sigma 1,\sigma 2}$ is 1 if each arc in $K_{\sigma 1,\sigma 2}$ is the unique s-geodesic between its nodes.

Proof. Suppose that each arc of the complete bipartite fuzzy graph $K_{\sigma 1,\sigma 2}$ is the unique s-geodesic between its nodes. Let $S \subseteq V_1 \cup V_2$. We have to consider the following cases.

Case(1): S comprises of two nodes from the same partition.
Suppose S comprises of two nodes u and v from the same partition say V_1. Then since there is no arc joining u to v in a bipartite fuzzy graph, there always exists a node (say x) from V_2 lying on an s-geodesic joining u and v. Thus $S^1 = (S) = \{u, v, x\}$ and since by assumption the arcs (u, x) and (v, x) are s-geodesics, we get $S^2 = (S^1) = S^1$. Therefore s-gin(S) = 1.
The same result holds if S comprises of two nodes from V_2.

Case(2): S comprises of two nodes, one from V_1 and the other from V_2.
Then since by assumption each arc is an s-geodesic, we get $S^1 = (S) = S$ and so s-gin(S) = 0. Now all the other subsets of $V_1 \cup V_2$ will contain either nodes from the same partition or from two different partitions and hence for all $S \subseteq V_1 \cup V_2$, we get s-gin(S) as 0 or 1 and so s-gin($K_{\sigma 1,\sigma 2}$) = max{0, 1} = 1.

Example 3.13. Consider the complete bipartite fuzzy graph G given in Fig.3.

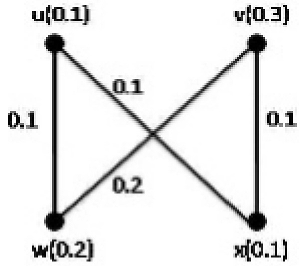

Fig.3

Here if S = {u, v} then S^1 = (S) = {u, x, v} ≠S. Also, S^2 = (S^1) = {u, x, v} = S^1. Therefore s-gin(S) = 1. Similarly if S = {w, x}, we get s-gin(S) = 1. Next, if S = {u, w} then S^1 = (S) = S and so s-gin(S) = 0. Note that for every subset S which consists of one node in first partition and another node in the second partition, s-gin(S) = 0. Now, if S = {u, x, w} then S^1 = (S) = {u, x, w} = S. Therefore s-gin(S) = 0. Again if S = {v, x, w} then S^1 = (S) = {v, x, w, u} = V (G) ≠S. Also, S^2 = (S^1) = V (G) = S^1. Hence s-gin(S) = 1. Similarly for all other S ⊆ V (G), it can be shown that s-gin(S) is either 0 or 1 and so s-gin(G) = max{0, 1} = 1.

Proposition 3.14. Let $K_{\sigma1,\sigma2}$= (V_1∪V_2, σ, μ) be a complete bipartite fuzzy graph with $|V_1| = 2$ and $|V_2| \geq 3$. Then the s-geodetic iteration number of $K_{\sigma1,\sigma2}$, s-gin($K_{\sigma1,\sigma2}$) = 2 if each arc of $K_{\sigma1,\sigma2}$has the same membership value.

Proof. Let u and v be the two nodes in V_1 and S ⊆ V_1∪ V_2. Consider the following cases:

Case(1): S = V_1. Then since each arc of $K_{\sigma1,\sigma2}$ has the same membership value, we get S^1 = (S) = V_1∪ V_2 and so S^2 = (S^1) = V_1∪ V_2 = S^1. Hence in this case, s-gin(S) = 1.
Case (2): S ⊆ V_2 and |S| = 2. Then in this case, S^1 = (S) and V_1⊆ (S) and since all other nodes in $K_{\sigma1,\sigma2}$lie on an s-geodesic joining u and v, we get S^2 = (S^1) = V_1∪ V_2. Clearly then S^3 = (S^2) = V_1∪ V_2 = S^2 and hence s-gin(S) = 2.

Case (3): S contains two nodes, one from V_1 and the other from V_2. Then, since each arc is an s-geodesic, we get S^1 = (S) = S. Hence s-gin(S) = 0. All other subsets of V_1∪ V_2 will be a combination of the above three cases and so for all S ⊆ V_1∪ V_2, s-gin($K_{\sigma1,\sigma2}$) = max{0, 1, 2} = 2.

Example 3.15. Consider the complete bipartite fuzzy graph G given in Fig.4 .

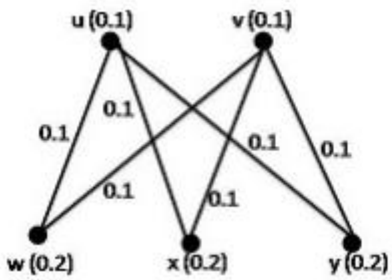

Fig.4

Here, if S = {u, v} then S^1 = (S) = V (G) and so S^2 = (S^1) = V (G) = S^1. Therefore s-gin(S) = 1. Now if S = {w, x} then S^1 = (S) = {w, u, v, x} and S^2 = (S^1) = V (G). Also S^3 = (S^2) = V (G) = S^2. Hence in this case, s-gin(S) = 2. Next suppose S = {u, w}, then clearly S^1 = (S) = S and so s-gin(S) = 0. It can be verified that for all other subsets S of V (G), s-gin(S) is either 0, 1 or 2. Hence, s-gin(G) = max{0, 1, 2} = 2.

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IV. s-GEODETIC NUMBER OF A FUZZY GRAPH [s-gn(G)]

Studies on the geodetic number of a crisp graph was done by Gary Chartrand, Harary and Zhang in [8]. The geodetic number of fuzzy graphs using g-distance was introduced by Suvarna and Sunitha in [28] and the same concept using μ-distance was later on developed by Linda and Sunitha in [13]. The concept of geodetic numbers using sum distance is defined below and some of the properties satisfied by them are exhibited.

Definition 4.1. A set S ⊆ V (G) such that every node of G is contained in an s-geodesic joining some pair of nodes in S is called an **s-geodetic cover(s-geodetic set)** of G. In other words if (S) = V (G), then S is an s-geodetic cover of G.

Example 4.2. Consider the fuzzy graph given in Fig.1.
If S = {v, x, w} then (S) = {u, v, x, w} = V (G). Therefore S is an s-geodetic cover of G.

Remark 4.3. A connected fuzzy graph has at least one s-geodetic cover.

Definition 4.4. The s-geodetic number of G, denoted by s-gn(G), is the minimum order of its s-geodetic covers and any cover of order s-gn(G) is an s-geodetic basis.

Example 4.5. Consider the fuzzy graph given in Fig.1. The set S = {v, x, w} is the unique s-geodetic basis and so s-gn(G) = 3.

Proposition 4.6. Let G : (V, σ, μ) be a connected fuzzy graph on n nodes. Then the s-geodetic number, s-gn(G) = n if and only if each pair of nodes in G is joined by an arc which is the unique s-geodesic between them.

Proof. Given G : (V, σ, μ) be a connected fuzzy graph on n nodes. First suppose that each pair of nodes in G is joined by an arc which is the unique s-geodesic between them.
Then $d_s(u, v) = \mu(u, v)$ for each arc (u, v) in G. Therefore no node lies on an s-geodesic between any two other nodes. Hence s-geodetic basis consists of all nodes of G. Thus s-gn(G) = n. Conversely, let s-gn(G) = n. Then the s-geodetic basis consists of all nodes in G. ie, S = V (G) is the s-geodetic cover with minimum cardinality. Hence no node of G lies on an s-geodesic between two other nodes. For if u is a node of G lying on an s-geodesic between some pair of nodes in G, then S − {u} is also an s-geodetic cover of G which is a contradiction to the fact that S is the s-geodetic cover with minimum cardinality. Hence each pair of nodes in G is joined by an arc which is the only s-geodesic between them.

Proposition 4.7. For any non-trivial connected fuzzy graph G on n nodes, 2 ≤ s-gn(G) ≤ n.
Proof. Any s-geodetic cover of a non-trivial connected fuzzy graph needs at least 2 nodes and so s-gn(G) ≥ 2. Also, clearly the set of all nodes of G is an s-geodetic cover of G and so s-gn(G) ≤ n. Thus 2 ≤ s-gn(G) ≤ n.

Remark 4.8. Clearly the set of two end-nodes of a path P_n is its unique s-geodetic basis and so s-gn(P_n) = 2.

Remark 4.9. For a complete fuzzy graph G on 2 nodes, s-gn(G) = 2. But the converse need not be true. Example 4.10. Let G be a complete fuzzy graph on 3 nodes as follows.

Example 4.10. Let G be a complete fuzzy graph on 3 nodes as follows.

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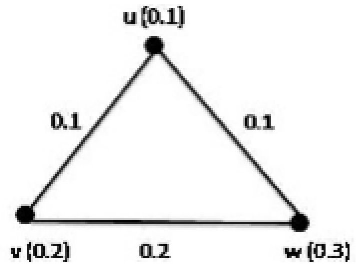

Fig.5

Here S = {v, w} is an s-geodetic basis and so s-gn(G) = 2.

Proposition 4.11. Let G : (V, σ, μ) be a fuzzy tree such that G^* is a tree. Then the set of all fuzzy end nodes of G form an s-geodetic basis for G and s-gn(G) is the number of fuzzy end nodes of G.

Proof. Let S be the set of all fuzzy end nodes of G. Clearly they are the end nodes of G^*. Let v be any internal node of G^*. Since in a tree, there exists a unique path between any two nodes, clearly v lies on an s-geodesic joining some pair of nodes in S. Since v is arbitrary, every internal node of G^* lies on an s-geodesic between some pair of nodes in S. Thus S is an s-geodetic cover of G. Also it is an s-geodetic set of minimum cardinality for if u is a fuzzy end node of G that does not belong to S, then u does not lie on any s-geodesic joining any pair of nodes in S. Therefore S is the s-geodetic basis for G and so s-gn(G)= number of fuzzy end nodes of G.

Corollary 4.12. Let G : (V, σ, μ) be a fuzzy tree having n nodes with $n \geq 3$ such that G^* is a tree. Then s-gn(G) = n − 1 only if G^* is a star graph.

Proof. Suppose G^* is a star graph on n nodes say $K_{1,n-1}$. Then by Proposition 4.11, the set of all fuzzy end nodes of G forms an s-geodetic basis of G. Hence s-gn(G) = n − 1.

Proposition 4.13. [20] A node w is a fuzzy cut node of G : (V, σ, μ) if and only if w is an internal node of every maximum spanning tree of G.

Proposition 4.14. [27] A fuzzy graph is a fuzzy tree if and only if it has a unique maximum spanning tree.

Proposition 4.15. [14] Let T be any spanning tree of a fuzzy graph G. Then T is a MST of G if and only if T contains no δ-arcs.

Using the above results, we get the following.

Corollary 4.16. An s-geodetic basis of a fuzzy tree G : (V, σ, μ) such that G^* is a tree contains none of the fuzzy cut nodes of G.

Proof. Let w be a fuzzy cut node of G. Then by Proposition 4.13, fuzzy cut nodes of a fuzzy graph are internal nodes of each of its maximum spanning trees. Hence using Proposition 4.14, we get w is an internal node of the unique maximum spanning tree T of G. Now, since by Proposition 4.15, each arc of T is strong, w being an internal node of T is not a fuzzy end node of G and so by Proposition 4.11, it follows that w is not a member of the s-geodetic basis of G. Since w is arbitrary, it follows that the s-geodetic basis of G contains none of the fuzzy cut nodes of G.

Remark 4.17. However in general, an s-geodetic basis of a fuzzy graph G may contain its fuzzy cut nodes.

Example 4.18. Consider the fuzzy graph given in Fig.6.

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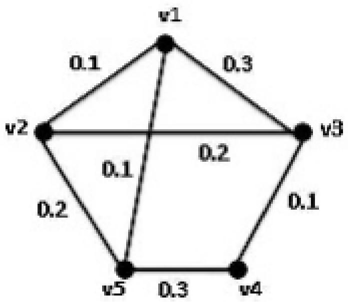

Fig.6

Here $S = \{v_3, v_5\}$ is an s-geodetic basis for G and v_5 is a fuzzy cut node of G.

Remark 4.19. It has been proved using geodesic distance that a fuzzy tree has a unique geodesic basis consisting of its fuzzy end nodes [4] .But for a fuzzy tree, using sum distance, s-geodetic basis need not be the set of fuzzy end nodes of G.

Example 4.20. Consider the fuzzy graph G given in fig.7.

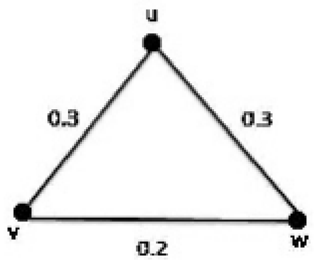

Fig.7

Here v and w are the fuzzy end nodes of G but {v, w} is not an s-geodetic cover since ({v, w}) = {v, w} ≠V (G) and the s-geodetic basis is {u, v, w}.

Proposition 4.21. For any connected fuzzy graph G, s-gn(G) = 2 if and only if there exists s-peripheral nodes u and v such that every node of G lies on an s-geodesic joining u and v. Also let P : $u = u_0, u_1, u_2, ..., u_n = v$ be an s-geodesic joining u and v. Then $d_s(u, v) = d_s(u_0, u_1) + d_s(u_1, u_2) + ... + d_s(u_{n-1}, u_n)$.

Proof. Let u and v be such that each node of G is on an s-geodesic joining u and v. Since G is non-trivial, s-gn(G) ≥ 2. Also since each node of G is on an s-geodesic between u and v, S = {u, v} is an s-geodetic basis and hence s-gn(G) = 2. Conversely let s-gn(G) = 2 and S = {u, v} be an s-geodetic basis of G. That is, S = {u, v}is an s-geodetic cover of G with minimum cardinality. Hence each node of G lies on some s-geodesic between u and v. Now to prove that u and v are s-peripheral nodes. i.e, $d_s(u, v) = d_s(G)$. Assume $d_s(u, v) < d_s(G)$. Then there exists s-peripheral nodes s and t such that s and t belongs to distinct s-geodesics joining u and v and $d_s(s, t) = d_s(G)$.
Then $d_s(u, v) = d_s(u, s) + d_s(s, v)$............(1)
$d_s(u, v) = d_s(u, t) + d_s(t, v)$.............(2)
$d_s(s, t) \leq d_s(s, u) + d_s(u, t)$.............(3)
$d_s(s, t) \leq d_s(s, v) + d_s(v, t)$...............(4)
Since $d_s(u, v) < d_s(s, t)$, (3) implies that $d_s(u, v) < d_s(s, u) + d_s(u, t)$.
Then by (1) we get $d_s(u, s) + d_s(s, v) < d_s(s, u) + d_s(u, t)$.
Therefore $d_s(s, v) < d_s(u, t)$.
Now by (4), $d_s(s, t) < d_s(u, t) + d_s(v, t)$.
Then again using (1) we get $d_s(s, t) < d_s(u, v)$, which is a contradiction.

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Thus u and v must be s-peripheral nodes.
Next given P : u = u_0, u_1, u_2, ..., u_n = v be an s-geodesic joining u and v.
Then $d_s(u_{i-1}, u_i) = \mu(u_{i-1}, u_i)$.
Therefore $d_s(u,v) = L(P) = \sum_{i=1}^{n} \mu(u_{i-1}, u_i) = \sum_{i=1}^{n} d_s(u_{i-1}, u_i)$.
Hence $d_s(u, v) = d_s(u_0, u_1) + d_s(u_1, u_2) + ... + d_s(u_{n-1}, u_n)$.

Proposition 4.22. If G : (V, σ, μ) is a non-trivial connected fuzzy graph on n nodes with diameter d, then s-gn(G) ≤ n−|W| where W is the non-empty set of all nodes, other than the s-peripheral nodes, lying on the diametral path of a pair of nodes in G.

Proof. Let u and v be the s-peripheral nodes of G for which $d_s(u, v) = d$ and let W be the set of all nodes other than u and v lying on the diametral path of G joining u and v. Now let S = V (G) − W. Then clearly since diametral paths are all s-geodesic paths, we get (S) = V (G) and consequently, s-gn(G) ≤ |S| = n − |W|.

Proposition 4.23. Let $K_{\sigma 1,\sigma 2} = (V_1 \cup V_2, \sigma, \mu)$ be a complete bipartite fuzzy graph on n nodes. Then
1. s-gn($K_{\sigma 1,\sigma 2}$) = 2, if $|V_1| = |V_2| = 1$.
2. s-gn($K_{\sigma 1,\sigma 2}$) = $|V_2|$, if $|V_1| = 1$ and $|V_2| \geq 2$.
3. s-gn($K_{\sigma 1,\sigma 2}$) = $|V_1|$, if $\sigma_1(u_i) < \sigma_2(w_j)$ $\forall u_i \in V_1$ and $\forall w_j \in V_2$ where $|V_1|, |V_2| \geq 2$ and $\sigma_1(u_i) \neq \sigma_1(u_k)$ for atleast one i and k.

Proof. -
1. Follows from Proposition 4.6.
2. Follows from Corollary 4.12.
3. Let $|V_1| = r$ and $|V_2| = s$, r, s ≥ 2 where $V_1 = \{u_1, u_2, ..., u_r\}$ and $V_2 = \{w_1, w_2, ..., w_s\}$ are bi-partitions of $K_{\sigma 1,\sigma 2}$.
Suppose that $\sigma_1(u_i) < \sigma_2(w_j)$ $\forall u_i \in V_1$ and $\forall w_j \in V_2$. Let u_p be a node of V_1 having the least non-zero membership value say *a* and let u_q be the node of V_1 having the next least membership value say *b*. Then clearly each edge adjacent to u_p has strength *a* whereas each edge adjacent to u_q has strength *b* where *a, b*∈ (0, 1]. Take S = {u_p, u_q}. We have $d_s(u_p, u_q) = d_s(u_p, w_j) + d_s(w_j , u_q) = a + b$, $(1 \leq j \leq s)$ which is the shortest sum distance between u_p and u_q. Therefore each node w_j, $(1 \leq j \leq s)$ lies on an s-geodesic joining u_p and u_q. That is, (S) = S $\cup V_2$. Thus these two nodes together with the remaining nodes of V_1 will form an s-geodetic cover of $K_{\sigma 1,\sigma 2}$. i.e, V_1 is an s-geodetic cover of $K_{\sigma 1,\sigma 2}$. We prove that V_1 is an s-geodetic basis of $K_{\sigma 1,\sigma 2}$. That is, we prove that V_1 is an s-geodetic cover of $K_{\sigma 1,\sigma 2}$ having minimum cardinality. In other words, if T is any set of nodes such that $|T| < |V_1| = r$, then we show that T is not an s-geodetic cover of $K_{\sigma 1,\sigma 2}$. Let us consider the following cases.

Case(1): If $T \subset V_1$, then there exists a node $u_i \in V_1$ such that $u_i \notin T$. Then the only s-geodesics containing u_i are u_i-w_j-u_k,$(k \neq i)$ and w_j-u_i-w_l ,$(l \neq j)$ and so u_i cannot lie on an s-geodesic joining 2 nodes of T. Thus T is not an s-geodetic cover of $K_{\sigma 1,\sigma 2}$.

Case(2): If $T \subset V_2$, then by a similar argument, T is not an s-geodetic cover of $K_{\sigma 1,\sigma 2}$.

Case(3): Now, if $|V_2| < |V_1| = r$, take T = V_2.
Then the shortest sum distance between any two nodes of V_2 is given by
$d_s(w_j , w_l) = d_s(w_j, u_p) + d_s(u_p, w_l) = a + a$......................(1)
Therefore (T) = $V_2 \cup \{u_p\} \neq V (K_{\sigma 1,\sigma 2})$. Hence T is not an s-geodetic cover of $K_{\sigma 1,\sigma 2}$.

Case(4): If $T \subset V_1 \cup V_2$ such that T contains at least one node from each of V_1 and V_2, then since $|T| < |V_1|$, there exists at least one node $u_i \in V_1$, say u_k, distinct from u_p such that $u_k \notin T$.
Now since $\sigma_1(u_k) > \sigma_1(u_p)$, each edge adjacent to u_k will have strength greater than a. Thus we get $d_s(w_j, u_k)+d_s(u_k, w_l) > a+a = d_s(w_j , w_l)$ (from (1)). Therefore u_k does not lie on any s-geodesic joining nodes of V_2. Hence $u_k \notin$ (T) and so T is not an s-geodetic cover of $K_{\sigma 1,\sigma 2}$. Thus in any case, T is not an s-geodetic cover of $K_{\sigma 1,\sigma 2}$. Hence V_1 is an s-geodetic basis of $K_{\sigma 1,\sigma 2}$ so that s-gn($K_{\sigma 1,\sigma 2}$) = $|V_1|$ = r.

Example 4.24. Consider the complete bipartite fuzzy graph $K_{\sigma 1,\sigma 2} = (V_1 \cup V_2, \sigma, \mu)$ given below.

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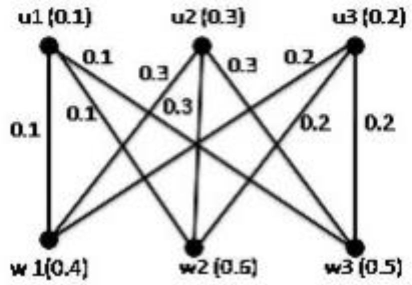

Fig.8

Let $V_1 = \{u_1, u_2, u_3\}$ and $V_2 = \{w_1, w_2, w_3\}$. Since $d_s(u_1, w_1) + d_s(w_1, u_3) = 0.1 + 0.2 = 0.3 = d_s(u_1, u_3)$, w_1 lies on the s-geodesic joining u_1 and u_3. Similarly it can be shown that all the other nodes of V_2 also lies on the s-geodesic joining u_1 and u_3. Therefore $S = V_1$ is an s-geodetic cover of $K_{\sigma1,\sigma2}$. Since no other proper subset of $K_{\sigma1,\sigma2}$is an s-geodetic cover, S is an s-geodetic basis of $K_{\sigma1,\sigma2}$ and hence $s\text{-}gn(K_{\sigma1,\sigma2}) = 3 = |V_1|$.

Remark 4.25. Let C_n, $n \geq 3$, be fuzzy cycles each of whose arcs are having same strength. When n is even, the set of any two s-peripheral nodes is an s-geodetic set of C_n. But when n is odd, no 2 nodes form an s-geodetic set and in fact there exists an s-geodetic set on 3 nodes. Therefore, for cycles having each arc of same strength, $s\text{-}gn(C_n) = \begin{cases} 2\,; & \text{when n is even} \\ 3\,; & \text{when n is odd} \end{cases}$.

Example 4.26. Consider the following fuzzy cycles C_1 and C_2 each of whose arcs are having same strength.

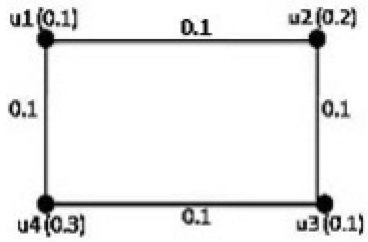

Fig.9.: C_1

Here $S = \{u_1, u_3\}$ is an s-geodesic basis of the fuzzy cycle C_1 and so $s\text{-}gn(C_1) = 2$.

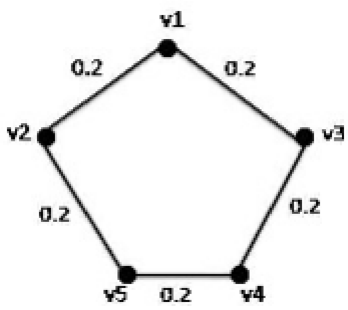

Fig.10: C_2

Also, $S = \{v_1, v_4, v_5\}$ is an s-geodesic basis of the fuzzy cycle C_2 and so $s\text{-}gn(C_2) = 3$.

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V. EXTREME s-GEODESIC FUZZY GRAPHS

Gary Chartrand and Ping Zhang in 2002 introduced the concept of Extreme geodesic graphs [7] in Graph Theory. Here we are extending these ideas to fuzzy graphs based on sum distance using s-geodesics.

Definition 5.1. A node v in a fuzzy graph G is called an **extreme node** if the fuzzy subgraph induced by its neighbors is a complete fuzzy graph.

Example 5.2. Consider the fuzzy graph G in Fig.11.

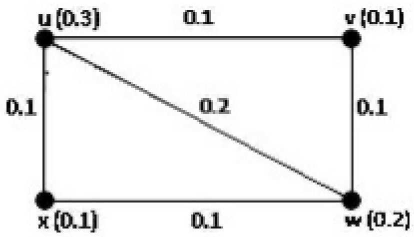

Fig.11

Here both v and x are extreme nodes since S = {u, w} are the neighbors of v and x where the fuzzy subgraph induced by S, < S > is a complete fuzzy graph.

Remark 5.3. Gary Chartrand et.al in [8] showed that every geodetic set of a crisp graph contains its extreme nodes. But the result need not be true in the case of s-geodetic sets in a fuzzy graph G.

Example 5.4. The fuzzy graph G given in Fig.11 has v and x as its extreme nodes. But S = {u, w} is an s-geodetic set of G that does not contain any of its extreme nodes.

Definition 5.5. The number of extreme nodes in a fuzzy graph G is called the **extreme order** of G and is denoted by ex(G). In Example 5.2, ex(G) = 2.

Proposition 5.6. For a connected fuzzy graph G on n nodes, $0 \leq ex(G) \leq n$.

Example 5.7. Consider the fuzzy graph G in Fig.12.

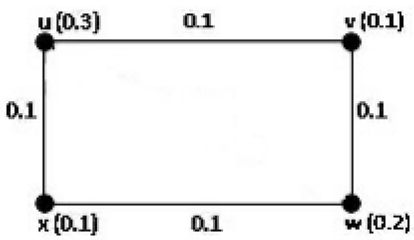

Fig.12

Here none of the nodes are extreme nodes of G. Therefore ex(G) = 0.

Example 5.8. Consider the complete fuzzy graph G given in Fig.13.

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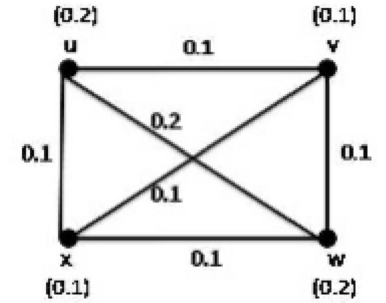

Fig.13

Here the node u is an extreme node since the fuzzy subgraph induced by the neighbors v, w and x of u is a complete fuzzy graph. Similarly v, w and x are all extreme nodes and so ex(G) = 4 =number of nodes of G.

Remark 5.9. Gary Chartrand and Ping Zhang in [7] showed that for a crisp graph G, $0 \leq ex(G) \leq g(G)$ where g(G) is the geodetic number of G. But this result is not true for a fuzzy graph G based on s-geodesic.

Example 5.10. In the complete fuzzy graph G given in Fig.13, the nodes u, v, w and x are all extreme nodes and so ex(G) = 4. But s-gn(G) = 2 since S = {u, w} is an s-geodetic basis of G. Therefore ex(G) > s-gn(G).

Definition 5.11. A fuzzy graph G is called an **extreme s-geodesic fuzzy graph** if its s-geodetic number s-gn(G) = ex(G). That is if G has an s-geodetic basis consisting of the extreme nodes of G.

Example 5.12. In Fig.11, ex(G) = 2. Also S = {v, x} is an s-geodetic basis since (S) = V (G). Thus s-gn(G) = 2 = ex(G) and so G is an extreme s-geodesic fuzzy graph.

Example 5.13. Consider the fuzzy graph G’ in Fig.14.

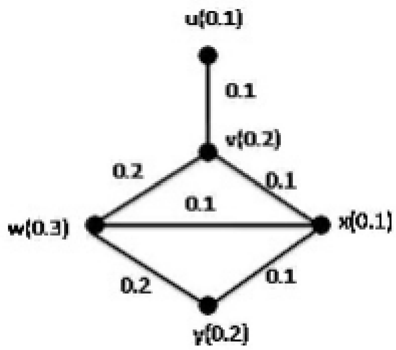

Fig.14

Here u and y are the only extreme nodes of G’ .Therefore ex(G’) = 2. But S = {u, y, w} is the only s-geodetic basis of G’ so that s-gn(G’) = 3 ≠ ex(G’). Hence G’ is not an extreme s-geodesic fuzzy graph.

Proposition 5.14. A complete fuzzy graph G : (V, σ, μ) on n nodes in which each pair of nodes is joined by an arc which is the unique s-geodesic between them is an extreme s-geodesic fuzzy graph.
Proof. By Proposition 4.6 it follows that s-gn(G) = n, the number of nodes of G. Also each node of a complete fuzzy graph is an extreme node and so s-gn(G) = ex(G) = n. Hence G is an extreme s-geodesic fuzzy graph.

Proposition 5.15. Every fuzzy tree G : (V, σ, μ) such that G^* is a tree is an extreme s-geodesic fuzzy graph.

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Proof. By Proposition 4.11, the s-geodetic number of a fuzzy tree G such that G^* is a tree is the number of fuzzy end nodes of G. In fact, the set of all fuzzy end nodes of G are the end nodes of G^* and consequently, they are the only extreme nodes of G. Therefore, ex(G) = s-gn(G), implying that the fuzzy tree G is an extreme s-geodesic fuzzy graph.

Remark 5.16. A path P is always an extreme s-geodesic fuzzy graph with ex(P) = 2 = s-gn(P).

Remark 5.17. A cycle C_n, $n \geq 4$, contains no extreme nodes and so C_n is not an extreme s-geodesic fuzzy graph.

Proposition 5.18. A complete bipartite fuzzy graph $K_{\sigma 1,\sigma 2} = (V_1 \cup V_2, \sigma, \mu)$ is an extreme s-geodesic fuzzy graph if
1. $|V_1| = |V_2| = 1$.
2. $|V_1| = 1$ and $|V_2| \geq 2$.

Proof. - 1. Follows from Proposition 5.14.
2. Follows from Proposition 5.15.

Remark 5.19. A complete bipartite fuzzy graph G on $n \geq 4$ nodes, containing partitions each of whose cardinality is greater than 1, contains no extreme nodes and so G is not an extreme s-geodesic fuzzy graph.

Definition 5.20. For a connected fuzzy graph G on n nodes, $n \geq 2$, the s-geodetic ratio of G is defined as s-r_g(G) = s−gn(G)/n.

Remark 5.21. Since by Proposition 4.7, $2 \leq$ s-gn(G) $\leq n$ for every nontrivial connected fuzzy graph G on n nodes, we get $0 <$ s-r_g(G) ≤ 1.

Proposition 5.22. Let C_n, $n \geq 3$, be fuzzy cycles each of whose arcs are having same strength. Then the s-geodetic ratio of C_n is as follows.

$$\text{s-}r_g(C_n) = \begin{cases} \frac{2}{n}; \text{ when n is even} \\ \frac{3}{n}; \text{ when n is odd} \end{cases}.$$

Proposition 5.23. The s-geodetic ratio of a fuzzy tree G : (V, σ, μ) on n nodes, $n \geq 3$, such that G^* is a star graph is given by s-r_g(G) = (n−1)/ n .

Proof. It follows from Corollary 4.12 that the s-geodetic number of G on n nodes, s-gn(G), is n − 1. Therefore, the s-geodetic ratio of G, s-rg(G) = s−gn(G)/ n = (n−1)/ n.

Definition 5.24. The extreme order ratio of a fuzzy graph G on n nodes with $n \geq 2$ is defined as r_{ex}(G) = ex(G)/ n.

Proposition 5.25. For an extreme s-geodesic fuzzy graph G, the s-geodetic ratio of G coincides with its extreme order ratio.

Proof. Let G be an extreme s-geodesic fuzzy graph on n nodes. Then by definition 5.11, ex(G) = s-gn(G).
⇒ ex(G) n = s−gn(G) n .
⇒r_{ex}(G) = s-r_g(G).

Proposition 5.26. If G : (V, σ, μ) is a complete fuzzy graph on n nodes in which each pair of nodes is joined by an arc which is the unique s-geodesic between them, then s-r_g(G) = r_{ex}(G) = 1.

Proof. By Proposition 5.14, it follows that s-gn(G) = n = ex(G), G being an extreme s-geodesic fuzzy graph. Therefore s-r_g(G) = s−gn(G)/n = n/n = 1 and r_{ex}(G) = ex(G)/n = n/n = 1. Thus s-r_g(G) = r_{ex}(G) = 1.

Proposition 5.27. For a path P on n nodes, s-r_g(P) = r_{ex}(P) = 2/n .

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Proof. A path P is an extreme s-geodesic fuzzy graph with ex(P) = 2 = s-gn(P). Therefore s-r_g(P) = s−gn(P)/ n = 2/ n and r_{ex}(P) = ex(P)/ n = 2/ n . Thus s-r_g(P) = r_{ex}(P) = 2/n .

The above result can be generalized as follows.

Proposition 5.28. If G : (V, σ, μ) is a fuzzy tree on n nodes such that G* is a tree with p end-nodes, then s-r_g(G) = r_{ex}(G) = p/n .

Proof. Using Propositions 4.11 and 5.15, we get s-gn(G) = ex(G) = p.
Therefore, s−gn(G)/ n = ex(G)/ n = p/ n .
⇒ s-r_g(G) = r_{ex}(G) = p/ n .

VI. MINIMUM s-GEODETIC FUZZY SUBGRAPH

The concept of Minimum geodetic subgraphs was introduced by Gary Chartrand, Frank Harary and Ping Zhang in [8]. Here we are introducing this concept in fuzzy graph using sum distance.

Definition 6.1. A fuzzy graph H is called a minimum s-geodetic fuzzy subgraph if there exists a fuzzy graph G containing H as an induced fuzzy subgraph such that V (H) is an s-geodetic basis of G.

Example 6.2. Consider the fuzzy graph H in Fig.15.

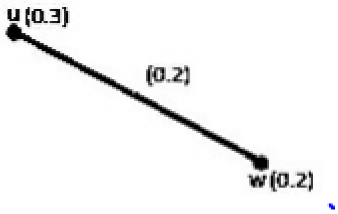

Fig.15

Clearly H is a minimum s-geodetic fuzzy subgraph of the fuzzy graph G in Fig.11.

Proposition 6.3. If H is a minimum s-geodetic fuzzy subgraph of a connected fuzzy graph G on n nodes, then 2 ≤ s-gn(H) ≤ s-gn(G) ≤ n.

Proof. Since H is a non-trivial fuzzy graph, by Proposition 4.7 we get 2 ≤ s-gn(H) ≤ |V (H)|. But since H is a minimum s-geodetic fuzzy subgraph of G, V (H) is an sgeodetic basis for G and so s-gn(G) = |V (H)|.
Therefore we get 2 ≤ s-gn(H) ≤ s-gn(G). Again by Proposition 4.7 we get 2 ≤ s-gn(G) ≤ n so that the result follows. Thus 2 ≤ s-gn(H) ≤ s-gn(G) ≤ n.

Proposition 6.4. Let H be a minimum s-geodetic fuzzy subgraph of a connected fuzzy graph G. If s-gn(G) = 2 then the nodes of H are s-peripheral nodes of G.

Proof. Since H is a minimum s-geodetic fuzzy subgraph of G, V (H) is an s-geodetic basis of G. Now by Proposition 4.21, if s-gn(G) = 2 then there exists s-peripheral nodes u and v such that every node of G lies on an s-geodesic joining u and v. Therefore if s-gn(G) = 2 then {u, v} is an s-geodetic basis of G. i.e, if s-gn(G) = 2 then V (H) = {u, v} where u and v are s-peripheral nodes of G.

Remark 6.5. The converse of Proposition 6.4 is not true. That is s-gn(G) need not be 2 even though the nodes of its minimum s-geodetic fuzzy subgraph are s-peripheral nodes of G.

Example 6.6. Consider the fuzzy graph H given in Fig.16.

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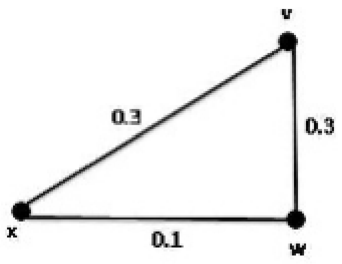

Fig.16

Clearly H is a minimum s-geodetic fuzzy subgraph of the connected fuzzy graph G given in Fig.17.

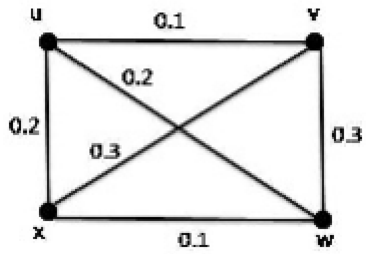

Fig.17

Here V (F) = {v, w, x} is an s-geodetic basis of G that contains s-peripheral nodes of G but here s-gn(G) = 3.

Proposition 6.7. The subgraph induced by the fuzzy end nodes of a fuzzy tree G : (V, σ, μ) such that G^* is a tree is the minimum s-geodetic fuzzy subgraph of G.
Proof. By Proposition 4.11, the set of all fuzzy end nodes of a fuzzy tree G such that G^* is a tree forms an s-geodetic basis of G. Thus the fuzzy subgraph induced by the collection of all fuzzy end nodes of G is a minimum s-geodetic basis of G.

Remark 6.8. The converse of Proposition 6.7 is not true. That is, a fuzzy graph G need not be a fuzzy tree with G^* as a tree even if its minimum geodetic fuzzy sub graph is the fuzzy sub graph induced by its fuzzy end nodes.

Example 6.9. Consider the fuzzy graph G in Fig.18.

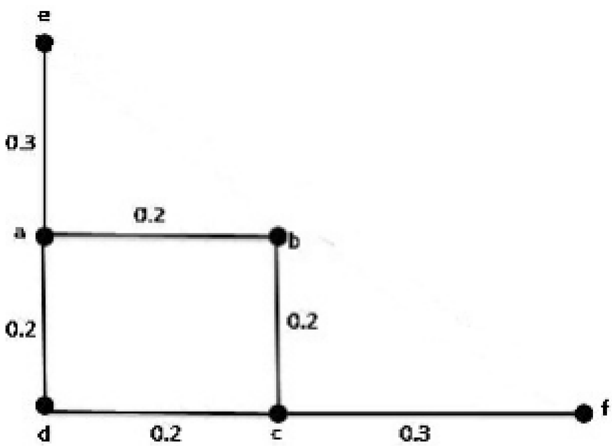

Fig.18

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Evidently {e, f} is an s-geodetic basis for G where both e and f are fuzzy end nodes of G. Hence the fuzzy sub graph induced by {e, f} is the minimum s-geodetic fuzzy sub graph of G, but G is not a fuzzy tree with G^* as a tree.

VII. CONCLUSION

In this paper, we introduced s-geodesic, s-geodetic closure, s-geodetic iteration number, s-geodetic cover and s-geodetic number of a fuzzy graph along with suitable examples and studied some of their properties. The upper and lower bounds for s-geodetic number of a fuzzy graph are obtained. The s-geodetic number of complete bipartite fuzzy graphs, fuzzy cycles and also of fuzzy trees subject to certain conditions is examined. The concept of extreme nodes in a fuzzy graph is defined leading to the introduction of a special type of fuzzy graph known as extreme s-geodesic fuzzy graph. An attempt to define a minimum s-geodetic fuzzy subgraph is also made.

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