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Internal Problems for the Biharmonic Equation on Plane

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ABSTRACT: The problem of determining the value of the biharmonic function in the ball is solved by its values and by the values of the derivative given at two radii or on a curve passing through the origin. Theorems of uniqueness of continuation and estimates of stability and regularization of the solution are obtained in the case when the values are given at two radii. Also considered is the possibility of continuation of the biharmonic function when the values are given on discrete sets with a limit point lying either on two radii or on a curve passing through the origin.

KEY WORDS: biharmonic function, ill-posed problems, uniqueness of continuation, stability estimate, regularization.

I. NTRODUCTION

Following the terminology of [1], the problem of determining the solution of differential equations (or systems of equations) with partial derivatives in a certain domain from its values on a certain set inside the domain will be called internal problems. The problems of continuation of physical fields lead to internal problems for differential equations, since real physical fields are described by solutions of partial differential equations. A characteristic feature of many internal problems is their incorrectness in the classical sense. Therefore, their study uses the approach proposed by A.N.Tikhonov [2]. With this approach, it is a priori assumed that a solution to the problem exists and that it belongs to a given set. The main point in proving the "conditionally well-posed" problem is to obtain a uniqueness theorem and an estimate of stability on the well-posedness set.

II. RELATED WORKS

The paper considers the problem of determining the orientation biharmonic functions $U(r,\varphi)$ in a circle

$$K=\{(r,\varphi):\ 0\le r\le 1,\qquad 0\le\varphi\le 2\pi\}$$

from the data of its value and the value of its Laplacians at two radii

$$U(r,\alpha_0)=f_0(r),\quad U(r,\alpha_1)=f_1(r);\qquad (1)$$
$$\Delta U(r,\alpha_0)=f_2(r),\qquad \Delta U(r,\alpha_1)=f_3(r).$$

Assuming that

$$|U(r,\varphi)|\le 1,\ \ |\Delta U(r,\varphi)|\le 1,\quad (r,\varphi)\in\overline{K}\qquad (2)$$

and angle difference $\alpha=\alpha_1-\alpha_0$ for any positive integer m satisfying this condition

$$|\sin m\,\alpha|\ge\frac{\theta}{m^{\sigma}}\qquad (3)$$

The conditional stability estimate is proved

$$|U(r,\varphi)|\le C\left[\frac{2}{1-p}\right]^{1-\omega}\varepsilon^{\omega},\quad (r,\varphi)\in K_\rho\qquad (4)$$

where K_ρ- is the circle of radius $\rho<p<1$. For this problem, a regularizing operator is constructed and its efficiency estimate is given.

Next, a sequence of points $E=\{(r_k,\varphi_k)\}_{k=1}^{\infty}\subset K$ is introduced which satisfies this the condition: E has a limit point at the origin; E is located a) at two radii $\varphi=\alpha_0$ and $\varphi=\alpha_1$, where the difference $\alpha=\alpha_1-\alpha_0$ satisfies condition (3) or b) on a curve $\varphi=\varphi(r),\ 0\le r<1$ passing through the origin and satisfying the condition

$$r=o\big(\varphi(r)\big),\ \ r\to 0\qquad (5)$$

For the problem of continuation of a biharmonic function $U(r,\varphi)$ from a set E , the following results are obtained. In the case where E satisfies I) - 2a), estimation analogous to (4) (Theorem 1). In the case when E satisfies condition I) – 2b), the uniqueness of the assigned problem (THEOREM 3).

Here we consider some problems about the report has to solve a biharmonic equation by their value pits and its Laplacian values on sets that lie within the domain of regularity.

I. Let $U(r,\varphi)$ (biharmonic function in the circle $K=\{(r,\varphi): 0\le r<1,\ \ 0\le\varphi\le 2\pi\}$ is continuous on $\overline{K}$ and does not comply with the equations (2). Suppose further value biharmonic functions are known to the two radii (2) where the $0\le r<1,\ \alpha_i (i=0,1)$ given numbers of the interval $[0,2\pi]$ and $f_i(r)\ (i=0,1,2,3)$ are the given functions. Consider the problem of extending a biharmonic function $U(r,\varphi)$ to the whole circle K according to the data (1).

III. CONCLUSION

THEOREM 1. Let for $0\le r<1,\ \ i=0,1,2,3$

$$|f_i(r)|<\varepsilon \tag{6}$$

and the difference $\alpha=\alpha_1-\alpha_0$ is such that for all natural m satisfies the inequality (3) with some $\theta>0,\ \sigma\ge 1$. Then for any point of a circle of K_p radius $p<1$

$$|U(r,\varphi)|\le C(\sigma,\theta,r,\rho)\left[\frac{2}{1-p}\right]^{1-\frac{1}{\pi}(\sqrt{p}-\sqrt{\rho})}\varepsilon^{\frac{1}{\pi}(\sqrt{p}-\sqrt{\rho})},\qquad (r,\varphi)\in K_p \tag{7}$$

To obtain this estimate, we use the well-known representation of the biharmonic function in the unit circle K [1]:

$$U(r,\varphi)=(r^2-1)u_1(r,\varphi)+u_2(r,\varphi) \tag{8}$$

where $u_1(r,\varphi),\ u_2(r,\varphi)$ - are harmonic functions in the unit circle. For harmonic functions is true analogous theorem [5] from which it follows estimate (8).

Let us give a regularization of the considered problem. If the expansion of functions $f_i(r)$ in a Taylor series has the form

$$f_i(r)=\sum_{m=0}^{\infty}c_m^i r^m,\qquad i=0,1,2,3$$

where $c_m^i=\frac{f_i^{(m)}(o)}{m!}$ the solution of the problem under consideration can be represented as

$$U(r,\varphi)=\sum_{m=0}^{\infty}\{\frac{(1-r^2)}{4}[(c_m^3\sin m\,\alpha_1-c_m^2\sin m\,\alpha_0)\cos m\,\varphi+(c_m^2\cos m\,\alpha_0-c_m^3\sin m\,\alpha_i)\sin m\,\varphi]+$$
$$+[((c_m^1-\frac{r^2-1}{4}c_m^3)\sin m\,\alpha_1-(c_m^0-\frac{r^2-1}{4}c_m^2)\sin m\,\alpha_0)\cos m\,\varphi+ \tag{8}$$
$$+((c_m^0-\frac{r^2-1}{4}c_m^2)\cos m\,\alpha_0-(c_m^1-\frac{r^2-1}{4}c_m^3)\sin m\,\alpha_i)\}\frac{r^m}{\sin m\alpha}$$

Consider a family of linear operators R_n depending on an integer parameter n, defined as follows

$$R_n f(r)=U_n(r,\varphi),$$

where $f(r)=(f_0(r),\ f_1(r),\ f_2(r),\ f_3(r))$ and $U_n(r,\varphi)$ the final amount of the right-hand side of (8). It is not hard to verify that the family R_n will be a regularizing family [2] and

$$\lim_{n\to\infty}R_n f(r)=U(r,\varphi)$$

Now let's move on to problems when data are specified on discrete sets. Consider a sequence of point s $E=\{(r_k,\varphi_k)\}_{k=1}^{\infty}\subset K$ satisfying the conditions:

1) E has a limit point at the origin;

2) $E$is located a) at two radii $\varphi=\alpha_0$ and $\varphi=\alpha_1$, where the difference satisfies condition (3) or b) on the curve

$$\varphi=\varphi(r)\qquad 0\le r<1,$$

passing through the origin and satisfying the condition

$$r=o\big(\varphi(r)\big),\quad r\to 0.$$

Consider the problem of continuing biharmonic functions $U(r,\varphi)$ satisfying the condition (2) with a plurality of E on the circle. In the case when E satisfies condition 1) and 2a) the following is true.

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THEOREM 2. Let for $i = 0,1$ and $k = 1,2,\ldots$ then $|U(r_k, \alpha_k)| < \varepsilon$ is for any point $(r, \varphi) \in K_\rho$

$$|U(r,\varphi)| \leq C(\sigma,\theta,r,\rho)\left[\frac{2}{1-p}\right]^{1-\frac{C_1(p-\rho)}{ln\frac{1}{\mu_n(\varepsilon)}}} \varepsilon^{\frac{C_2(p-\rho)}{ln\frac{1}{\mu_n(\varepsilon)}}} \qquad (9)$$

where $r < \rho < p < 1$, $\mu_n(E) = min\{\mu_n(E_0),\ \mu_n(E_1)\}$, $E_i = E|_{\varphi=\alpha_i}(i = 0,1)$, and the number n is determined from the relation

$$\left(\frac{\mu_{n+1}(E)}{C_1}\right)^{n+1} < \varepsilon \leq \left(\frac{\mu_n(E)}{C_2}\right)^n,$$

C_1, C_2 - some constants.

Theorem 2 immediately implies the uniqueness of the problem posed in the case when E satisfies conditions 1) and 2a).

THEOREM 3. Let the sequence E satisfy conditions 1) and 2b) and

$$u(r,\varphi)|_E = 0.$$

Then for any point

$$(r,\varphi) \in KU(r,\varphi) \equiv 0.$$

REFERENCES

[1] Tikhonov A. N., Samarski A. A. Equations of mathematical of physics. Courier Corporation, 2013 y.

[2] Tikhonov, A. N.; Arsenin, V. Y.. Solutions of ill-Posed Problems. New York: Winston.1977.

[3] Lavrentiev M.M., Romanov V.G. and Shishatskii S.P. Ill-Posed Problems of Mathematical Physics and Analysis. American Mathematical Society, Providence, 1986.

[4] Lavrentiev M.M. Some Improperly Posed Problems in Mathematical Physics. Springer, Berlin, 1967.

[5] Abdukarimov A., Shamsutdinova N.Sh.. Materials of the International Conference «Inverse and ill-posed problems of mathematical physics», Novosibirsk, 5-12 August 2012.