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Algorithms for Stable Parametric Identification of Non-Stationary Systems Based On Orthogonal Polynoms

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ABSTRACT: Algorithms for stable parametric identification of nonstationary systems based on orthogonal polynomials are presented. To obtain a stable solution of the identification equation, we used the decomposition of the matrix operator on the right into elementary orthogonal rotation matrices and on the left into permutation matrices. The above expressions make it possible to obtain a stable solution of the considered equation, which improves the accuracy of algorithms for stable parametric identification of nonstationary systems based on orthogonal polynomials.

KEY WORDS: non-stationary system, parametric identification, stable algorithms, orthogonal polynomials.

I.INTRODUCTION

The current state of the theory and practice of identification is characterized by the intensive development of methods focused on the use of computers [1-6]. However, despite the high level of theoretical research, the experience of successful practical application of their results in the construction of mathematical models of real dynamic objects is small. This is largely due to the specificity of the identification problem - its belonging to the class of inverse problems [7-11].

II. SIGNIFICANCE OF THE SYSTEM

A characteristic feature of inverse problems is their incorrectness, as a result of which many known computational schemes are unstable even with insignificant errors in specifying the initial data. That is why the problem of incorrectness is of fundamental importance when identifying dynamic objects. The foregoing determines the need for the development of special regularization algorithms that ensure the receipt of stable solutions of the identification problem corresponding to real physical objects.

III. LITERATURE SURVEY

Currently, the original methods for solving inverse problems are being successfully developed, which allow algorithmic selection of solutions based on additional information about them [9-13]. To reduce the influence of the inaccuracy of the initial data on the identification results when solving practical problems, methods based on the use of test signals of a special type [2, 14], smoothing of information signals [15, 16], and expansion of the sought mathematical models of objects in a series in terms of orthogonal systems have become widespread functions [17,18].

IV. METHODOLOGY

At present, when expanding the required mathematical models of objects in a series in terms of orthogonal systems of functions, so-called classical orthogonal polynomials are often used [19, 20], that is, polynomials of Chebyshev, Legendre, Chebyshev - Hermite, Chebyshev - Laguerre and general Jacobi polynomials. Recently, there are more and more new possibilities of using classical orthogonal polynomials in solving various technical problems [20, 21].

Chebyshev $T_i(z)$ polynomials are defined as follows

$$T_i(z) = \cos(i \cos^{-1} z), \quad -1 \le z \le 1.$$

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For identification purposes , it is convenient to replace the variable

$$t = \beta(1-z)/2$$

and go to shifted Chebyshev polynomials

$T_0(t) = 1,$

$T_1(t) = 1 - 2t/\beta,$

$T_2(t) = 8(t/\beta)^2 - 8(t/\beta) + 1,$

… … …

$$T_{i+1}(t) = (2 - 4t/\beta)T_i(t) - T_{i-1}(t).$$

The orthogonality condition has the form [20,21]

$$\int_0^\beta \frac{T_i(t)T_j(t)}{(\beta t - t^2)^{1/2}} dt = \begin{cases} 0, & i \neq j; \\ \pi/2, & i = j \neq 0; \\ \pi, & i = j = 0. \end{cases}$$

so how

$$T_i(t) = \cos[i \cos^{-1}(1 - 2t/\beta)],$$

then

$$T_i(t)T_j(t) = \frac{1}{2}[T_{i+j}(t) + T_{i-j}(t)].$$

Hence it follows that

$$T_i(t)T_j(t) = \sum_{k=0}^{m-1} S_{ijk} T_k(t), \qquad (1)$$

Where

$$S_{000} = 1,$$

$$S_{ijk} = \frac{2}{\pi} h_{ijk},$$

$$h_{ijk} = \int_0^\beta \frac{T_i(t)T_j(t)T_k(t)}{(\beta t - t^2)^{1/2}} dt \quad = \frac{1}{2}\int_0^\beta \left[\frac{T_{i+j}(t)T_k(t)}{(\beta t - t^2)^{1/2}} + \frac{T_{i-j}(t)T_k(t)}{(\beta t - t^2)^{1/2}} \right] dt$$

$$= \begin{cases} \pi, & i = j = k = 0 \\ \pi/2, & i = 0,\ j = k \neq 0;\ j = 0, i = k \neq 0 \\ \pi/4, & i + j = k \neq 0 (i \neq 0, \quad j \neq 0) \\ \pi/4, & |i - j| = k \neq 0 (i \neq 0, \quad j \neq 0) \\ \pi/4, & i = j \neq 0, \quad k = 0 \\ 0, & otherwise. \end{cases}$$

An arbitrary function $f(t)$ is approximated by a segment of the Chebyshev series [21]

$$f(t) = \sum_{i=0}^{m-1} f_i T_i(t) = f^T T(t),$$

Where

$$f = [f_0, f_1, f_2 ..., f_{m-1}]^T,$$

$$T(t) = [T_0(t), T_1(t), T_2(t), ... T_{m-1}(t)]^T.$$

You can show [18,19] , that

$$\int_0^t T(t)dt = PT(t),$$

where

$$P=\beta\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{8} & 0 & -\frac{1}{8} & \cdots & 0 & 0 & 0 \\ -\frac{1}{6} & \frac{1}{4} & 0 & \cdots & 0 & 0 & 0 \\ -\frac{1}{16} & 0 & \frac{1}{8} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{-1}{2(m-1)(m-3)} & 0 & 0 & \cdots & \frac{1}{4(m-3)} & 0 & \frac{-1}{4(m-1)} \\ \frac{-1}{2m(m-2)} & 0 & 0 & \cdots & 0 & \frac{1}{4(m-2)} & 0 \end{bmatrix} \quad (2)$$

Consider a linear nonstationary system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad (3)$$

where $x-n-$th dimensional state vector, $u-r-$th dimensional control vector, and the initial vector $x(0)$ are assumed to be known. The problem is posed of identifying unknown elements of matrices $A(t)$ and $B(t)$ both by observations of the input and output and the known initial state.

Suppose that $x(\mathrm{t}), u(\mathrm{t}), A(\mathrm{t}), B(\mathrm{t})$ are integrable on $[0,\beta]$ and are represented in the form

$$A(t)=\sum_{i=0}^{m-1} A_i T_i(t); \quad B(t)=\sum_{i=0}^{m-1} B_i T_i(t);$$
$$u(t)=\sum_{i=0}^{m-1} u_i T_i(t); \quad x(t)=\sum_{i=0}^{m-1} x_i T_i(t). \quad (4)$$

Using (1), we obtain

$$A(t)x(t) = [y_0, y_1, y_2,..., y_{m-1}]T(t) \overset{\Delta}{=} y^T T(t), \quad (5)$$
$$B(t)u(t) = [z_0, z_1, z_2,..., z_{m-1}]T(t) \overset{\Delta}{=} z^T T(t),$$

where

$$y_k = \sum_{i=0}^{m-1}\sum_{j=0}^{m-1} A_i x_j s_{ijk} \quad (k = 0,1,2,..., m-1),$$
$$z_k = \sum_{i=0}^{m-1}\sum_{j=0}^{m-1} B_j u_j s_{ijk} \quad (k = 0,1,2,..., m-1).$$

Integrating (3) from 0 to t, we obtain

$$x(t) - x(0) = \int_0^t A(t)x(t)dt + \int_0^t B(t)u(t)dt. \quad (6)$$

Substituting (5) into (6) and using (2), we obtain

$$x^T - \lceil x(0), 0, 0,..., 0 \rceil = y^T P + z^T P. \quad (7)$$

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We use test signals for identification. Let $u_v(t)$, the test signal, and the reaction $x_v(t)$ of the system to $u_v(t)$, $v = 1, 2, 3, ..., q \geq q \geq (n + r)$. Then from (7) we obtain:

$$[\alpha_{vo}, \alpha_{v1}, \alpha_{v2}, ..., \alpha_{vm-1}] = \tilde{A}^T \tilde{x}_v P + \tilde{B}^T \tilde{u}_v P, \tag{8}$$

Where

$$[\alpha_{v0}, \alpha_{v1}, \alpha_{v2}, ..., \alpha_{v,m-1}] = [x_{v0}, x_{v1}, x_{v2}, ..., x_{v,m-1}] - [x_v(0), 0, 0, ..., 0],$$

$$\tilde{A}^T = [A_0, A_1, A_2, ..., A_{m-1}],$$

$$\tilde{B}^T = [B_0, B_1, B_2, ..., B_{m-1}],$$

$$\tilde{x}_v = \begin{bmatrix} \sum_{j=0}^{m-1} x_{vj} s_{0j0} & \sum_{j=0}^{m-1} x_{vj} s_{0j1} & \cdots & \sum_{j=0}^{m-1} x_{vj} s_{0j,m-1} \\ \sum_{j=0}^{m-1} x_{vj} s_{1j0} & \sum_{j=0}^{m-1} x_{vj} s_{1j1} & \cdots & \sum_{j=0}^{m-1} x_{vj} s_{1j,m-1} \\ \vdots & \vdots & & \vdots \\ \sum_{j=0}^{m-1} x_{vj} s_{m-1,j,0} & \sum_{j=0}^{m-1} x_{vj} s_{m-1,j,1} & \cdots & \sum_{j=0}^{m-1} x_{vj} s_{m-1,j,m-1} \end{bmatrix},$$

$$\tilde{u}_v = \begin{bmatrix} \sum_{j=0}^{m-1} u_{vj} s_{0j0} & \sum_{j=0}^{m-1} u_{vj} s_{0j1} & \cdots & \sum_{j=0}^{m-1} u_{vj} s_{0j,m-1} \\ \sum_{j=0}^{m-1} u_{vj} s_{1j0} & \sum_{j=0}^{m-1} u_{vj} s_{1j1} & \cdots & \sum_{j=0}^{m-1} u_{vj} s_{1j,m-1} \\ \vdots & \vdots & & \vdots \\ \sum_{j=0}^{m-1} u_{vj} s_{m-1,j,0} & \sum_{j=0}^{m-1} u_{vj} s_{m-1,j,1} & \cdots & \sum_{j=0}^{m-1} u_{vj} s_{m-1,j,m-1} \end{bmatrix}$$

Now relation (8) becomes the equation

$$F\theta = L, \tag{9}$$

where

$$F = [(\tilde{x}_v P)^T \otimes I_n, (\tilde{u}_v P)^T \otimes I_n],$$

$$\theta = [A_0^T, A_1^T, A_2^T, ..., A_{m-1}^T, B_0^T, B_1^T, B_2^T, ..., B_{m-1}^T]^T,$$

$$L = [\alpha_{v0}^T, \alpha_{v1}^T, \alpha_{v2}^T, ..., \alpha_{v,m-1}^T]^T$$

and $\otimes$ - the symbol of Kronecker's work.
To obtain sustainable solutions of the equation (9) will be used decomposition of the matrix into the product of two matrices

$$F = UV, \tag{10}$$

where U and V are rank of r, and

$$VV^T = I_r, \tag{11}$$

and main minor order r of the matrix U is different from zero. Here I_r is the identity matrix of the order r.
Decomposition (10) lies at the basis of many computational algorithms solving problems of linear algebra [22-25]. It can be obtained by multiplying the matrix F from the right to the basic orthogonal matrix (rotation or reflection) and the left at the matrix permutation , e.g., with the help of normalized process [22]. In this case, the rows of the matrix V

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will be orthogonal, so that equality (11) will hold. In addition, the main minor order of the matrix U is a maximum among all minors of the same order.
Let us introduce the following notation:

$$U = \begin{Vmatrix} U_1 \\ U_2 \end{Vmatrix}, \qquad V = \begin{Vmatrix} V_1 \\ V_2 \end{Vmatrix}, \qquad L = \begin{Vmatrix} L_1 \\ L_2 \end{Vmatrix},$$

where , U_1, V_1 - square matrix of the order r ,U_2, V_2 - matrix size $(m-r)\times r$ and $r\times(n-r)$ accordingly, L_1 and L_2 - the vectors of the first r component of the free member of the system L (9).
Following the [22, 23] can show that d A solvability system (9) is necessary and sufficient to

$$U_2 U_1^{-1} L_1 = L_2$$

Representing the matrix F in the cellular form

$$F = \begin{Vmatrix} U_1 \\ U_2 \end{Vmatrix} \begin{Vmatrix} V_1 & V_2 \end{Vmatrix} = \begin{Vmatrix} U_1 V_1 & U_1 V_2 \\ U_2 V_1 & U_2 V_2 \end{Vmatrix}$$

and considering that

$$FF^+F = F ,$$

the pseudoinverse matrix for $r = n < m$ can be written in the following form:

$$F^+ = \begin{Vmatrix} U_1^T V_1^{-1} & 0 \\ V_2^T U_1^{-1} & 0 \end{Vmatrix}$$

Then the general solution of system (9) is given by the equality

$$\theta = F^+ L + R_{np} b,$$

where $R_{np} = I_n - Q^T Q$ is the right annihilator , b is an arbitrary n dimension vector.

V. CONCLUSION AND FUTURE WORK

The above expressions make it possible to obtain a stable solution to equation (9), which improves the accuracy of algorithms for stable parametric identification of nonstationary systems based on orthogonal polynomials.

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