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Stable Algorithms for Diagnosing Nonlinear Dynamic Systems in the Presence of Disturbing Factors

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ABSTRACT: Stable algorithms for diagnosing nonlinear dynamic systems in the presence of disturbing factors are presented. In this case, the diagnostic process includes the procedure for the formation of a residual as a result of a mismatch between the behavior of the system being diagnosed and its reference model. To obtain a stable representative value of the extended state vector, a regular algorithm based on the Hermite method is used. The obtained regular algorithm allows the programmed motion to be carried out according to the desired law of change in time of the state vector and ensures the convergence of the system motion to a certain neighborhood with respect to the reference motion.

KEY WORDS: nonlinear dynamic system, stable diagnostic algorithms, disturbing factors.

I. INTRODUCTION

Currently, a significant number of different methods for solving diagnostic problems have been developed, united by the general concept of analytical redundancy. According to this concept, diagnostics is carried out based on checking the analytical dependencies that exist between the control and output signals of the diagnosed system (DS) measured at a certain time interval. The diagnostic process includes the formation of a residual as a result of the mismatch between the behavior of the DS and its reference model, available to the developer, and making a decision based on the results of the residual analysis.

II. SIGNIFICANCE OF THE SYSTEM

Most of the methods are oriented towards the use of linear [1-2] or bilinear [3] models of diagnosed systems. Well-known nonlinear methods are characterized either by restrictions on the existence of a solution [6-8], or do not allow to fully realize the potential of active methods of achieving robustness [7-9].

III. LITERATURE SURVEY

In [10], a method for synthesizing a residual generator in the form of nonlinear parity relations is proposed, which allows combining the advantages obtained from using a nonlinear DS model with the effect of using well-known optimization approaches to ensure robustness. The main disadvantage of the method is that its implementation can be associated with significant computational costs. In order to eliminate it, in the same work, a modification of the method is considered, which leads to obtaining a residual generator in the form of a bank of less accurate quasilinear parity relations.

IV. METHODOLOGY

Consider a nonlinear parametric DS model

$$x(t+1) = f^*(x(t), u(t), \zeta(t), p(t)), \qquad y(t) = h^*(x(t), \zeta(t), p(t)) \tag{1}$$

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where $x(t) \in X \subseteq R^{n^*}$ – the state vector, $u(t) \in U \subseteq R^m$ – the vector control $y(t) \in Y \subseteq R^l$ – output vector, $p(t) \in R^g$ – the vector of the model parameters, $\zeta(t)$ – vector that reflecting the effect of destabilizing factors, f^* *and* h^* – respectively, the nonlinear vector function dynamics and output arguments for differentiable, t = 0,1, 2, ... - discrete time. The defect-free operation of the system corresponds to the nominal value p^0 of the vector of model parameters.

We represent the vector $\zeta(t)$ as consisting of two subvectors $-\gamma(t)$ и $\vartheta(t)$: $\zeta(t) = col\left(\gamma(t)^T,\ \vartheta(t)^T\right)$. The vector $\vartheta(t) \in \Theta \subseteq R^\nu$ takes into account the disturbing influences on the dynamics from the system and the limited accuracy of measuring the control and output vectors. The vector is used to set indefinite , unknown in advance, constant or slowly changing in time , coefficients of the model. It is assumed that its dynamics is described by the equation

$$\vartheta(t+1) = \vartheta(t) + \upsilon(t), \tag{2}$$

where is the $\upsilon(t) \in V \subseteq R^\nu$ – unknown vector.

Model (1), supplemented by the equation (2), normally called s in ayut system parametric model with extended state vector $q(t) = col\left(x(t)^T,\ \vartheta(t)^T\right)$, or enhanced parametric model (PRM), which if $p(t) = p^0$, $\gamma(t) = 0$ and $\upsilon(t) = 0$ called nominal RPM. We write the equation of this model in the form

$$q(t+1) = f(q(t), u(t)),\quad y(t) = h(q(t)); \tag{3}$$

the dimension of the vector q is denoted by n, $n = n^* + \nu$, $q(t) \in Q \subseteq R^n$.

From [12] it follows that for the nominal model (3) there exists a vector of nonlinear parity relations of order k_0, containing exactly $s - n$, $s = \sum_{j=1}^{l}(k_j + 1)$, functionally independent components and satisfying the strict equality

$$P(t, k_0) = \begin{bmatrix} \phi_1(\underline{u}(t),\ \underline{y}(t)) \\ \dots\dots\dots\dots\dots \\ \phi_{s-n}(\underline{u}(t),\ \underline{y}(t)) \end{bmatrix} = 0; \tag{4}$$

functions $\phi_j(\underline{u}(t), \underline{y}(t))$, $j = 1,2,\dots, s-n$ are also defined in [10].

Let's introduce a vector of adaptive ratios of dimension parity $s - n$:

$$P_*(t, k_0) = \Omega\left(\underline{y}(t) - H^*\left(q^0(t), \underline{u}(t)\right)\right), \tag{5}$$

where is the Ω – matrix of the corresponding dimension, the $q^0(t)$ – so called representative value of the extended state vector, calculated at each moment of time t and which is an estimate of the value of this vector at the moment of $t-1$ time. The values $q^0(t)$ will also be used to determine the scope $\Delta(t) \subseteq Q$ of the current value of the extended state vector $q(t)$. Expression (5) differs from the traditional linear parity relations obtained in [1-2] both in the method of calculating the matrix Ω and in the nonlinear dependence on the control vector. The specific form of this relationship is set in each cycle t based on the values of the state vectors $x(t-1)$ and undefined coefficients $\vartheta(t-1)$, the estimated n as a representative basis $q^0(t)$ values.

In the vicinity of the current value of the state vector $q(t)$

$$\underline{y}(t) = H^*\left(q^0(t), \underline{u}(t)\right) = H^*\left(q(t), \underline{u}(t)\right) - H^*\left(q^0(t), \underline{u}(t)\right)$$

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can be expanded into a Taylor series. By neglecting the higher-order expansion terms, one can obtain an approximate equality following from the definition of the matrix $C_j(t,k)$:

$$\underline{y}(t)-H^*\left(q^0(t),\underline{u}(t)\right)=C(t,k_0)\left(q(t)-q^0(t)\right), \tag{6}$$

where the matrix

$$C(t,k_0)=\begin{bmatrix} C_1(t,k_1) \\ .... \\ C_l(t,k_l) \end{bmatrix}$$

is a function of vectors $q(t)$ and $\underline{u}(t)$, so we can write

$$P_*(t,k_0)=\Omega C(t,k_0)\left(q(t)-q^0(t)\right)$$

By analogy with relation (4), for quasilinear adaptive parity relations, the equality

$$P_*(t,k_0)=0 \tag{7}$$

for all possible $q(t)\in\Delta(t)$. In general, it is impossible to guarantee the existence of a matrix Ω that ensures fulfillment of this condition, but it can be so that condition (7) is satisfied in the best way in a certain sense for a large number of values of the extended state vector at time moment *t*.

Let $q^j(t),\ j=1,2,...,N,\ N\geq s-$ the set of representative values of the vectors $q(t)$ in $\Delta(t)$. We put

$$D^j(t,k_0)=C(t,k_0)\Big|_{q(t)=q^j(t)}\left(q^j(t)-q^0(t)\right),\quad P_*^j(t,k_0)=\Omega D^j(t,k_0).$$

We introduce the criterion

$$J_1=\sum_{j=1}^{N}\left\|P_*^j(t,k_0)\right\|_2^2,$$

where the symbol $\|\cdot\|_2$ denotes the Euclidean norm of the vector. The problem of finding the matrix is Ω formulated as the problem of minimizing the J_1 criterion. The known solution of the latter is reduced [2] to the singular value decomposition of the matrix

$$D^j(t,k_0)=\left[D^1(t,k_0):D^2(t,k_0):...:D^n(t,k_0)\right],$$

which can be done using the MATLAB package and has the form

$$D(t,k_0)=WGV^T, \tag{8}$$

where *W* and *V* are orthogonal matrices, and

$$W=\left[W^1_{s\times(s-n)}:W^2_{s\times n}\right],$$

$$G=\left[\sum_{s\times s}:0_{s\times(N-s)}\right],\quad \sum=\mathrm{diag}\left(\sigma_1,\sigma_2,...,\sigma_s\right)$$

and the singular values of the matrix $D(t,k_0)$ are ordered so that $\sigma_1\leq\sigma_2\leq...\leq\sigma_s$. Then the choice $\Omega^T=W^1$ provides the best approximation to the exact parity relations (7). The criterion value obtained in this case has the form

$$J_1=\sum_{i=1}^{s-n}\sigma_i^2$$

We assume that the approximate equality (6) holds in the surroundings $\Delta(t)$ is representative of the values $q^0(t)$. Let's replace the vector on the right-hand side of this $q(t)$ equality by its representative value $q^0(t+1)$, since, by definition, the latter is an estimate of this vector. Based on the least squares method [14], we obtain

$$q^0(t+1)=q^0(t)+C^+\left(\underline{y}(t)-H^*\left(q^0(t),u(t)\right)\right), \tag{9}$$

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where pseudo-inverse to $C(c,k_0)$ the Moore - Penrose matrix

$$C^{+}=\left[C(t,k_0)^T C(t,k_0)\right]^{-1} C(t,k_0)^T$$

calculated with the measured $\underline{u}(t)$ and substituted instead of $q(t)$ the vector $q^0(t)$ as the initial value $q^0(t)$ in real time.

Representative values $q^j(t),\ j=1,2,\dots,N$ are selected in the field $\Delta(t)$ on the basis $\Delta_i^{\min}$ and $\Delta_i^{\max}$, *i* = 1, 2,..., *n*. Within the meaning of $\Delta_i^{\min}$ and $\Delta_i^{\max}$, respectively, the minimum and maximum difference between the values of the *i*- th component of the RPM in adjacent measures are set [10,12]. Next, a vector interval $\Delta(t)=col(\Delta_1(t),\Delta_2(t),\dots,\Delta_n(t))$ with components

$$\Delta_i(t)=\left[q_i^0(t)+\Delta_i^{\min},\ q_i^0(t)+\Delta_i^{\max}\right],\quad i=1,2,..,n$$

and vectors are formed $\Delta^{\min}$ and $\Delta^{\max}$:

$$\Delta^{\min}=col\left(\Delta_1^{\min},\ \Delta_2^{\min},\dots,\Delta_n^{\min}\right),$$
$$\Delta^{\max}=col\left(\Delta_1^{\max},\ \Delta_2^{\max},\dots,\Delta_n^{\max}\right).$$

From the obtained vector interval [$\Delta^{\min}$, $\Delta^{\max}$], *N* different vectors $\delta^1\delta^1,..,\delta^N;\delta^j\in\left[\Delta^{\min},\Delta^{\max}\right],\ j=1,2,\dots,N$ are selected :

$$\delta^j=\Delta^{\min}+\frac{j-1}{N-1},\left(\Delta^{\max}-\Delta^{\min}\right),\ j=1,2,\dots,N. \tag{10}$$

After that, the representative vectors are found

$$q^j(t)=q^0(t)+\delta^j,\qquad j=1,2,\dots,N. \tag{11}$$

The construction of vectors $\delta^1,\delta^2,\dots,\delta^N$ is usually carried out in advance before carrying out the diagnostic process using relations (10). In contrast, the calculation of representative values $q^0(t)$ and $q^j(t),\ j=1,2,..,N$, the state vector RPM (3) is carried out directly in each cycle *t* on the basis of the relations (9) and (11) respectively, because of their dependence on the current values and control output vectors.

Expression (9) contains a pseudo inverse matrix C^+ to form a representative value $q^0(t+1)$. The quality of the control processes $q^0(t+1)$ of the synthesized control system depends on the accuracy of the determination. In view of this circumstance, it becomes necessary to use efficient algorithms for pseudo-inversion of overdetermined matrices.

It is known [15-17] that the problem of calculating a pseudo inverse matrix is generally unstable with respect to errors in specifying the original matrix. In this case, the errors of the initial data naturally depend on the accuracy of the experimental studies, and the characteristics of the calculated process depend on the degree of adequacy of the model to the real process. The influence of rounding errors made during the implementation of the computational procedure on the accuracy of the desired solution can be analyzed based on known methods of analysis and balance of accuracy.

In the case under consideration, it is advisable to use the Hermite method [18, 19], which is economical and has a sufficiently high accuracy. According to this method, the pseudo inverse matrix is determined based on the expression

$$C^{+}=C^T M_R^{-} C C^T,$$

Where

$$M=(CC^T)^2,$$

a M_R^- - reflective g -back (generalized inverse) of the matrix *M*.

One of the possible direct methods for constructing g - inverse matrices [18] uses the reduction of the $(m\times n)$ -matrix *N* by two-sided multiplication by non-degenerate matrices to the form:

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$$R = PNQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

Here $rank\ N$, and zero blocks have sizes $r \times (n-r)$, $(m-r) \times r$ and $(m-r) \times (n-r)$.

If the matrices P and Q in equality (12) are known, then any $(n \times m)$-matrix of $\hat{R}$ block structure

$$\hat{R} = \begin{bmatrix} I_r & U \\ V & W \end{bmatrix}$$

generates g - the inverse for the matrix N by the formula

$$G = Q\hat{R}P \qquad (13)$$

Thus, the considered method provides for a two-fold reduction to the normal lowercase form. In the Hermite algorithm, the matrix *M is* reduced to the form (13) as follows [16 -20]: first, *M is* reduced to the normal row form M_1, which can be described by the relation:

$$EM = M_1, \qquad (14)$$

where E is a non-degenerate matrix; then the adjoint matrix is M_1^T also reduced to the normal row form R by the non-degenerate matrix F :

$$FM_1^T = R \qquad (15)$$

In this case, the matrices M and R are Hermitian, and R is a diagonal matrix with r ones and others - zeroes on the main diagonal.

Using this circumstance, we can obtain from (14) and (15) the equality

$$R = EMF^T.$$

According to (14), this implies that

$$M_R^- = F^T RE$$

V. CONCLUSION AND FUTURE WORK

The presented stable algorithms for diagnosing nonlinear dynamical systems in the presence of destabilizing factors make it possible to carry out programmed motion according to the desired law of change in time of the state vector and ensure the convergence of the system motion in a certain neighborhood with respect to the reference motion.

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