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Classification of Second-Order Surfaces in the Galilean Space

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ABSTRACT: This article highlights the classification of second-order surfaces in the Galilean space, indicating surfaces that are different from the Euclidean space that we know. Second-order surfaces were divided into seventeen types. Distinguished surfaces are different from some surfaces in Euclidean space and form a separate class. Thus, they cannot be transform from one surface to another by changing.

KEYWORDS: Second-order surface; Galilean space; Galilean transformation; classification; parallel transformation.

I. INTRODUCTION

It is known that, second-order surfaces in the Euclidean space were classified and their types were identified.[1]

The formula for transformation in the Galilean space is as follows [2]:

$$\begin{cases} X = x + a \\ Y = h_1 x + \cos\alpha y - \sin\alpha z + b \\ Z = h_2 x + \sin\alpha y + \cos\alpha z + c \end{cases} \qquad (1)$$

The classification of second-order lines in Galilean space was studied by Makarova [3]. The following article presents invariants with respect to second-order lines in Galilean space [4]. In this paper, we classify second-order surfaces with respect to transformation (1) and classify them into types to show second-order surfaces.

II. PRELIMINARIES

Let two vectors $\vec{X}(x_1, y_1, z_1)$ and $\vec{Y}(x_2, y_2, z_2)$ be given in the affine space A_3.

Definition 1[2;5]. Galilean spaces are affine spaces in which the scalar product of the vectors $\vec{X}(x_1, y_1, z_1)$ and $\vec{Y}(x_2, y_2, z_2)$ is defined as follows:

$$\begin{cases} (XY)_1 = x_1 x_2; \quad if \quad x_1 \neq 0 \ or \ x_2 \neq 0 \\ (XY)_2 = y_1 y_2 + z_1 z_2; \ if \ x_1 = x_2 = 0. \end{cases}$$

In the Galilean space R_3^1, the norm of the vector is defined as the square root of the scalar product of the vector by itself, that is

$$\| \vec{X} \| = \begin{cases} |x_1|, \ npu \ x_1 \neq 0; \\ \sqrt{y_1^2 + z_1^2}, \ npu \ x_1 = 0. \end{cases}$$

In R_3^1, the distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ will be equal to the norm of the vector $\overrightarrow{AB}$

$$|\overrightarrow{AB}| = \begin{cases} |x_2 - x_1| \quad npu \quad x_1 \neq x_2; \\ \sqrt{(y_2 - y_1)^2 + (z_2 - z_1)^2} \quad npu \ x_1 = x_2. \end{cases}$$

In the Galilean space, equation of motion is defineed in the form of equation (1). In order to determine this, let's look at the motion in the Galilean plane.

The motion of the Galilean plane is a linear transformation[4]:

$$\begin{cases} x' = x + a \\ y' = hx + y + b \end{cases} \qquad 0 < h < +\infty$$

consisting of parallel transfer to the vector $\vec{a} = (a;b)$ and transformation matrix $A = \begin{pmatrix} 1 & 0 \\ h & 1 \end{pmatrix}$, where $DetA = 1$. The matrix A will be an element of the Heisenberg group [6]. When in the linear transformation $a = b = 0$, then

$$\begin{cases} x' = x \\ y' = hx + y. \end{cases}$$

If $x = x_0$ is straight parallel to the Oy axis, then the linear transformation will have the following form:

$$\begin{cases} x' = x_0 \\ y' = hx_0 + y \end{cases}$$

This means that $x' = x_0$ the straight line does not change, from the equality $y' = hx_0 + y$ it follows that the straight line slides at a distance hx_0 along the straight line itself.

III. MAIN RESULTS

Let a second-order surface equation be given:

$$a_{11}X^2 + a_{22}Y^2 + a_{33}Y^2 + 2a_{12}XY + 2a_{13}YZ + 2a_{23}YZ + 2a_1X + 2a_2Y + 2a_3Z + a = 0 \qquad (2)$$

To simplify the equation of a second-order surface, it is necessary to obtain the coordinate axes conveniently. We first bring the coordinate head to the center of the surface by performing the following transformation:

$$\begin{cases} X = x + a \\ Y = y + b \\ Z = z + c \end{cases}$$

In this case:

$$a_{11}x_1^2 + a_{22}y_1^2 + a_{33}z_1^2 + 2a_{12}x_1y_1 + 2a_{13}x_1z_1 + 2a_{23}y_1z_1 + a' = 0 \qquad (3)$$

There:

$$a' = \frac{\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_1 \\ a_{21} & a_{22} & a_{23} & a_2 \\ a_{32} & a_{32} & a_{33} & a_3 \\ a_1 & a_2 & a_3 & a \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{12} \\ a_{12} & a_{22} & a_{12} \\ a_{12} & a_{12} & a_{33} \end{vmatrix}}$$

We now perform the transformation in Equation (3).

$$\begin{cases} x_1 = x \\ y_1 = h_1x + \cos\alpha y - \sin\alpha z \\ z_1 = h_2x + \sin\alpha y + \cos\alpha z \end{cases} \quad (4)$$

$$a_{11}x^2 + a_{22}(h_1x + cos\alpha y - sin\alpha z)^2 + a_{33}(h_2x + sin\alpha y + cos\alpha z)^2 +$$
$$+2a_{12}x(h_1x + cos\alpha y - sin\alpha z) + 2a_{13}x(h_2x + sin\alpha y + cos\alpha z) +$$
$$+2a_{23}(h_1x + cos\alpha y - sin\alpha z)(h_2x + sin\alpha y + cos\alpha z) + a = 0$$
$$(a_{11} + a_{22}h_1 + a_{33}h_2 + 2a_{12}h_1 + 2a_{13}h_2 + 2a_{23}h_1h_2)x^2 +$$
$$+(a_{22}cos^2\alpha + a_{33}sin^2\alpha + 2a_{23}cos\alpha sin\alpha)y^2 +$$
$$+(a_{22}sin^2\alpha + a_{33}cos^2\alpha - 2a_{23}cos\alpha sin\alpha)z^2 +$$
$$+(2a_{22}h_1cos\alpha + 2a_{33}h_2sin\alpha + 2a_{12}cos\alpha + 2a_{13}sin\alpha + 2a_{23}h_1sin\alpha + 2a_{23}h_2cos\alpha)xy +$$
$$+(-2a_{22}h_1sin\alpha + 2a_{33}h_2cos\alpha - 2a_{12}sin\alpha + 2a_{13}cos\alpha + 2a_{23}h_1cos\alpha - 2a_{23}h_2sin\alpha)xz +$$
$$(-2a_{22}cos\alpha sin\alpha + 2a_{33}cos\alpha sin\alpha + 2a_{23}cos^2\alpha - 2a_{23}sin^2\alpha)yz.$$

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Now we set the coefficients in front of the terms xy, yz and xz to zero.

$$\begin{cases} (2\mathrm{a}_{22}\cos\alpha + 2\mathrm{a}_{23}\sin\alpha)\mathrm{h}_1 + (2\mathrm{a}_{33}\sin\alpha + 2\mathrm{a}_{23}\cos\alpha)\mathrm{h}_2 + 2\mathrm{a}_{12}\cos\alpha + 2\mathrm{a}_{13}\sin\alpha = 0 \\ (2\mathrm{a}_{23}\cos\alpha - 2\mathrm{a}_{22}\sin\alpha)\mathrm{h}_1 + (2\mathrm{a}_{33}\cos\alpha - 2\mathrm{a}_{23}\sin\alpha)\mathrm{h}_2 + 2\mathrm{a}_{13}\cos\alpha - 2\mathrm{a}_{12}\sin\alpha = 0 \\ -2\mathrm{a}_{22}\cos\alpha\sin\alpha + 2\mathrm{a}_{33}\cos\alpha\sin\alpha + 2\mathrm{a}_{23}\cos^2\alpha - 2\mathrm{a}_{23}\sin^2\alpha = 0 \end{cases}$$

If we find the unknowns $\mathrm{h}_1, \mathrm{h}_2$ and α in this system, we get the following:

$$\mathrm{h}_1 = \frac{\begin{vmatrix} \mathrm{a}_{13} & \mathrm{a}_{12} \\ \mathrm{a}_{33} & \mathrm{a}_{23} \end{vmatrix}}{\begin{vmatrix} \mathrm{a}_{22} & \mathrm{a}_{23} \\ \mathrm{a}_{23} & \mathrm{a}_{33} \end{vmatrix}}, \mathrm{h}_2 = \frac{\begin{vmatrix} \mathrm{a}_{12} & \mathrm{a}_{13} \\ \mathrm{a}_{22} & \mathrm{a}_{23} \end{vmatrix}}{\begin{vmatrix} \mathrm{a}_{22} & \mathrm{a}_{23} \\ \mathrm{a}_{23} & \mathrm{a}_{33} \end{vmatrix}}, \mathrm{tg}2\alpha = \frac{2\mathrm{a}_{23}}{\mathrm{a}_{22}-\mathrm{a}_{33}}.$$

If we simplify the findings by the above equation, we get the following form:

$$\frac{\begin{vmatrix} a_{11} & a_{12} & a_{12} \\ a_{12} & a_{22} & a_{12} \\ a_{12} & a_{12} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix}} x^2 + \lambda_1 y^2 + \lambda_2 z^2 + \frac{\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_1 \\ a_{21} & a_{22} & a_{23} & a_2 \\ a_{32} & a_{32} & a_{33} & a_3 \\ a_1 & a_2 & a_3 & a \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{12} \\ a_{12} & a_{22} & a_{12} \\ a_{12} & a_{12} & a_{33} \end{vmatrix}} = 0 \qquad (5)$$

Here$\lambda_1$and$\lambda_2$are the roots of the equation $\lambda^2 + (a_{22} + a_{33})\lambda + \begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix} = 0$

We can indicate the following definitions:$I_1 = a_{22} + a_{33}$, $I_2 = \begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix}$,

$I_3 = \begin{vmatrix} a_{11} & a_{12} & a_{12} \\ a_{12} & a_{22} & a_{12} \\ a_{12} & a_{12} & a_{33} \end{vmatrix}$ $I_4 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_1 \\ a_{21} & a_{22} & a_{23} & a_2 \\ a_{32} & a_{32} & a_{33} & a_3 \\ a_1 & a_2 & a_3 & a \end{vmatrix}$.

What geometric position Equation (5) represents depends on the signs of $I_1, I_2, I_3, I_4, \lambda_1, \lambda_2$.

1)If the indicators of $I_3, \lambda_1, \lambda_2$ are the same and if$I_4 < 0$, Equation (5) represents the real ellipsoid.

2) If the indicators of$I_3, \lambda_1, \lambda_2$are the same and if$I_4 > 0$, Equation (5) representsthe abstract ellipsoid.

3)) If the indicators of $I_3, \lambda_1, \lambda_2$ are the same and$I_4 = 0$, it represents the absurd cone. 4) If the indicators of $\lambda_1, \lambda_2$are the same and I_3has an opposite indicator to them, and if $I_4 = 0$, Equation (5) represents the first type of cone.

5) If the indicators of $\lambda_1, \lambda_2$are opposite and$I_3 \neq 0$, if $I_4 = 0$, Equation (5) represents the second type of cone.

6) If the indicators of $\lambda_1, \lambda_2$are the same and$I_3$is opposite to them, and if$I_4 > 0$, Equation (5) represents the first type of single-phase hyperboloid.

7) If the indicators of λ_1, λ_2 are opposite and$I_3 \neq 0$, and $I_4 < 0$, Equation (5) represents the econd type of single-phase hyperboloid.

8) If the indicators of $\lambda_1, \lambda_2$are the same and $I_3$is opposite to them, and if $I_4 < 0$, then Equation (5) represents the first type of two-phase hyperboloid.

9) If the indicators of $\lambda_1, \lambda_2$are opposite and$I_3 \neq 0$, and if $I_4 > 0$,then Equation (5) represents the second type of two-phase hyperboloid.

Let the equation of the second-order surface of the form (1) be given. Let's assume the same:

$$I_3 = 0$$

In this case, by rotating and sliding the coordinate head without changing it, we can bring Equation (1) to the following form:

$$\lambda_1 X^2 + \lambda_2 Z^2 + 2a_1' X + 2a_2' Y + 2a_3' Z + a \qquad (6)$$

Then, keeping the direction of the axes, we bring the origin to a point (a, b, c). Let the following:

$$\begin{cases} X = x + a \\ Y = y + b \\ Z = z + c \end{cases}$$

If we define the left-hand side of equation (1) by $f(x, y, z)$, then (6) looks like this:

$$\lambda_1 y^2 + \lambda_2 z^2 + 2x f_x(a, b, c) + 2y f_y(a, b, c) + 2z f_z(a, b, c) + f(a, b, c) = 0 (7)$$

Here, $f_x, f_y, f_z$are private derivatives.

If we choose the coordination heads(a, b, c)points as follows:

$$f_y(a, b, c) = 0, f_z(a, b, c) = 0, f(a, b, c) = 0.$$

Then the equation (1) is supposed to be as follows:

$$\lambda_1 y^2 + \lambda_2 z^2 + 2x f_x(a, b, c) = 0$$

or

$$f_x(a, b, c) = C$$

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The result will be

$$\lambda_1 y^2 + \lambda_2 z^2 + 2Cx = 0 \qquad (8)$$

1) If the indicators of λ_1, $\lambda_2$are the same, equation (8) representsthe first type of elliptical paraboloid.
2) If the indicators of λ_1, $\lambda_2$are opposite, equation (8) represents the first type of paraboloid.

If$f_x(a,b,c) = 0$, equation (7) will be:

$$\lambda_1 y^2 + \lambda_2 z^2 + 2y f_y(a,b,c) + 2z f_z(a,b,c) + f(a,b,c) = 0 \qquad (9)$$

or in short:

$$\varphi(y,z) = 0.$$

It is clear that the constructor of such an equation represents a cylinder parallel to the axis OX. If the indicators of λ_1, and $\lambda_2$are the same, equation (9) represents the first type of elliptical (real or abstract) cylinder; if the indicators of λ_1, and λ_2 are different, equation (9) represents the first type of hyperbolic cylinder. If one of λ_1, and λ_2 are equal to zero, equation (9) represents the first type of parabolic cylinder. In special cases, equation (9) represents two intersecting or parallel planes. Since this equation (9) is an expression for y and z, and since the geometry of the OYZ plane is the same as the Euclidean geometry, we did not dwell on its simplification.

Let us be given a second-order surface equation of the following form:

$$a_{11}X^2 + a_{22}Y^2 + a_{33}Z^2 + 2a_{23}YZ + 2a_1X + 2a_2Y + 2a_3Z + a = 0 \qquad (10)$$

If in the equation (10), $a_{11} \neq 0$is equal to$I_2 = 0$, equation (10) can be shaped as follows:

$$a_{11}X^2 + NZ^2 + 2a_1Z + 2a_2'Y + 2a_3'Z + a = 0 \qquad (11)$$

Here,$N = \frac{a_{22}^2 + a_{23}^2}{a_{22}}$.

Now, keeping the direction of the axes, we bring the head of coordinations to a point (a,b,c), that is

$$\begin{cases} X = x + a \\ Y = y + b \\ Z = z + c \end{cases}$$

If we define the left side of equation (10) as $f(x,y,z)$, equation (11) will be in the folowing form:

$$a_{11}x^2 + Nz^2 + 2x f_x(a,b,c) + 2y f_y(a,b,c) + 2z f_z(a,b,c) + f(a,b,c) = 0 \qquad (12)$$

If we define a point (a,b,c) that is a head of coordination as follows:

$$f_x(a,b,c) = 0,\ f_z(a,b,c) = 0,\ f(a,b,c) = 0$$

In this case,if we suppose equation (10):

$$a_{11}x^2 + Nz^2 + 2y f_y(a,b,c) = 0$$

or

$$f_y(a,b,c) = C,$$

it will be in the form of

$$a_{11}x^2 + Nz^2 + 2Cy = 0 \qquad (13)$$

1) If the indicators of$a_{11}$and$N$are the same, equation (13) represents the second type of elliptical paraboloid.
2) If the indicators of a_{11} and N are opposite, equation (13) represents the second type of hyperbolic paraboloid.

If $f_y(a,b,c) = 0$, equation (12) will be in the following form:

$$a_{11}x^2 + Nz^2 + 2x f_x(a,b,c) + 2z f_z(a,b,c) + f(a,b,c) = 0 \qquad (14)$$

or in short:.

$$\varphi(x,z) = 0.$$

It is known that the constructor of such an equation represents a cylinder parallel to the OY axis. We can write the expression (14) as follows.

$$a_{11}x^2 + Nz^2 + \Delta = 0 \qquad (15)$$

Here, it is satisfactory$\Delta = \frac{\begin{vmatrix} a_{11} & 0 & f_x(a,b,c) \\ 0 & N & f_z(a,b,c) \\ f_x(a,b,c) & f_z(a,b,c) & f(a,b,c) \end{vmatrix}}{a_{11}N}$.

3) If the indicators of $a_{11}$and$N$are the same and$\Delta$is opposite to them, equation (15) represents the second type of elliptical cylinder.
4) If the indicators of$a_{11}$and$N$are opposite, equation (15) represents a hyperbolic cylinder.
5) If$N = 0$ in the equation (14), then equation (14) represents a parabolic cylinder.

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6) If equation (14) is an expression that only depends on x, that is $\varphi(x) = 0$, then equation (14) represents two (real or abstract) special planes.

7)

IV. .CONCLUSION

To sup up, the second-order surfaces in Galileo's space have different surfaces than the second-order surfaces in Euclidean space. The second-order surfaces separated by the following transformation (1) appear to be a separate class from each other. These separated surfaces are not transferred and can be separated. The internal geometry of the above surfaces is different.

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