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Piecewise-Polynomial HAAR Wavelets And Their Application

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ABSTRACT: This article presents piecewise polynomial Haar wavelets and shows that they can be used to interpolate functions and signals. An algorithm for interpolating functions in the wavelet bases on piecewise constant and piecewise linear Haar functions developing, and the results of estimating interpolation errors are also presented. It is shown that when interpolating using piecewise constant Haar wavelets, the error will increase sharply, to reduce the error, it is recommended to use piecewise linear Haar wavelets and as a result, an error reduction is achieved.

KEYWORDS: wavelet transform, interpolation, interpolation errors, Haar wavelet, piecewise constant wavelet, piecewise linear wavelet, Orthogonal wavelet, Digital processing, scalar product, Fourier transforms, B-spline, absolute estimates, relative error.

I. INTRODUCTION

The state and prospects development of information technologies in the 21st century are characterized by the wide practical use of digital signal processing technology, one of the most dynamic and fastest growing technologies in the world of telecommunications and information technology in society. Digital signal processing is, in fact, real-time computer science, designed to solve the problems of receiving, processing, reducing redundancy and transmitting information at a set speed[2].

The basic principles theory of digital signal processing were laid and tested on the theory that discrete systems and unitary transformations. To build models of signals received from real objects, traditional harmonic functions are widely used. This is due to the fact that many signals received from real objects can be easily represented by a combination of sinusoidal and cosine oscillations, for which the Fourier analysis apparatus are used. The result of this transition from time to frequency functions[5]. However, the representation of a temporal function by sinusoidal and cosine functions is only one of many representations. Any complete system of orthogonal functions can be applied for expansion into series that correspond to Fourier series. Algorithms for fast Fourier transforms were developed, and a theory of binary-orthogonal transformations with local and integral bases were created. The task developers of algorithms and devices for digital signal processing was to minimize computational and hardware costs with limited memory resources and permissible calculation errors.

II. FORMULATION OF THE PROBLEM

There are algorithms for fast conversion of Haar wavelets and they are widely used in solving practical problems.
Orthogonal Haar wavelet is determined by the following formula [4]:
Haar wavelets attract the attention of specialists for two reasons:

$$har_k(x) = har_{pj}(x) = \begin{cases} +1 & x \in \overset{-}{h}_{pj} \\ -1 & x \in \overset{+}{h}_{pj} \\ 0 & x \in h_{pj} \end{cases}$$

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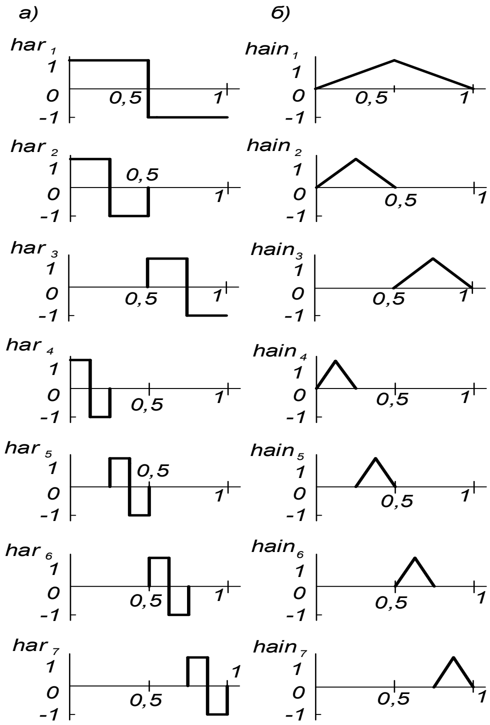
1. Reducing the number of coefficients necessary for approximation (with a given accuracy) with respect to the total number of binary segments.

2. The lack of “long” operations for calculating the coefficients. Only operations of addition, subtraction and shift are used.

The drawback of Haar's rectangular orthogonal bases the weak convergence of the series in piecewise constant functions, i.e. need to memorize several hundred coefficients for many functions in order to ensure errors of the order 0.1%.

The search for methods to reduce the volume of coefficient tables and improve the “smoothness” indicators obviously leads to systems piecewise polynomial basis functions of a higher degree. Most simply, piecewise linear basis functions are obtained as a result of integration with a variable upper limit of the orthogonal piecewise constant Haar functions[5].



1.1-picture a) piecewise constant, b) Haar piecewise linear wavelets.

On 1 A.-fig. The graphs of piecewise constant Haar wavelets, and 1.b-fig. piecewise linear Haar wavelet charts[8].
The wavelet signal conversion process is based on two types of functions: the shift function and the scale function, i.e. they are based on the same mother wavelet - mixing in time according to the signal and is built with a change in the time scale:

$$\psi_{ab}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right), \quad (a,b) \in R, \ \psi(t) \in L^2(R)$$

In problems of digital signal processing, to reveal details and local properties of signals, they are used from the wavelet function, and scaling functions are used to approximate signals. When choosing wavelet functions, attention to the smoothness index, transfer dimensions, and zero values.

We denote V^0 by the set of constant functions on the segment $[0,1]$, i.e. sets to the linear vectors.

In this case, the scale function V^0 belongs to the following set:

$$\phi(t) = \phi_{0,0}(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & or \end{cases} \qquad (1)$$

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when $i=0$ it is a large-scale function

The set V^1 on the intervals $\left[0,\frac{1}{2}\right]$ and $\left[\frac{1}{2},1\right]$ is the set of constant functions and it forms linear vectors. Scaling functions V^1 belong to the set and refers to the quality of wavelet functions:

$$\phi_{1,0}(t)=\phi(2t)=\begin{cases}1, & 0\le t<\frac{1}{2} \\ 0, & or\end{cases} \text{ и}$$

$$\phi_{1,1}(t)=\phi(2t-1)=\begin{cases}1, & \frac{1}{2}\le t<1 \\ 0, & or\end{cases} \quad (2)$$

and when $i=1$ is a scaling function

This function is at intervals $[0,1]$, $\left[0,\frac{1}{2}\right]$ и $\left[\frac{1}{2},1\right]$ is constant. For this, each element of the set V^0 belongs to set V^1, i.e. has the ratio $V^0\subset V^1$. The set V^2 can also be defined. The set V^2 is a lot of functions at intervals $\left[0,\frac{1}{4}\right]$, $\left[\frac{1}{4},\frac{1}{2}\right]$, $\left[\frac{1}{2},\frac{3}{4}\right]$, $\left[\frac{3}{4},1\right]$. Likewise the set V^n is the set of scale functions i.e.

$$\phi_{n,j}(t)=\phi(2^n t-j),\ j=0,1,...,2^n-1$$

$$0\le 2^n t-j<1,\ \frac{j}{2^n}\le t<\frac{j+1}{2^n}$$

$$\phi_{n,j}(t)=\begin{cases}1, & \frac{j}{2^n}\le t<\frac{j+1}{2^n} \\ 0, & otherwise\end{cases},\quad j=0,1,...2^n-1 \quad (3)$$

$$V^0\subset V^1\subset...\subset V^n\subset...$$

when $i=n$ is a scaling function, where $0\le 2^n t-j<1,\ \frac{j}{2^n}\le t<\frac{j+1}{2^n}$ are intervals change functions of scaling, funtions $\phi_{n,j}(t)$ belong to the sets of scaling functions V^n , here there are many scalar products of vectors, meaning it is a Euclidean space[1]. In this case, as a scalar product we take them in the form

$$(f,g)=\int_0^1 f(t)g(t)\,dt \quad (4)$$

Using this formula, the coefficients of the scaling function are calculated: C_n.

Then $\phi_{n,j}(t)=\sqrt{2^n}\phi(2^n t-j),\ j=0,1,...,2^n-1$ using formulas (3) and (4) we define the Haar wavelet coefficients: $C_n=\int_0^1 \phi_n(x)f(x)\,dx$ (5)

The formula for determining the Haar wavelet coefficients[7].

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$$f(x) \cong \sum_{n=0}^{\infty} C_n \phi_n(x) \quad (6)$$

where $f(x)$ interpolated function on piecewise constant wavelets.

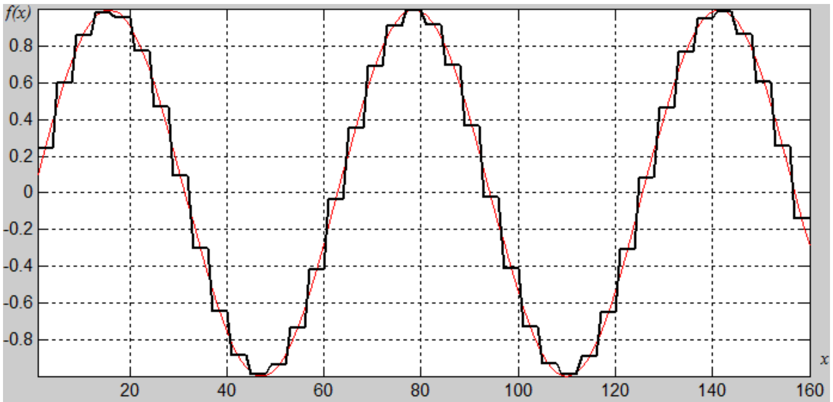

Fig. 2. Results of function interpolation $f(x) = \sin x$ in the bases of piecewise constant Haar wavelets

When solving many problems, the possibilities of piecewise constant wavelets are insufficient, this is explained by their weak approximation properties (weak smoothness, large error). Therefore, it is advisable to switch to piecewise linear wavelets. analysis of the existing literature shows that the method for determining the coefficients of piecewise linear Haar wavelets does not exist. Because this type of wavelets are not widely used[10].

III.ALGORITHM FOR CALCULATING COEFFICIENTS IN PIECEWISE LINEAR HAAR BASES

Lets consider the approximating series in piecewise linear Haar functions[5, 6, 9, 10]:

$$f(x) \cong \sum_{k=0}^{n-1} C_k \, hain_k(x) \quad (7)$$

The disadvantage of this series is the lack of a quick coefficient calculation algorithm. This disadvantage can be eliminated by applying a parabolic spline. If we take the second derivative of a parabolic spline interpolating on $[0,1]$ functions $f(x)$, then it will be a piecewise constant function with changes in the values of the steps at the nodes of the spline, and a series in piecewise constant orthogonal basis functions. We write, for example, the expansion of the derivative spline in a Haar series:

$$S_2''(x) \cong \sum_{k=0}^{n-1} C_k \, har_k(x)$$

In accordance with the theorems on bounded convergence and on the integration of closed systems as a result of integration both parts, we obtain:

$$S_2'(x) = 2^p \int_0^x S_2''(u)\,du = \sum_{k=0}^{n-1} C_k \, hain_k(x) + S_2'(0) \, (8)$$

when it follows that the coefficients of series expansion in the Haar orthogonal functions the second derivative of a parabolic spline interpolating the function in binary rational nodes are the coefficients of the expansion of the first derivative of the spline with respect to $har(x)$ -function ,the main spline *is (x)* - functions. The coefficient linear part of the expansion is determined as the value of the first derivative spline S2 (x) at x = 0, and the constant component as the value of the spline at this point[3, 8, 9, 10].

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If we take as an approximating function a parabolic B-spline, then the function $y = f(x)$ can be expressed through B-splines in the following form:

$$f(x) \cong b_{-1}B_{-1}(x) + b_0 B_0(x) + b_1 B_1(x) \quad (9)$$

where $f(x)$ - interpolated function, Bi (x) -parabolic basis spline, b_i -coefficients are determined by the formula:

$$b_i = \frac{1}{8}\left(-f_{i-1} + 10 f_i - f_{i+1}\right) \quad (10)$$

or a first-order derivative of a function is determined through a first-order derivative of a B-spline function in this way:

$$f'(x) \cong b_{-1}B'_{-1}(x) + b_0 B'_0(x) + b_1 B'_1(x) \quad (9.1)$$

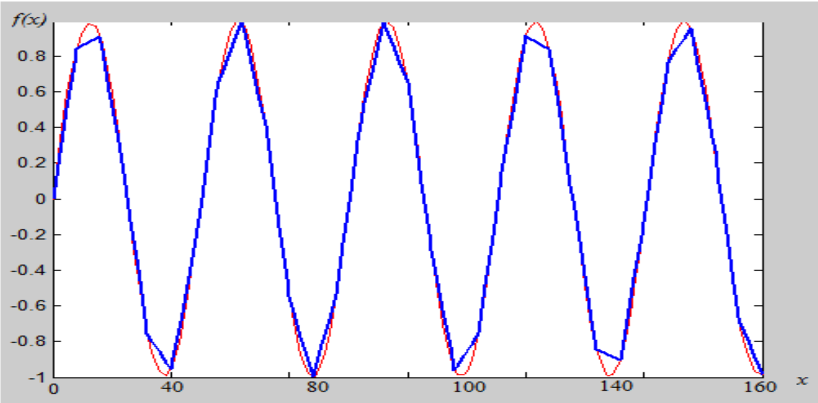

Fig. 3. Function Interpolation Results $f(x) = \sin x$ with piecewise linear wavelets of Haar.

We give estimates of the interpolation uncertainties in the bases of piecewise constant and piecewise linear wavelets.

Let a continuous function be given $f(x)$ on the segment of $[a,b]$. Segment $[a,b]$ we divide into node points

$$a \le x_0 < x_1 < ... < x_i < ... < x_n \le b .$$

$$h = x_{i+1} - x_i = const \quad (11)$$

where h - distance between points of nodes.

IV. ESTIMATION OF INTERPOLATION ERRORS IN PIECEWISE-CONSTANT AND PIECEWISE-LINEAR BASES

For varying degrees of polynomials, an expression for the methodological error of interpolation is known. For example, for polynomials of degree zero (piecewise constant functions), the formula for estimating the error is determined by[1,2, 3, 5, 10]:

$$|P(x) - f(x)| \le \frac{1}{2}\max\left|f'(x)\right|h$$

For polynomials of the first degree (piecewise linear functions) the error estimation formula is:

$$|P(x) - f(x)| \le \frac{1}{8}\max\left|f''(x)\right|h$$

We give estimates of the absolute and relative error of interpolation for piecewise constant wavelets:

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$$\Delta_1 = \max_{a \le x \le b} |f(x_i) - har(x_i)| = 0.08013$$

$$\delta_1 = \frac{|f(x_i) - har(x_i)|}{f(x_i)} = 0.08012 \quad (13)$$

Δ_1 - absolute error of piecewise constant wavelets;

δ_1 - relative error of piecewise constant wavelets.

Here are estimates absolute and relative error of interpolation for piecewise linear wavelets:

$$\Delta_2 = \max_{a \le x \le b} |f(x_i) - hain(x_i)| = 0.00791$$

$$\delta_2 = \frac{|f(x_i) - hain(x_i)|}{f(x_i)} = 0.00783 \quad (14)$$

Δ_2 - absolute error of piecewise constant wavelets;

δ_2 - relative error of piecewise constant wavelets.

V. CONCLUSION

Thus, piecewise constant and piecewise linear Haar wavelets are widely used in digital processing tasks. However, the disadvantage of piecewise constant Haar wavelet bases on weak convergence of the series in piecewise constant functions, i.e. need to memorize several hundred coefficients for many functions in order to ensure errors of the order 0.1%. The search for methods to reduce the volume of coefficient tables and improve the “smoothness” indicators obviously leads to systems of piecewise polynomial basis functions of a higher degree. The transition to piecewise linear wavelet bases on Haar makes it possible to reduce the interpolation error by 10.2 times for the yyy function.

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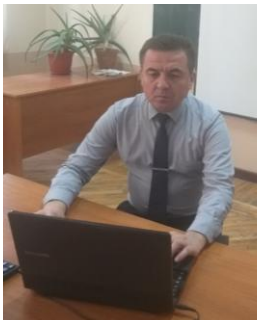

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