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Determination of the Value of Parameter μ of the Model $X = \mu + \varepsilon$ by GHM

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ABSTRACT: In continuation to the study on formulation of arithmetic–geometric mean (abbreviated as *AGM*) by Gauss and of arithmetic-harmonic mean (abbreviated as AHM), which have recently been found to be applicable in evaluating the value of parameter from observed data containing the parameter itself and random error, an attempt has here been made on formulating of another formulation of average termed as geometric–harmonic mean (abbreviated as *GHM*) with an attempt to derive that this formulation can be a suitable one for determining the value of parameter from observed data containing itself and random error. This paper describes the formulation of *GHM* and the justification of its suitability for evaluating the value of the parameter μ of the model

$$X = \mu + \varepsilon$$

by GHM along with some numerical applications.

KEYWORDS: GHM, numerical data, parameter, random error, determination of parameter.

I. INTRODUCTION

There had been lot of researches on the construction of tables of random numbers by reputed researchers like *Tippett* Several research have already been done on developing definitions of average [1 , 2], a basic concept used in developing most of the measures used in analysis of data. Pythagoras [3], the pioneer of researchers in this area, constructed three definitions / formulations of average namely Arithmetic Mean, Geometric Mean & Harmonic Mean which are called Pythagorean means [4 , 5 , 14 , 18]. A lot of definitions / formulations have already been developed among which some are arithmetic mean. geometric mean, harmonic mean, quadratic mean, cubic mean, square root mean, cube root mean, general p mean and many others [6 − 19]. Kolmogorov [20] formulated one generalized definition of average namely Generalized f - Mean. [7 , 8]. It has been shown that the definitions/formulations of the existing means and also of some new means can be derived from this Generalized f - Mean [9 , 10]. In an study, Chakrabarty formulated one generalized definition of average namely Generalized f_H – Mean [11]. In another study, Chakrabarty formulated another generalized definition of average namely Generalized f_G – Mean [12 , 13] and developed one general method of defining average [15 – 17] as well as the different formulations of average from the first principles [19].

In many real situations, observed numerical data

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

are found to be composed of a single parameter μ and corresponding chance / random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_N$$

i.e. the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , N$$

[21 – 29].

The existing methods of estimation of the parameter μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [31 – 52] cannot provide appropriate value of the parameter μ [21 – 23]. In some recent studies, some methods have been developed for determining the value of parameter from observed data containing the parameter itself and random error [21 – 30 , 53 – 60]. In continuation to the study on formulation of average starting from Pythagorean means, Gauss developed one formulation of average from the definitions of arithmetic mean and geometric mean. This definition later on was termed as arithmetic–geometric mean (abbreviated as *AGM*) [61 – 62]. Recently, this formulation of average (namely *AGM*) has been applied in evaluating the value of parameter from observed data containing the parameter itself and random error [63 – 64].

In continuation to the study on formulation of arithmetic–geometric mean (abbreviated as *AGM*) by Gauss and arithmetic-harmonic mean (abbreviated as AHM), which have recently been found to be applicable in evaluating the value of parameter from observed data containing the parameter itself and random error [65 , 66], an attempt has here been made on formulating of one measure of average termed as geometric–harmonic mean (abbreviated as *GHM*) with an attempt to derive that this formulation can be a technique of determining the value of parameter from observed data containing itself and random error. This paper describes the formulation of *GHM* and the derivation of the technique along with numerical application.

II. GEOMETRIC-HARMONIC MEAN (*GHM*)

Let g_0 & h_0 be respectively the *GM* (Geometric Mean) & the *HM* (Harmonic Mean) of the N numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

From the inequality of Pythagorean means [4 , 5] namely

$$AM > GM > HM$$

(where *AM* means Arithmetic Mean),
it follows that

$$g_0 > h_0$$

provided $x_1 , x_2 , \ldots\ldots\ldots , x_N$ are positive and not all equal.

Let $\{g''_n\}$ & $\{h''_n\}$ be two sequences defined respectively by

$$g''_{n+1} = (g''_n \,.\, h''_n)^{1/2}$$

$$\& \quad h''_{n+1} = \{½({g''_n}^{-1} + {h''_n}^{-1})\}^{-1}$$

where the square root takes the principal value.

From the Pythagorean inequality mentioned above, one can conclude that

$$h''_n < g''_n$$

and thus $g''_{n+1} = (h''_n \,.\, g''_n)^{1/2} < (g''_n \,.\, g''_n)^{1/2} = g''_n$

i.e. $g''_{n+1} < g''_n$

This means that the sequence $\{ g''_n \}$ is non-increasing.

Moreover, the sequence $\{ g''_n \}$ is bounded below by the smallest of

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

(which follows from the fact that both the geometric mean and the harmonic mean of these numbers lie between the smallest and the largest of them).

Therefore, by monotone convergence theorem [67 , 68], there exists a finite number M_{GH} such that

g''_n converges to M_{GH} as n approaches infinity.

Again, h''_n can be expressed as

$$h''_n = g''_{n+1})^2 / g''_n$$

This implies that the limiting value of h''_n as n approaches infinity is M_{GH}.

Therefore,

h''_n converges to M_{GH} as n approaches infinity.

Thus, the two sequences $\{g''_n\}$ & $\{h''_n\}$ converge to the same point M_{GH} as n approaches infinity.

This common converging point M_{GH} can be termed / named / regarded as the Geometric-Harmonic Mean (abbreviated as *GHM*) of the N numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

Accordingly, *GHM* can be defined as follows:

If g_0 & h_0 are respectively the *GM* & the *HM* of n numbers (or values or observations) viz.

$$x_1 , x_2 , \ldots\ldots , x_n$$

Then the two sequences $\{g^{//}_n\}$ & $\{h^{//}_n\}$ defined respectively by

$$g^{//}_{n+1} = (g^{//}_n . h^{//}_n)^{1/2}$$

$$\& \quad h^{//}_{n+1} = \{½(g^{//\,-1}_n + h^{//\,-1}_n)\}^{-1}$$

where the square root takes the principal value, converge to a common limit M_{GH} which can be termed as the Geometric-Harmonic Mean (abbreviated by *GHM*) of

$$x_1 , x_2 , \ldots\ldots , x_n$$

and is denoted here by $GHM(x_1 , x_2 , \ldots\ldots , x_n)$ i.e.

$GHM(x_1 , x_2 , \ldots\ldots , x_n) = M_{GH}$

III. *GHM* AS A TECHNIQUE OF EVALUATION OF μ

If the observations

$$x_1 , x_2 , \ldots\ldots , x_N$$

are composed of some parameter μ and random errors then the observations can be expressed as

$$x_i = \mu\, e_i \quad , \quad (i = 1 , 2 , \ldots\ldots , N)$$

where

$$e_1 , e_2 , \ldots\ldots , e_N$$

are the random errors, which assume positive and negative values in random order, associated to

$$x_1 , x_2 , \ldots\ldots , x_N$$

respectively.

In this case,

$$G(x_1 , x_2 , \ldots\ldots , x_N) \rightarrow \mu \text{ as } N \rightarrow \infty$$

where $G(x_1 , x_2 , \ldots\ldots , x_N) = \left(\prod_{i=1}^{N} x_i \right)^{1/N}$

Again since the observations

$$x_1 , x_2 , \ldots\ldots , x_N$$

consist of μ and random errors,

therefore, the reciprocals

$$x_1^{-1} , x_2^{-1} , \ldots\ldots , x_N^{-1}$$

are composed of μ^{-1} and random errors different from the respective random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

provided $x_1 , x_2 , \ldots\ldots , x_N$ are all different from zero.

In this case thus

$$x_i^{-1} = \mu^{-1} + \varepsilon_i^{/} \quad , \quad (i = 1 , 2 , \ldots\ldots , N)$$

where

$$\varepsilon_1^{/} , \varepsilon_2^{/} , \ldots\ldots , \varepsilon_N^{/}$$

are the random errors, which assume positive and negative values in random order, associated to are the random errors associated to

$$x_1^{-1} , x_2^{-1} , \ldots\ldots , x_N^{-1}$$

respectively..

In this case,

$$H(x_1 , x_2 , \ldots\ldots , x_N) \rightarrow \mu \text{ as } N \rightarrow \infty$$

where $H(x_1 , x_2 , \ldots\ldots\ldots , x_N) = (\frac{1}{N} \sum_{i=1}^{N} x_{\mathrm{i}}{}^{-1})^{-1}$

This implies that the common converging value of

$$G(x_1 , x_2 , \ldots\ldots\ldots , x_N) \ \ \& \ \ H(x_1 , x_2 , \ldots\ldots\ldots , x_N)$$

is the value of μ.

It is to be noted that the converging value may not be possible to be obtained for a finite set of observed values namely

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

In order to obtain the value of μ, in this case, let us write

$$G(x_1 , x_2 , \ldots\ldots\ldots , x_N) = G_0$$

$$\& \ \ H(x_1 , x_2 , \ldots\ldots\ldots , x_N) = H_0$$

and then define the two interdependent sequences $\{G_n\}$ & $\{H_n\}$ as

$$G_{n+1} = ½\,(G_n . H_n)^{1/2}$$

$$\& \ \ H_{n+1} = \{½\,(G_n{}^{-1} + H_n{}^{-1})\}^{-1}$$

Then, both of G_n & H_n converges to some real number C as n approaches infinity.

Now, it is required to verify whether this C is equal to μ.

From the model it is obtained that

$$G_0 = \mu + \delta_0 \ \ \& \ \ H_0 = \mu + e_0$$

Pythagorean inequality implies that $G_0 > H_0$ i.e. $\delta_0 > e_0$

Thus $G_1 = \mu + \delta_1$ where $\delta_1 = ½\,(\delta_0 + e_0) < \delta_0$

In general, corresponding to G_{n+1}, it holds that

$$\delta_{n+1} = ½\,(\delta_n + e_n) < \delta_n$$

This implies, δ_n converges to 0 i.e. G_n converges to μ.

By the existence of GHM, H_n also converges to μ.

Thus, the *GHM* of

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

is the value of μ. .

IV. NUMERICAL EXAMPLE: APPLICATION TO NUMERICAL DATA

Observed data considered here are the data on each of annual maximum & annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013. The objective here is to evaluate the central tendency of each of annual maximum & annual minimum of surface air temperature at Guwahati

A. Annual Maximum of Surface Air Temperature at Guwahati

Observed data considered here are the data on each of annual maximum & annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013. The objective here is to evaluate the central tendency of each of annual maximum & annual minimum of surface air temperature at Guwahati

B. Annual Maximum of Surface Air Temperature at Guwahati

From the observed data on annual maximum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013 [63 , 64], the *GM* & the *HM* have been found to be
37.192287148576076781925812747586 & 37.175398903562627634836294491501
respectively.

Here the observed values can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and random errors.

Evaluation of Value of μ (the central tendency of annual maximum)
Let us write
G_0= 37.192287148576076781925812747586 & H_0 = 37.175398903562627634836294491501
In this case the iterations give the values which are given in the following table (**Table – 1**):

Table – 1

n	G_n	H_n
0	$\underline{37.1}92287148576076781925812747586$	37.175398903562627634836294491501
1	$\underline{37.183}842067276499922160771566921$	$\underline{37.183}841108483672358606539675987$
2	$\underline{37.18384158788008}3050050439193677$	$\underline{37.1838415878800}79959717222765898$
3	$\underline{37.1838415878800815048838309797}87$	$\underline{37.1838415878800815048838309797}84$
4	$\underline{37.183841587880081504883830979786}$	$\underline{37.183841587880081504883830979786}$

The digits in G_n and H_n, which are agreed, have been underlined in the above table.

The *GHM* of

37.192287148576076781925812747586 & 37.175398903562627634836294491501

is the common limit of these two sequences which is 37.183841587880081504883830979786 .

Thus the value of μ, the central tendency of annual maximum of surface air temperature at Guwahati, obtained by *GHM*, is 37.183841587880081504883830979786 Degree Celsius.

C. Annual Minima of Surface Air Temperature at Guwahati

From the observed data on annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013 [63 , 64], the *GM* & the *HM* have been found to be
7.2597176194576185608709616351297 & 7.1543933802823525209849744707569
respectively.
In this case also, the observed values can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and random errors.

Determination of Value of μ (the central tendency of annual minimum)
In this case the iterations give the values which are given in the following table (**Table – 2**):

Table – 2

n	G_n	H_n
0	$\underline{7}.2597176194576185608709616351297$	$\underline{7}.1543933802823525209849744707569$
1	$\underline{7.206}8630956447857997320179691161$	$\underline{7.206}6706965561339698683103736099$
2	$\underline{7.20676689}54583999665077195028225$	$\underline{7.20676689}48163400482724767591701$
3	$\underline{7.2067668951373700073}829478903856$	$\underline{7.2067668951373700073}757976497748$
4	$\underline{7.2067668951373700073793727700802}$	$\underline{7.2067668951373700073793727700802}$

The digits in A_n and H_n, which are agreed, have been underlined in the above table.

The *GHM* of

7.2597176194576185608709616351297 & 7.1543933802823525209849744707569

is the common limit of these two sequences which is 7.2067668951373700073793727700802 .

Thus the value of μ, the central tendency of annual minimum of surface air temperature at Guwahati, obtained by *GHM*, is 7.2067668951373700073793727700802 Degree Celsius.

V. CONCLUSION

In the methods developed so far, for determining the value of parameter from ob served data containing the parameter itself and random error, a finite set of observed data may not be sufficient for obtaining the value of the parameter. However, the application of *GHM* can yield the value of the parameter even if the set of observed data is small.
Moreover, the application of *GHM* in determining the value of parameter in this situation involves lesser computational tasks than those involved in the methods developed so far for the same purpose.
It seems that there is scope of developing more formulation(s) of average based on the other combinations of the three Pythagorean means namely arithmetic mean, geometric mean and harmonic mean.

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AUTHOR'S BIOGRAPHY

(Dr. Dhritikesh Chakrabarty in the sea beach at Hualien city, Taiwan, during his visit in National Dong Hwa University there for presenting invited paper in The 3rd International Conference on Fuzzy Systems and Data Mining (FSDM 2017), November 24th – 27th, 2017)

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