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Synthesis of Adaptive Matrix Regulators with an Implicit Reference Model Based on the Polynomial Approach

H.Z. Igamberdiyev, T.V. Botirov, U. F. Mamirov

Academician of the Republic of Uzbekistan, professor of Tashkent State Technical University, doctor of technical sciences, Tashkent, Uzbekistan,
Associate Professor, Department of Automation and Control, Navoi State Mining Institute, Navoi, Uzbekistan
Associate Professor, Department of Information processing and management systems, Tashkent State Technical University, Tashkent, Uzbekistan,

ABSTRACT: The article deals with the formation of new algorithms for the synthesis of adaptive matrix regulators with an implicit reference model based on the polynomial approach. Algorithms for controlling the object under consideration using the procedure of pseudo-rotation of matrices are presented. For pseudo-rotation of matrices, recurrent algorithms based on the fringing method are used. These relations allow us to perform recurrent pseudo-rotation of matrices in the control algorithm and thereby implement control actions in an adaptive control system for a matrix object under conditions of uncertainty.

KEYWORDS: polynomial approach, implicit reference model, matrix controller, synthesis of adaptive systems, pseudo-circulation.

I. INTRODUCTION

One of the main methods of synthesis of linear control system controllers is a method that provides a given location of the poles of the transfer function of a closed system [1-4]. Regulators with a reference model considered in [5-7] have much in common with the method of synthesizing regulators with a given pole arrangement, which allow ensuring equality or proximity of the output signals of the system. In both cases, the controller parameters are calculated based on operations with transfer functions and reduced to solving polynomial Diophantine equations.

II. RELATED WORK

Let's consider the methods of constructing regulators with a reference model in terms of synthesis based on the polynomial calculus of matrix regulators that combine the properties of regulators with a given pole position and a reference model. The solution of this problem is based on the method of synthesis of regulators considered in [8,9].

III. FORMULATION OF THE PROBLEM

Let the control object be described by the following autoregressive moving average model:

$$A(z^{-1})y(t) = z^{-k}B(z^{-1})u(t), \tag{1}$$

where $y(t)$ - output signal of the object, $y(t) \in R$, $u(t)$ - object input signal, $u(t) \in R$, k – discrete delay, $k \in N; z^{-1}$ – backward shift operator $y(t) = y(t-1)$; $A(z^{-1})\,u\,B(z^{-1})$ – polynomial matrices of z^{-1} view's:

$$A(z^{-1}) = I_n + \sum_{i=1}^{\bar{n}} A_i A(z^{-1}), \qquad B(z^{-1}) = I_n + \sum_{i=1}^{\bar{n}} B_i A(z^{-1}),$$

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where I_n - the identity matrix of dimension $[n \times n]$; $A_i \in R, 1, n$; $B_i \in R^{n\times m}$, $i = \overline{0, n}$.

It is assumed that the polynomial matrices $A(z^{-1})$, $B(z^{-1})$ are mutually Prime on the left. To the polynomial matrices $A(z^{-1})$ and $B(z^{-1})$, we compare the mutually simple right-hand polynomial matrices $A*(z^{-1})$ and $B*(z^{-1})$ [4]:

$A(z^{-1})^{-1}B(z^{-1}) = B*(z^{-1})A*(z^{-1})^{-1}$

$$A*(z^{-1}) = I_m + \sum_{i=1}^{m} A_i^*(z^{-1}), A_i^* \in R^{m\times m},\ i = \overline{1, n}; \tag{2}$$

$$B*(z^{-1}) = \sum_{i=0}^{n} B_i^*(z^{-1}), B_i^* \in R^{n\times m},\ i = \overline{1, n}.$$

Let the object (1) be controlled by a linear controller, which is described by the equation:

$$R(z^{-1})u(t) = Q(z^{-1})g(t) - S(z^{-1})y(t) \tag{3}$$

where $g(t)$ – master control, $g(t)\ R,\ R(z^{-1}),\ Q(z^{-1}),\ S(z^{-1})$ polynomial matrices from z^{-1} type :

$R_i \in R^{m\times m},\ i = \overline{1, n_R};\ Q_i \in R^{m\times 1},\ i = \overline{0, r};\ S_i \in R^{m\times n},\ i = \overline{0, n_S};$

The matrix transfer function of the system (1), (3) can be represented as:

$$W(z^{-1}) = z^{-k}B^*(z^{-1})R(z^{-1}) + z^{-k}S(z^{-1})B^*(z^{-1}))^{-1}Q(z^{-1}),$$
$$W(z^{-1}) = z^{-k}R*(z^{-1})(R(z^{-1})A*(z^{-1}) +)g(t) - S(z^{-1})y(t). \tag{4}$$

The parameters of the polynomial matrices $R(z^{-1})S(z^{-1})$ are determined from the condition for assigning the desired poles of the system (1), (3):

$$R(z^{-1})A^*(z^{-1}) + z^{-k}S(z^{-1})B^*(z^{-1})D(z^{-1}),$$

where $D(z^{-1}) = I_m + \sum_{i=1}^{n_D} D_i z^{-1}, D_i \in R^{mxm}, i = \overline{1, n_D}$ - a stable matrix polynomial from z^{-1} that determines the desired distribution of poles on the complex plane.

Since the polynomial matrices $A^*(z^{-1})$ and $B^*(z^{-1})$ are mutually Prime on the right, the solution of the matrix polynomial Diophantine equation (5) always exists [4]. Denote

Since the polynomial matrices 1 and 2 are mutually Prime on the right, the solution of the matrix polynomial Diophantine equation (5) always exists [4]. Denote:

$$C(z^{-1}) = B^*(z^{-1})adj)D(z^{-1}) = \sum_{i=0}^{n_C} C_i z^{-1}, C_i \in R^{nxm}, i = \overline{0, n_C},$$

$$\det(D(z^{-1}) = d(z^{-1}), d(z^{-1}) = 1 + \sum_{i=1}^{n_d} z^{-i}.$$

Matrix transfer function of the system (1), (3), (5) can be expressed as:

$$W(z^{-1}) = z^{-k}\frac{C(z^{-1})Q(z^{-1})}{d(z^{-1})}. \tag{6}$$

We assume that the desired dynamic characteristics of the system under consideration are determined by its implicit reference model, which is described by the equation:

$$y_m(t) = W_m(z^{-1})g(t),$$

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where y(t) – output signal of the reference model, $y_m(t) \in R^n$; $Wm(z^{-1})$ – matrix transfer function of the reference model of the form:

$$W_m(z^{-1}) = z^{-k} \frac{H(z^{-1})}{f(z^{-1})} \tag{7}$$

where $H(z^{-1}) = \sum_{i=0}^{n_H} H_i z^{-1}$, $H_i \in R^{nx1}$, $i = \overline{0, n_H}$; $f(z^{-1}) = 1 + \sum_{i=1}^{n_f} f_i z^{-1}$ – stable polynomial.

The use of an implicit reference model provides a flexible tool for forming a reference transition process that can be selected as optimal in the sense of minimizing the optimality criterion set during design.

Denote

$$s = \max(n_C + r + l, n_d, n_H + 1, n_f)$$

and without limiting generality in order to simplify the further presentation, we will consider the powers of polynomials $C(z^{-1}), d(z^{-1}), H(z^{-1})$ and $f(z^{-1})$ to be equal $s - r - 1, s, s - 1, s,$, respectively, by defining polynomials of lesser degree by terms with zero coefficients. The following task is set: to determine the polynomial matrix $Q(z^{-1})$ of the controller that best approximates the matrix transfer functions of the system (6) and its reference model (7).

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IV. SOLUTION OF THE TASK

We introduce the following notation:

$$E(z^{-1}) = W(z^{-1}) - W_m(z^{-1}) = z^{-k} \sum_{i=0}^{\infty} E_i z^{-1}, E_i \in R^{nx1},$$

$$\overline{Q} = [Q_0^T, Q_1^T, ..., Q_r^T]^T; \overline{Q}_1 = [\overline{Q}_1^T ..., \overline{Q}_r^T]^T;$$

T - the symbol of transposition;

$$\overline{H} = [H_0^T, H_1^T, ..., H_{s-1}^T]^T; \quad \overline{C} = \underbrace{[C_o^T, C_1^T, ..., C_{s-r-1}^T 0]^T}_{ns}; \quad \overline{Q} = [Q_0^T, Q_1^T, ..., Q_r^T]^T;$$

$$L = \begin{pmatrix} C & . & . & 0 \\ C_1 & . & . & C_0 \\ . & . & . & C_1 \\ & . & . & \\ C_{s-r-1} & & . & \\ 0 & . & & C_{s-r-1} \end{pmatrix}; \quad U = \begin{pmatrix} I_n I_n ... I_n \\ I_q \end{pmatrix}, q = ns;$$

$$V_h = \begin{pmatrix} g_0 & & & 0 \\ g_1 & . & & \\ & . & . & g_0 \\ & & . & g_1 \end{pmatrix}$$ - semi-infinite dimension matrix $[\infty \times s]$,

$h(z^{-1}) = 1 + \sum_{i+1}^{s} h_i z^{-i}$ - stable polynomial and $\frac{1}{h(z^{-1})} = \sum_{i=0}^{\infty} g_i z^{-i}$,

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$$\overline{V_h} = \begin{pmatrix} g_0 I_n & & 0 \\ g_1 I_n & \dots & g_0 I_n \\ . & \dots & g_1 I_n \\ . & & . \\ . & & . \end{pmatrix}$$ - block semi-infinite matrix of dimension $[\infty \times sn]$,

$$P_h = \begin{pmatrix} -h_1 & -h_2 & \dots & -h_s \\ 1 & & & . \\ & . & & . \\ & & . & . \\ 0 & & & 10 \end{pmatrix};$$

$$T(u,v) = \{t_{ij}^{uv}\}_{i,j=1} = \left[\begin{pmatrix} 1 & & & 0 \\ v_1 & . & & 0 \\ . & . & . & \\ . & & . & . \\ . & & & . \\ v_{s-1} & \dots & & v_1 1 \end{pmatrix} \begin{pmatrix} 1 & u_1 & \dots & u_{s-1} \\ & & & \\ & & & \\ & & & u_1 \\ & & & 1 \end{pmatrix} - \begin{pmatrix} u_s & & & \\ u_{s-1} & & 0 & \\ . & & & \\ . & & & \\ u_1 & \dots & u_{s-1} & u_s \end{pmatrix} \begin{pmatrix} v_s & v_{s-1} & v_1 & \\ & & & \\ 0 & & & v_{s-1} \\ & & & v_s \end{pmatrix} \right]^{-1},$$

$u(z^{-1}) = 1 + \sum_{i=1}^{s} u_i z^{-1}$ и $v(z^{-1}) = 1 + \sum_{i=1}^{s} s_i z^{-1}$ - stable polynomials; $\overline{T}(u,v)$ - block matrix of dimension $[sn \times sn], \{T(u,v)\}_{ij} = t_{ij}^{uv} I_n; K$ - nonsingular matrix, satisfying the relation $K^T K = \overline{T}(d, d_j; M = (K^T)^{-1} \overline{T}(d,f)$.

As a criterion for the proximity of the matrix transfer functions of the system (6) and its implicit reference model (7), we use the following functional, which is the square of the Frobenius norm of the matrix $E(z^{-1})$:

$$J = tr \frac{1}{2\pi j} \oint_{|z|=1} E(z)^T E(z^{-1}) \frac{dz}{z}. \tag{8}$$

Otherwise, this functionality can be represented as follows:

$$J = tr(\overline{V}_d L \overline{Q} - \overline{V}_f \overline{H})^T (V_d L \overline{Q} - \overline{V}_f \overline{H}) = tr(\overline{Q}^T L^T \overline{V}_d^T - 2\overline{Q}^T L^T \overline{V}_d^T \overline{V}_f \overline{H} + \overline{H}^T V_f^T \overline{V}_f \overline{H}). \tag{9}$$

We define the matrix parameters [10] by denoting

$R_0 = V_d^T V_d$, $Q^0 = V_d^T V_f$, $S^0 = V_f^T V_f$.

Then the matrix X is Toeplitz and $X^{-1} = T(u,v)$ Now functional (9) can be expressed as:

$$J = tr(\overline{Q}^T L^T \overline{T}(d,d) L \overline{Q}^T - 2\overline{Q}^T \overline{T}(d,f) \overline{H} + \overline{H}^T \overline{T}(f,f) \overline{H}). \tag{10}$$

Hence, the matrix $\overline{Q}$ minimizing the functional (8) is defined by the expression:

$$\overline{Q} = (KL)^+ M \overline{H}, \tag{11}$$

where " + " is the pseudo-message sign [11-17].

If $m \geq n$, it is possible to determine the parameters of the polynomial matrix $Q(z^{-1})$ that minimize the functional (8), provided that the output signals of the system and its implicit reference model coincide asymptotically in response to a constant setting influence:

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$$\frac{C(1)Q(1)}{d(1)}=\frac{H(1)}{f(1)}. \qquad (12)$$

Denote

$$F=\frac{d(1)}{f(1)}C(1)^{+}H(1).$$

Then the relation (12) can be represented as:

$$Q_0=-(\sum_{i=1}^{r}Q_i)+F. \qquad (13)$$

Matrix $\overline{Q}$ under condition (12) can be expressed:

$$\overline{Q}=U\overline{Q}_1+\begin{pmatrix} I_n \\ 0 \end{pmatrix}F.$$

Substituting this expression in relation (10) and minimizing the functional by the parameters of matrix $\overline{Q}_1$, we determine the optimal matrix $\overline{Q}_1$

$$\overline{Q}_1=(KLU)^{+}(M\overline{H}-K\overline{C}F). \qquad (14)$$

Substituting this expression in equation (10) and minimizing the functional parameters of the sensor 1 and 1 determine the best matrix If the parameters of the object (1) are constant (or slowly changing), but is unknown in magnitude, the known value of the discrete lag k degree n of polynomial matrices $A^*(z^{-1})$ and $B^*(z^{-1})$ then you can use the following algorithm indirect adaptive control [1,4]:

1. Identification of parameters of polynomial matrices $A^*(z^{-1})$ and $B^*(z^{-1})$ based on equation (1) based on one of the discrete identification methods.
2. Calculating parameters of polynomial matrices $A^*(z^{-1})$ and $B^*(z^{-1})$ based on the relation (2).
3. Calculating parameters of polynomial matrices $R^*(z^{-1})$ and $S^*(z^{-1})$ based on the relation (5)
4. Calculation of parameters of the polynomial matrix $Q(z^{-1})$ by formula (11) (formulas (14), (13))
5. Calculation of the control effect using the formula

$$u(t)=(I_m-R(z^{-1})u(t)+Q(z^{-1})g(t)-S(z^{-1})y(t).$$

Next, install $t=t+1$ and return to step 1.

It should be noted that the matrices K, $M\overline{H}$ do not depend on the parameters of the object (1) and can be determined in advance at the design stage of the control system

To form optimal matrices (11) and (14), it is advisable to use recurrent formulas for pseudo-rotation of matrices $\Lambda=KL$ and $\Omega=KLU$. Based on the theory and methods of pseudo-circulation of matrices [11-16], we present a pseudo-circulation algorithm for matrix Λ , which can also be used for calculating the pseudo-inverse matrix Ω^{+}.

Известно [11,14], что для каждой действительной 1-матрицы 2 существует единственная действительная псевдообратная матрица 3, удовлетворяющая следующим свойствам It is known [11,14] that for every real $(n+1)s\times ms$ -matrix Λ_m , there exists a unique real pseudo-inverse matrix U that satisfies the following properties:

$$U\Lambda U=U,\ (\Lambda U)^T=\Lambda U,\ \Lambda U\Lambda=\Lambda,\ (U\Lambda)^T=U\Lambda. \qquad (15)$$

In practice, it is not uncommon for additional information in the form of ($ms\times 1$) - matrix λ_{m+1} to be added to the existing ($(n+1)s\times ms$)- matrix Λ_m as its last row, so that the newly formed matrix Λ_{m+1} can be represented in the form:

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$$\Lambda_{m+1} = \left| \begin{array}{c} \Lambda_m \\ \hdashline \lambda_{m+1}^T \end{array} \right|. \tag{16}$$

It is known [15,16] that the pseudo-inverse matrix U_{m+1} for matrix Λ_{m+1} in the form (16) can be represented as:

$$U_{m+1} = \left\| V_m^T \vdots v_{m+1} \right\|, \tag{17}$$

where $V_m^T = U_m - v_{m+1}\lambda_{m+1}^T U_m$.

From the first relation (15), using the partition U_{m+1} in the form (17), we can write

$$U_{m+1} = U_{m+1}\Lambda_{m+1}U_{m+1} = [V_m^T \Lambda_m + v_{m+1}\lambda_{m+1}]U_{m+1}. \tag{18}$$

From here $I = V_m^T \Lambda_m + v_{m+1}\lambda_{m+1}^T$.

Based on (18), we find $U_m\Lambda_m = V_m^T \Lambda_m + v_{m+1}\lambda_{m+1}^T U_m \Lambda_m$, from which we can come to the expression

$$V_m^T = U_m - v_{m+1}\lambda_{m+1}^T U_m, \tag{19}$$

Substituting (19) in (17), we find

$$U_{m+1} = U_m - v_{m+1}\lambda_{m+1}^T U_m \vdots v_{m+1}, \tag{20}$$

for $m=1$ we have $U_1 = (\lambda_1^T \lambda_1)^{-1}\lambda_1$.

When using the algorithm (20), there can be two cases. It is possible that the row: λ_{m+1}^T does not increase the rank of the matrix Λ_m . Then v_{m+1} can be defined from the expression [13,16]:

$$v_{m+1}^T = (\lambda_{m+1}^T U_m U_m^T \lambda_{m+1} + 1)^{-1} \lambda_{m+1}^T U_m U_m^T. \tag{21}$$

Entering the designation

$$S_m = U_m U_m^T = (\Lambda_m^T \Lambda_m)^{-1} \tag{22}$$

and substituting (22) in (21) and transposing, we find

$$v_{m+1} = S_m \lambda_{m+1} (\lambda_{m+1}^T S_m \lambda_{m+1} + 1)^{-1}.$$

In this case, matrix S_m is defined by the expression

$$S_{m+1} = S_m - v_{m+1}\lambda_{m+1}^T S_m,$$

If row λ_{m+1}^T increases the rank of matrix Λ_m , column v_{m+1} can be determined based on the expression [14,15-18]:

$$v_{m+1} = (I - U_m^T \Lambda_m^T)\lambda_{m+1}[\lambda_{m+1}^T (I - U_m^T \Lambda_m^T)\lambda_{m+1}]^{-1}.$$

By entering the designation $T_m = I - U_m^T \Lambda_m^T$ find $v_{m+1} = T_m \lambda_{m+1} (\lambda_{m+1}^T T_m \lambda_{m+1})^{-1}$.

In this case, matrix T_m can be defined as follows

$$T_{m+1} = T_m - v_{m+1}\lambda_{m+1}^T T_m.$$

by $m=1$ have $T_1 = I - U_1\Lambda_1$.

In expressions (16)-(22) U_{m+1} – pseudo-return matrix for the matrix composed of the first $m+1$ lines of source; λ_{m+1}^T is $(m+1)$ row of the original matrix; v_{m+1} is the $(m+1)$-th column of the matrix pseudo-return; T_{m+1},

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S_{m+1} – symmetric matrix when a vector-column with v_{m+1} vector-row λ_{m+1}^T; I – identity matrix; T – the sign of transposition; $\vdots$ – the sign of separation m columns of the matrix U_{m+1} of the ($m+1$)-th column

V. CONCLUSION

The considered pseudo-circulation algorithm is direct and uses the advantages inherent in the fringing method [13,17,18]. When using this algorithm, it is possible to control the correctness of calculations after each step , taking into account the symmetry of matrices T_m and S_m , as well as other relations, and determine the rank of the original matrix by the number of non-zero values of vector v_m .

The given relations allow us to perform recurrent pseudo-circulation of matrices and in expressions (11) and (14), and thereby implement control actions in an adaptive control system for a matrix object under conditions of uncertainty.

In this way, adaptive matrix controllers with a given pole position and an implicit reference model are synthesized. An implicit reference model is defined as a reference matrix transfer function that determines the desired behavior of the signal at the output of the control system. The synthesis of regulators is carried out using well-known methods of polynomial calculus, which require a relatively small amount of calculations, and includes polynomial factorization, the solution of a matrix polynomial equation, and a system of linear algebraic equations

REFERENCES

[1] Isermann R. Digital control system. M.: Mir, 1984.
[2] Prager D. L., Wellstead P. E. Multivariate pole-addignment self-tuning regulators // IEEE Proceedings. 1980. V. 128. Part D. No. 1. P. 9-18.
[3] Mikles J. A multivariable self-tuning controller based on pole-placement desing // Automatica. 1990. V. 26. No. 2. P. 293-302.
[4] Kraus F. J., Kučera V. Linear quadratic and pole placement iterative design / European Control Conference (ECC), Karlsruhe, 1999, pp. 653-658, doi: 10.23919/ECC.1999.7099379.
[5] Astrom K.J. Robustness of design method based on assignment of poles and zeros // IEEE Trans. Automat. Contr. 19480. V. AC-25. P. 582-584.
[6] Tsypkin Ya. z. non-Minimal-phase in discrete adaptive control systems // Results of science and technology. Ser. Technical Cybernetics. M.: VINITI, 1989. T. 26. - P. 3-40.
[7] Tsypkin Ya. z. Optimal discrete control systems for non-minimal phase objects / / AIT. 1991. No. 11. - P. 96-118.
[8] Yadykin I. B. Optimal tuning of linear regulators// Docl. ANR 1985, Vol. 285, No. 3, Pp. 574-576.
[9] Yadykin I. B. Principles of construction, architecture and software of automated systems for setting up industrial regulators // Computer engineering and control systems. Moscow; Sofia, 1989. Issue 1. - Pp. 23-36.
[10] Porat V., Friedlander V. The output error method for reduced order controller design / / IEEE Trans. Automat. Contr. 1984. V. AC-7. P. 629-631.
[11] Voevodin B. B., Kuznetsov Yul. Matrixes and calculations. M.: Nauka, 1984.
[12] Gantmacher F. R. Theory of matrices. - M.: Nauka, 1988. - 552 p.
[13] Golub J., Van Loach CH. Matrix calculations: Per. s Engl. - M.: Mir, 1999. -548 p.
[14] Horn R., Johnson CH. Matrix analysis: Per. s Engl. - M.: Mir., 1989 – - 655s.
[15]Demmel J. Computational linear algebra. Theory and applications: TRANS. from English-M.: Mir, 2001 -430 p.
[16] Zhdanov A. I. Introduction to methods for solving incorrect problems: - Ed. Samara state. aerospace University, 2006. -87 p.
[17] Mamirov U. F. Algorithms for stable control of a matrix object under parametric uncertainty / / journal " Chemical technology. Control and management". - Tashkent, 2018. – № 1-2 (79). –Pp. 164-168.
[18] Lawson CH., Henson R. Numerical solution of problems of the least squares method / TRANS. from English-M.: Nauka. GL. ed. Phys. - math. lit., 1986. -232 p.